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# Short and Length-Independent Group Delay Supported by Topological Edge States in Finite-Size Su–Schrieffer–Heeger Chains

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**Received:** 16 March 2026 | **Revised:** 20 September 2026 | **Accepted:** 28 September 2026

**Keywords:** edge states | group delay | microwave resonator | Su–Schrieffer–Heeger model | topology

## ABSTRACT

In order to transport information with topological protection, we explore experimentally the short group delay using edge states in one-dimensional (1D) Su–Schrieffer–Heeger (SSH) chains. The group delay is investigated in both one- and two-dimensional (2D) models with topological non-trivial band structures. The fast transport is inherited with the wavefunction localization, giving a stronger effective coupling strength between the mode and the measurement leads. Also the group delay in one-dimension is independent of the system size. To verify the assertion, we implement a chain of split-ring resonators (SRRs) and their complementary ones with controllable hopping strengths. By performing measurements on the group delay of non-trivially topological edge states with pulse excitations, the group delay between two edge states is directly observed with chain length up to 20. Along the route to harness topology to protect optical information, our experimental demonstrations provide a crucial guideline for utilizing photonic topological devices.

## 1 | Introduction

Composed of dimers with staggered hopping amplitudes, a topological phase transition can be revealed in the Su–Schrieffer–Heeger (SSH) model owing to the topological invariant characterized Zak phase defined as the integral of the one-dimensional (1D) Berry connection over the 1D Brillouin zone. [1, 2]. By utilizing the SSH model, people have illustrated the difference between bulk and boundary, chiral symmetry, adiabatic equivalence, topological invariants, and bulk–boundary correspondence [3]. Through

the analogy in single-particle Hamiltonian, topologically non-trivial zero or  $\pi$  modes can also be observed in photonic systems, through a periodically setting of the confining potentials [4–8]. With the topologically protected edge states, we can implement new types of lasing modes [9, 10] and optical control [11, 12] even under continuous deformations. As the edge modes in a 1D SSH model are zero-dimensional, by definition, they do not have a group velocity. However, for the practical implementation, instead of infinite chains, only a finite number of dimers can be fabricated. Moreover, to harness topology to protect optical

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information [13], a natural question arises on the corresponding transport properties in the supported non-trivially topological states. It is the common belief that the wavefunctions of supported edge states in a finite-size chain should remain staying localized strongly at their respective boundaries and decay exponentially in the bulk, with a penetration depth  $\xi$  depending on the contrast between coupling strengths.

Prior studies have extensively explored signal transmission and communication between localized excitations across photonic and electromagnetic (EM) platforms. For example, topological whispering-gallery modes have been employed to protect unidirectional wave propagation in circular microcavities [14–17], facilitating edge-to-edge spatial coupling [18, 19]. These efforts have primarily focused on subwavelength non-radiative coupling, and disorder-robust energy transfer. Alternatively, efficient edge-to-edge state transfer has been demonstrated by dynamically modulating internal intersite couplings, leveraging topological pumping protocols in 1D coupled-waveguide arrays [20–23]. Although capable of transfer over extended spatial ranges, these approaches rely on active, time-dependent modulations or adiabatic parameter sweeps.

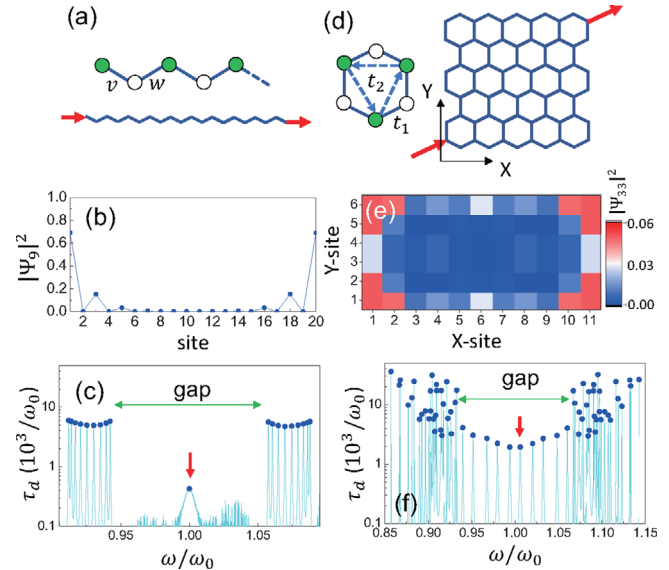
On the contrary, in this paper, we reveal that one can transport information between two supported edge states with non-trivially topological protection with a longer distance, and achieving a much shorter group delay. In an open, port-coupled topological network, the scattering group delay associated with the topological edge state does not scale with chain size, which is a major property of the group delay by the bulk modes. Because the edge state is spatially localized at the boundaries, its strong spatial overlap with external leads provides enhanced port coupling, enabling an order-of-magnitude reduction in latency relative to bulk modes while maintaining robust protection against internal structural disorder.

Experimentally, the existence of non-trivially optical topological edge states in a chain of dimers with split-ring resonator (SRR) and its counterpart, the complementary split-ring resonator (CSRR), is illustrated with a proper setting on the intra- and inter-cell coupling strengths between SRRs and CSRRs [24]. By extracting the amplitude and phase from the transmission spectroscopy, a photonic band gap in the passband is measured when the inversion symmetry is broken [25, 26]. In addition to verifying edge-state transmission in the regime where the system size significantly exceeds the penetration depth, the corresponding group delay was extracted from continuous-wave (CW) scattering data and confirmed via time-domain pulse measurements. Experimentally, we demonstrate that topological edge modes exhibit significantly shorter group and pulse delays compared to bulk modes.

## 2 | Theoretical Results

The  $N$ -site SSH model can be described by the Hamiltonian

$$H_{SSH} = \sum_{j=1}^N \omega_0 a_j^\dagger a_j + \sum_{j=1}^{N-1} g_j a_{j+1}^\dagger a_j + h.c. \quad (1)$$



**FIGURE 1** | (a) The SSH model with chain length  $N = 20$  and  $v/\omega_0, w/\omega_0 = -0.01597, 0.07256$ .  $|w/v| = 4.54$ . (b) The wavefunction of the edge mode  $\Psi_9$  supported in this finite-sized SSH model. (c) Calculated delay time  $\tau_d$  as a function of normalized frequency ( $\omega/\omega_0$ ) with intrinsic loss  $\kappa_i/\omega_0 = 10^{-4}$  and the coupling loss  $\kappa_{c,0}/\omega_0 = 5 \times 10^{-3}$ . The edge mode  $\Psi_9$  is indicated by the red arrow. (d) The Haldane model with  $N = 11 \times 6$  and coupling strengths  $t_1/\omega_0, t_2/\omega_0 = 0.05, 0.02j$ . (e) The wavefunction of an edge mode  $\Psi_{33}$  supported in a finite-size Haldane model. (f) Calculated  $\tau_d$  as a function of normalized frequency with  $\kappa_i/\omega_0 = 10^{-5}$  and  $\kappa_{c,0}/\omega_0 = 5 \times 10^{-3}$ . The edge mode  $\Psi_{33}$  is indicated by the red arrow.

where  $a_j$  is the annihilation operator for site  $j$ , and  $g_j = v$  and  $w$  for odd and even  $j$ , respectively (Figure 1a). This staggered coupling in the “diatomic basis” opens a band gap in the infinite chain case. The SSH model is known for its topological features and undergoes a transition from a trivial phase to a topological phase as it is tuned across  $w/v = 1$ . In the topological phase ( $w/v > 1$ ), the model supports protected edge modes whose energy lies at the center of the band gap. Outside this band center, one recovers the typical Bloch solutions expected for periodic systems. For a finite-size system a non-zero value of the energy difference  $\Delta_e$  occurs for “near-zero” modes even when  $w/v > 1$ . Given  $w/v > 1$ ,  $\Delta_e$  can be approximated by  $\Delta_e = 2w(\frac{w}{v})^{-N/2} [1 - (\frac{w}{v})^{-2}] / [1 - (\frac{w}{v})^{-N}]$ , in which  $N$  is the chain length (number of lattice sites) [27]. Associated with a finite  $\Delta_e$ , the corresponding modes are no longer isolated at one end, but become a symmetric or anti-symmetric combination of the isolated edge modes and characterized by the penetration depth  $\xi = 2/\ln(w/v)$  as presented in Figure 1b [27]. In this context,  $\Delta_e/2$  can be viewed as a coupling energy of the localized modes on each edge, scaled exponentially by  $N$ .

To analyze the transport properties of the 1D chain, we model a setup where measurement leads (or feedlines) are coupled to the ends of the chain. This configuration, combined with the gapped band structure of the SSH model, creates a system that resembles a typical molecular junction, with the chain itself acting as a long junction barrier. We can describe the photon transport to and from the leads by adding corresponding hopping terms to the

Hamiltonian:

$$H = H_{SSH} + g_l a_1 + g_r a_N + h.c., \quad (2)$$

where  $g_l$  is the hopping strength between the first resonator and the left lead, and  $g_r$  is the hopping strength between the last resonator and the right lead. For the transmission-type lead configuration shown in Figure 1a, the transmission complex amplitude  $T(\omega)$  is related to the transmission susceptibility  $\chi_T(\omega)$  by  $T(\omega) = -i\sqrt{g_l g_r} \chi_T(\omega)$ . The susceptibility is defined as [28, 29]:

$$\chi_T(\omega) = i \int_0^\infty dt e^{i\omega t} \left\langle \left[ a_N(t), \left( a_1 + a_1^\dagger \right) \right] \right\rangle \quad (3)$$

Here,  $(a_1 + a_1^\dagger)$  represents the generalized force applied to the system at the first site, and  $a_N(t)$  is the system's response observed at the last site. Similarly, the reflection susceptibility,  $\chi_R(\omega)$ , can be calculated by replacing the response operator  $a_N(t)$  with  $a_1(t)$  in Equation (3).

The group delay time for the photon transport via a resonance mode can be extracted from the phase data of  $T(\omega)$  as detailed in Ref. [30]. By denoting  $\kappa_c$  and  $\kappa_i$  as the coupling decay rate and the intrinsic decay rate, respectively, and  $\kappa = \kappa_c + \kappa_i$  as the total decay rate, the maximum (group) delay time associated with transmission is  $\tau_{d,m} = 2/(\kappa_i + \kappa_c) = 2/\kappa$  at the resonance frequency  $\omega = \omega_r$ . Conversely, the reflected signal exhibits a negative delay, which indicates a pulse “advance” with  $\tau_{a,m} = -2\kappa_c/(\kappa_i + \kappa_c)$ . Intriguingly, the difference between the maximum delay and the maximum advance depends only on the system's intrinsic loss rate  $\tau_{d,m} - \tau_{a,m} = 2/\kappa_i$ . This result shows that achieving larger delay or advance times is contingent on using low-loss resonators.

In a system of coupled resonators, the projection of an eigenmode  $|\Psi_s\rangle$  onto the contact sites plays a critical role in governing transmission amplitude  $T$  and reflection amplitude  $R$ , by modifying the effective coupling decay rates for collective modes. For  $R$ , the effective coupling rate is  $\kappa_c = \kappa_{c0} |\psi_1|^2$ , in which  $\psi_1 = \langle 1 | \Psi_s \rangle$  is the wavefunctions at lattice site 1, and  $\kappa_{c0}$  is a geometry-dependent coupling strength. For  $T$ , the total coupling decay rate involves both the input and output ports as  $\kappa_c = \kappa_{in} + \kappa_{out} = \kappa_{c0}(|\psi_1|^2 + |\psi_N|^2)$ . Leveraging the parity symmetry of the modes ( $\psi_N = e^{i\phi_s} \psi_1$ ), this can be approximated as  $\kappa_c \sim 2\kappa_{c0} |\psi_1|^2$ .

The effective  $\kappa_c$  modifies the delay times for bulk modes to be  $\tau_{d,m} = 2/(\kappa_i + \kappa_c) = 2/(\kappa_i + 2\kappa_{c0} |\psi_1|^2)$ . Close to the edge modes, there is the small splitting  $\Delta_e$  comparable to  $\kappa$  between them, and the maximum delay time is  $\tau_{d,m} = 2\gamma/(\kappa_i + \kappa_c) \sim 2\gamma/(\kappa_i + \kappa_{c0})$ , in which  $\gamma$  is a factor decreases from 2 to 1, as  $\Delta_e$  increases over  $\kappa$ . This doubling of the delay time is attributed to the superposition of the  $2\pi$  phases contributed by each of the two hybridized edge modes. On the other hand, the maximum advance time for bulk modes is  $\tau_{a,m} = -2\kappa_{c0} |\psi_1|^2 / \kappa_i (\kappa_i + \kappa_{c0} |\psi_1|^2)$ . For edge modes  $\tau_{a,m} \sim -2/\kappa_i$ , by assuming  $\kappa_{c0} > \kappa_i$ , and the edge modes have significantly larger wavefunctions at the edge, with  $|\psi_1|^2 \sim 1$ .

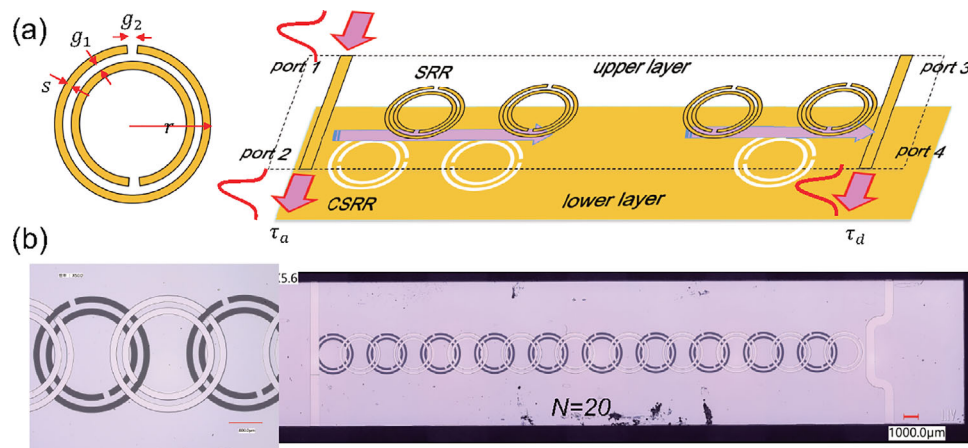
In short, the key difference in delay times between edge and bulk modes stems from the wavefunction's amplitude at the contact site,  $|\psi_1|$ , and is easy to understand in the ideal case  $\kappa_i = 0$ . For

bulk modes, we assume these behave like Bloch waves, with population equally distributed along the chain,  $|\psi_1|^2 \sim 1/N$ . The maximum delay time becomes  $\tau_{d,m} = 1/(\kappa_{c0} |\psi_1|^2) \sim N/\kappa_{c0}$ , scaling linearly with  $N$ . For edge modes, the wavefunction is localized at the boundary, giving  $|\psi_1|^2 \sim 1$ . This results in a maximum delay time of  $\tau_{d,m} \sim 2\gamma/\kappa_{c0}$ , which is  $N$ -independent. Although the long delay time for bulk modes is a direct consequence of their small effective coupling rate, a classical interpretation is that the bulk mode transport is akin to a sequential, site-by-site hopping process, while edge mode transport resembles a single-step tunneling event. Meanwhile, the amplitude of the transmission,  $T$ , provides an estimate of the transfer efficiency for either process.

Topological protection against bulk disorder extends directly to the edge-mode group delay, which can be formally understood via first-order perturbation theory. We consider two standard types of defects localized at site  $n_d$ : an on-site defect  $\Delta H = \Delta\omega |n_d\rangle\langle n_d|$  and a coupling defect  $\Delta H = \Delta g |n_d + 1\rangle\langle n_d| + h.c.$ . The first-order shift in eigenfrequency  $\omega_s$  for an eigenstate  $|\Psi_s\rangle$  is given by  $\Delta\omega_s = \Delta\omega |\langle n_d | \Psi_s \rangle|^2$  for on-site defects and  $\Delta\omega_s = 2\text{Re} [\Delta g \langle n_d + 1 | \Psi_s \rangle^* \langle n_d | \Psi_s \rangle]$  for coupling defects. Because the edge-mode wavefunction  $|\Psi_s\rangle$  is exponentially localized near the boundary, the spatial overlap  $|\langle n_d | \Psi_s \rangle|$  vanishes rapidly as  $\propto e^{-n_d/\xi}$  for any bulk site  $n_d$  beyond the localization length  $\xi$ . Consequently, bulk disorder produces virtually zero perturbation to the edge-mode resonance frequency.

Furthermore, under strong external port coupling ( $\kappa_{c0} \gg \kappa_i$ ), variations in the group delay are governed by perturbations to the boundary wavefunctions,  $\langle 1 | \Delta \Psi_s \rangle$  and  $\langle N | \Delta \Psi_s \rangle$ . From first-order non-degenerate perturbation theory, the eigenstate correction is  $|\Delta \Psi_s\rangle = \sum_{j \neq s} \langle \Psi_j | \Delta H | \Psi_s \rangle / (\omega_s - \omega_j) |\Psi_j\rangle$ . When  $|\Psi_s\rangle$  is a localized edge state and  $|\Psi_j\rangle$  is a bulk state, the matrix element  $\langle \Psi_j | \Delta H | \Psi_s \rangle = \Delta\omega \langle n_d | \Psi_j \rangle^* \langle n_d | \Psi_s \rangle$  (or  $\Delta g [\langle n_d + 1 | \Psi_j \rangle^* \langle n_d | \Psi_s \rangle + h.c.]$ ) is exponentially suppressed whenever the defect  $n_d$  is located deep in the bulk. Consequently, bulk disorder exerts negligible influence on the boundary overlaps and the resulting edge-mode group delay.

A critical scenario occurs when a defect is positioned near a boundary ( $n_d \leq \xi \approx 1$ ), where mixing within the quasi-degenerate subspace of the two edge states must be accounted for. In a finite system, the symmetric and antisymmetric linear combinations of the left and right edge modes ( $|\Psi_{\text{edge},L}\rangle$  and  $|\Psi_{\text{edge},R}\rangle$ ) have unperturbed eigenfrequencies of  $-\Delta_e/2$  and  $+\Delta_e/2$ . An on-site defect near the left boundary introduces the mixing of eigenstates with strength  $\eta \equiv \Delta\omega |\langle n_d | \Psi_{\text{edge},L} \rangle|^2$ , yielding perturbed eigenfrequencies  $\omega = (\eta \pm \sqrt{\Delta_e^2 + \eta^2})/2$ , defining an enlarged splitting of  $\Delta'_e = \sqrt{\Delta_e^2 + \eta^2}$ . For an intersite coupling defect, the mixing strength is given by  $\eta' = 2\text{Re} [\Delta g \langle n_d + 1 | \Psi_{\text{edge},L} \rangle^* \langle n_d | \Psi_{\text{edge},L} \rangle]$ , with the corresponding splitting obtained by substituting  $\eta \rightarrow \eta'$ . This effective splitting  $\Delta'_e$  plays a critical role in dictating the peak group delay time. Under strong disorder ( $\eta \gg \Delta_e$ ), the hybridized states decouple back into localized left and right edge modes. When the effective splitting exceeds the resonance linewidth ( $\Delta'_e > \kappa$ ), spectral peak splitting occurs, which reduces the maximum delay time by approximately half. Consequently, unless a defect is located directly at or adjacent to the boundary



**FIGURE 2** | (a) SSH model realized in a finite-size chain by combining SRRs and CSRRs. The two leads at the front and end nodes are implemented by two transmission lines, denoted with the input port 1, reflection port 2, and transmission ports 3 and 4, respectively. (b) The micrographs for the sapphire-based sample, in which the lighter regions represent aluminum (Al) metallization. Scale bars represent 800 and 1000  $\mu\text{m}$  for the left and right panels, respectively.

such that  $\eta$  (or  $\eta'$ )  $> \kappa > \Delta_e$ , the edge-mode group delay remains virtually unchanged, exhibiting robust topological immunity compared to the high sensitivity of bulk modes.

Following Equation (3), the group delay  $\tau_d = d\phi/d\omega$  is calculated and plotted in Figure 1c, with blue symbols indicating  $\tau_{d,m}$  at each resonance. The distribution of these resonances allows for the clear identification of the band gap and the topological edge mode, which is indicated by a red arrow. The central finding asserts that the edge mode transport has a much smaller group delay time than the bulk mode transport. As detailed in the [Supporting Information](#), the protection against disorder are verified by exact numerical calculations with disorder strengths of  $\Delta\omega = \Delta g = 0.01$ . The relative shift in delay time ( $\Delta\tau_{d,m}/\tau_{d,m}$ ) for edge transmission is  $\sim 0.2\%$  for on-site defects and  $\sim 2\%$  for coupling defects when placed deep in the bulk. In contrast, for a boundary defect at  $n_d = 1$  (or  $N$ ), the defect-induced splitting significantly reduces the normalized peak delay  $\tau_{d,m}\omega_0$  from  $\sim 400$  to  $\sim 250$ .

To confirm that this reduction in group delay is not restricted to the 1D case, we performed a similar calculation for a two-dimensional (2D) Haldane model, which have been proposed to produce topological effects in the optics [31]. With hopping strengths  $t_1/\omega_0, t_2/\omega_0 = 0.05, 0.02j$ , the finite system consists of  $5 \times 5$  honeycombs and features a total of 66 modes (Figure 1d). Figure 1e illustrates the wavefunction distribution ( $|\Psi_s|^2$ ) for one of these edge modes  $s = 33$ . The group delay for this model is shown in Figure 1f. The distribution of resonances reveals the band gap and the 10 corresponding edge modes, in which the  $s = 33$  mode is marked by a red arrow. The result confirms again that the group delay for the topological edge modes is significantly smaller than those of the bulk modes.

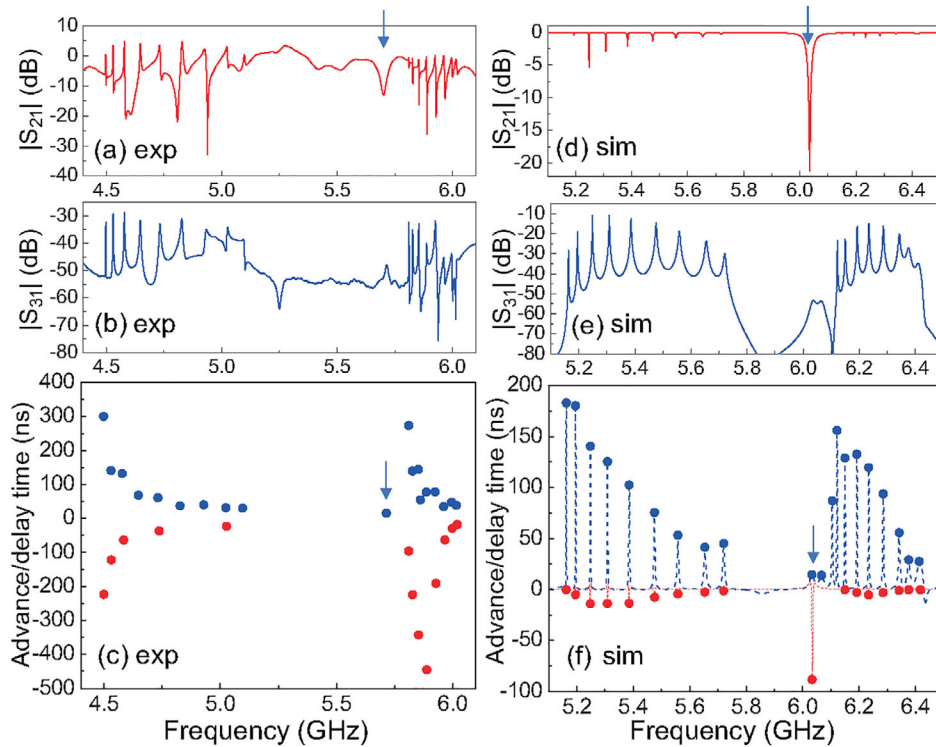
### 3 | Experimental Methods

To demonstrate the transport with short group delay supported by the topological edge states, we build 1D chains combining SRRs and CSRRs, fabricated on the front and back layers of

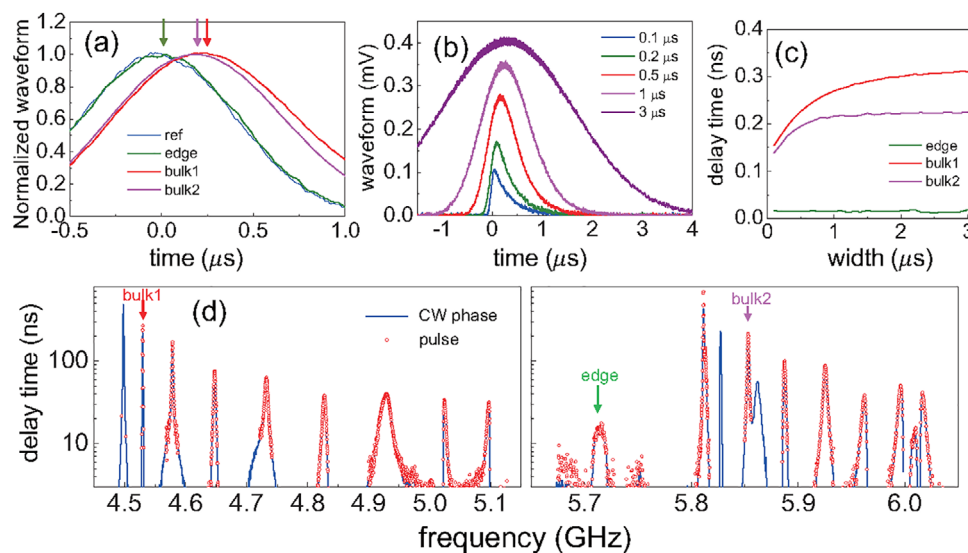
a substrate, respectively, as illustrated in Figure 2. Following the dimer structure, we manipulate the intra- and inter-cell coupling strengths by varying the orientation of SRRs and CSRRs. This control is possible because the coupling strength between SRRs is highly dependent on angle rotations [32–34], and the coupling between an SRR and a CSRR is greatly impacted by their orientation texture [35, 36]. We fabricated two different types of samples: The superconducting Al resonators are fabricated on the substrate sapphire at a operation frequency of about 6 GHz, and the normal conducting Cu resonators are fabricated on the Roger 4003 C printed circuit board (PCB) at a operation frequency of about 3 GHz. The Al resonators are designed with 1.406 mm in diameter( $r$ ), 0.148 mm in line width( $s$ ), gap between two rings( $g_1$ ), and split for the ring gaps( $g_2$ ), and 3.7 mm for the lattice constant. The Cu resonators are designed with  $r = 7.6$  mm,  $s = g_1 = g_2 = 0.4$  mm, and 10 mm for the lattice constant. To investigate the non-trivial topological case, we set the orientation angles of the SRR and CSRR to  $(30^\circ, 110^\circ)$ . For these selected angles, the resulting coupling ratio  $w/v$  is 4.67 for the Al resonators and 4.54 for the Cu resonators. To perform direct measurements of the transmission and reflection spectra, we introduce two leads at the first and last nodes of the chain. With the help of the transmission lines, we define the input (port 1), reflection (port 2), and transmission (ports 3 and 4) ports. Full-wave electromagnetic simulations were performed using the ANSYS HFSS [37] driven modal solver.

### 4 | Experimental Results

Figure 3a–c presents the experimental results for the  $N = 20$  superconducting resonator chain. The plots show the reflection amplitude( $|S_{21}|$ ), transmission amplitude( $|S_{31}|$ ), and the advance/delay time at the eigenmodes, respectively. The measured band structure reveals a lower band ranging from 4.5 to 5.1 GHz, a higher band from 5.6 to 6.1 GHz, and an edge mode in the band gap at 5.4 GHz. The group delay time for the edge mode is much smaller than those associated with the bulk modes. In this sample, the advance time is also long, with a magnitude comparable to the delay time. Furthermore, consistent with



**FIGURE 3** | (a) The measured reflection amplitude  $|S_{21}|$  for a chain of 20 superconducting resonators. The edge mode is indicated by the blue arrow. (b) The measured transmission amplitude  $|S_{31}|$ . (c) The maximum advanced time (red) and delay times (blue) as a function of frequency. The edge mode is indicated by the blue arrow. (d) The simulated reflection amplitude  $|S_{21}|$ . The edge mode is indicated by the blue arrow. (e) The simulated transmission amplitude  $|S_{31}|$ . (f) The maximum advanced time (red) and delay times (blue) as a function of frequency. The edge mode is indicated by the blue arrow.



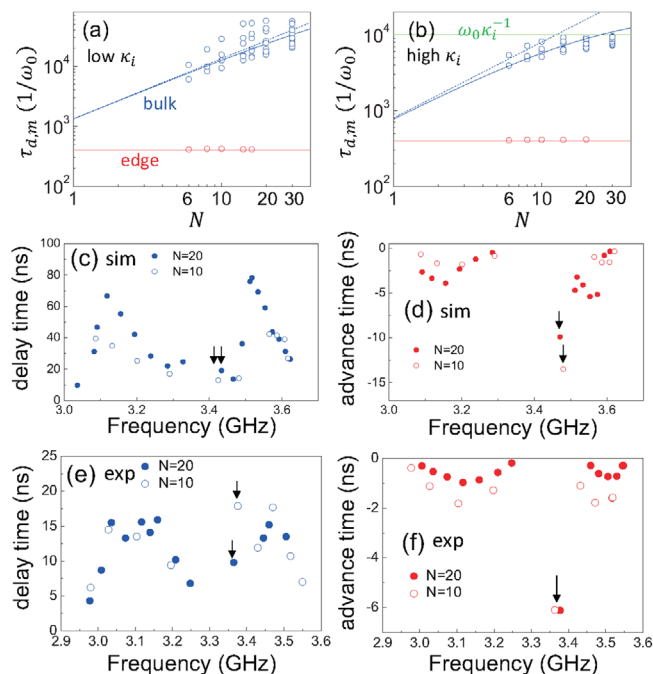
**FIGURE 4** | (a) Normalized transmitted waveforms of 1- $\mu$ s pulses for the reference at 5.1296 GHz (blue), the edge mode at 5.715 GHz (green), bulk mode 1 at 4.5298 GHz (red), and bulk mode 2 at 5.8533 GHz (magenta). Vertical arrows indicate the mode-dependent pulse peak delays. (b) Transmitted waveforms (in mV) through bulk mode 1 with  $t_p$  from 0.1 to 3  $\mu$ s. (c) Group delay as a function of pulse width for the edge mode (green), bulk mode 1 (red), and bulk mode 2 (magenta). (d) Group delay as a function of frequency derived from CW phase data (blue curve) and time-domain pulse measurements (red symbols). Vertical arrows highlight the resonance frequencies of the three examined modes. Excellent agreement between the two methods confirms sufficient mode bandwidth for distortionless 1- $\mu$ s pulse transmission.

simulated results, both advance and delay times trend shorter as the frequency increases.

To verify that the group delay extracted from CW phase spectroscopy accurately represents physical signal latency, we performed time-domain transmission measurements using Gaussian pulses (width  $t_p = 1 \mu\text{s}$ ) with carrier frequencies tuned to individual resonant modes. As shown in Figure 4a, the normalized  $S_{31}$  transmitted signals exhibit distinct pulse delay times for the edge mode (5.715 GHz), bulk mode 1 (4.5298 GHz), and bulk mode 2 (5.8533 GHz), relative to an off-resonance reference pulse at 5.1296 GHz. Edge-mode transmission is substantially faster than bulk-mode transport, yielding measured pulse delays of 15 ns for the edge mode compared to 270 ns and 217 ns for bulk modes 1 and 2, respectively. Figure 4b illustrates Gaussian pulse transmission through bulk mode 1 for pulse durations  $t_p$  ranging from 0.1 to 3  $\mu\text{s}$ . For pulse widths below 0.5  $\mu\text{s}$ , the narrow resonance bandwidth of the bulk mode induces envelope distortion and artificially reduces the measured delay. Figure 4c summarizes the pulse delay as a function of  $t_p$  across all three modes. While bulk-mode delays decrease rapidly for  $t_p < 1 \mu\text{s}$ , the edge-mode pulse remains undistorted down to  $t_p = 0.1 \mu\text{s}$ , maintaining a constant delay of 15 ns. Such pulse distortion occurs when  $t_p < 1/B$ , where the operational bandwidth  $B$  is determined by the resonance linewidth  $\kappa$ . The measured linewidths for the edge mode, bulk mode 1, and bulk mode 2 are  $\kappa = 30, 9.4$ , and  $8.2 \text{ MHz}$ , respectively. Finally, Figure 4d provides a comprehensive comparison between the CW phase-derived group delays and the measured pulse delays ( $t_p = 1 \mu\text{s}$ ) across the lower passband (left panel), the edge mode, and the upper passband (right panel). The excellent quantitative agreement between the two methods confirms that the resonant modes provide sufficient bandwidth to accommodate 1- $\mu\text{s}$  pulses without significant distortion.

The delay time as a function of chain length  $N$  for edge modes and bulk modes can be predicted by transmission amplitude calculations as presented in Figure 5a,b, respectively using  $\kappa_i/\omega_0 = 10^{-5}$  and  $10^{-4}$ , and both  $\kappa_{c0}/\omega_0 = 5 \times 10^{-3}$ . The red line shows the expected delay time for edge mode,  $\tau_{d,m} = 2/\kappa_{c0}$ . The blue dashed curves indicates the linear dependence of  $\tau_{d,m}$  and  $N$ , while the blue solid line includes the correction with a finite  $\kappa_i$ . The green line is the expected upper bound of delay time when the intrinsic loss is large. In Figure 5c,d, we present the delay times obtained by finite-element simulations with the normal conductor structures with  $N = 10$  and  $20$ . In simulations, the delay time of bulk mode group transport increases with chain length, while the edge mode group delay remain similar for  $N = 10$  and  $N = 20$ , as expected by theory for low-loss systems.

Experimentally, the length dependence of the group delays is investigated using the normal conducting samples, which have a larger intrinsic loss. In a high-loss regime where  $\kappa_i \gg \kappa_{c0}$ , the delay time is dominated by the intrinsic loss  $\tau_{d,m} \sim 2/\kappa_i$ , insensitive to the chain length  $N$  for both bulk and edge modes. However, the scaling of the advance time can be deduced as  $\tau_{a,m} \sim -2\kappa_{c0}|\psi_1|^2/\kappa_i^2$ . For bulk modes, this advance time scales as  $|\psi_1|^2 \sim 1/N$ , while for edge modes,  $\tau_{a,m}$  is  $N$ -independent. As one can see in Figure 5e,f, the aforementioned expectations are verified except that the edge mode has an unexpected long delay time of 117.5 ns for  $N = 10$ , that may be impacted by unwanted substrate modes. Overall, substantial internal losses in these



**FIGURE 5** | (a) The delay time as a function of chain length  $N$  for edge modes (red) and bulk mode (blue) determined by calculated transmission amplitude. The intrinsic loss and coupling rates are  $\kappa_i/\omega_0 = 10^{-5}$  and  $\kappa_{c0}/\omega_0 = 5 \times 10^{-3}$ . (b) The delay time as a function of chain length  $N$  with a higher intrinsic loss and coupling rates  $\kappa_i/\omega_0 = 10^{-4}$  and  $\kappa_{c0}/\omega_0 = 5 \times 10^{-3}$ . The green line indicates the upper limit of the delay time due to the intrinsic loss. (c) The delay times deduced from the  $S_{31}$  data obtained by finite-element simulations. The edge modes are indicated by the arrows. (d) The advance time deduced from the  $S_{21}$  data obtained by finite-element simulations. (e) The delay times deduced from the  $S_{31}$  data from the  $N = 10$  and  $N = 20$  normal conducting samples. Vertical arrows indicate the results for edge modes. (f) The advance time deduced from the  $S_{21}$  data from the  $N = 10$  and  $N = 20$  normal conducting samples.

normal conducting devices preclude direct experimental verification of  $N$ -independent edge-mode group delays. Consequently, experimentally demonstrating length-scaling behavior requires low-loss multi-length platforms, leaving superconducting device testing as a promising direction for future work.

Beyond pulse delay, key performance metrics such as power attenuation and spectral bandwidth strongly impact practical signal quality. In particular, to ensure undistorted pulse transport that aligns with the expected scattering group delay, the temporal pulse width must satisfy  $t_p > 1/B (\sim 1/\kappa) > 1/\Delta_e$ . Consequently, optimizing edge-mode transmission performance requires carefully balancing the dimerization ratio  $w/v$  and the chain length  $N$ . To clarify the constraints governing edge-state transmission as  $N$  scales, we performed exact numerical calculations with representative system parameters ( $\kappa_i/\omega_0 = 10^{-4}$  and  $\kappa_{c0}/\omega_0 = 5 \times 10^{-3}$ ) across chain lengths from  $N = 6$  to  $20$  as detailed in [Supporting Information](#). Peak transmittance decays exponentially with  $N$  for both state types, with the edge mode exhibiting an insertion loss rate of  $-6.57 \text{ dB/site}$  compared to  $-1.73 \text{ dB/site}$  for the bulk modes. The total linewidth from the resonance spectrum remains nearly constant, decreasing slightly from

$4.95 \times 10^{-3} \omega_0$  ( $N = 6$ ) to  $4.70 \times 10^{-3} \omega_0$  ( $N = 20$ ), as it is predominantly determined by external port coupling ( $\kappa_{c0}$ ). The delay-bandwidth product ( $\text{DBP} = \tau_{d,m} \kappa$ ) remains constant across system sizes, shifting only from 1.99 ( $N = 6$ ) to 1.94 ( $N = 20$ ). Finally, the speed-throughput figure of merit ( $\text{FoM} = |T|^2 / \tau_{d,m}$ ), formally defining the operational trade-off window where latency advantages outweigh transmission losses, decreases exponentially with  $N$  due to power transmittance decay.

## 5 | Conclusions

In conclusion, we have investigated the short group delay supported by the edge modes in low dimensional topological models, which is originated from the large effective coupling between the edge mode and measurement leads. Regarding experimental demonstrations, we implement a chain of interleaved splitting resonators and complementary split ring resonators with a proper setting in the orientation texture. The existence of non-trivially topological edge states, which are zero modes, is not only demonstrated through the transmission spectra, but also verified through the observation of a short group delay. Theoretical analysis reveals that in an open, port-coupled topological network, the scattering group delay of the edge state is independent of system size. The strong spatial overlap between the edge wavefunction and external leads provides enhanced port coupling, yielding an order-of-magnitude reduction in latency relative to bulk modes while maintaining robust protection against internal structural disorder. The group delay revealed and verified in our experiments provides a crucial guideline for utilizing photonic topological devices, which demonstrates the possibility to transport optical information with topological protection [38–42]. While practical pulse transport via edge modes in the 1D SSH model is constrained by low transmission efficiency at large system sizes, the benefit of reduced group delay persists in higher-dimensional topological systems, where boundary transport is significantly more efficient. Consequently, verifying fast edge-mode transport in 2D topological platforms represents a promising direction for future research.

## Acknowledgments

The authors are grateful to the National Center for High-performance Computing for computer time and facilities. The authors have used the cryostat facility at Instrument Center, National Chung Hsing University. Fruitful discussions with M. Randeria, Dongning Zheng and H. Shimada are acknowledged. This work was financially supported by the National Science and Technology Council, Taiwan under Grant Nos. 114-2112-M-005-011, 108-2923-M-007-001-MY3 and 109-2112-M-007-019-MY3), Office of Naval Research Global, US Army Research Office. G.W. acknowledges the financial supports from National Natural Science Foundation of China (11604231); Natural Science Foundation of Jiangsu Province (BK20160303).

## Conflicts of Interest Statement

The authors declare no conflicts of interest.

## Data Availability Statement

This is a data availability statement.

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### Supporting Information

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