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Interference-induced enhancement of field entanglement in a microwave-driven V-type single-atom laser

Research Article

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Abstract: We demonstrate the generation of two-mode continuous-variable (CV) entanglement in a V-type three-level atom trapped in a doubly resonant cavity using a microwave field driving a hyperfine transition between two upper excited states. By numerically simulating the dynamics of this system, our results show that the CV entanglement with large mean number of photons can be generated even in presence of the atomic relaxation and cavity losses. More interestingly, it is found that the intensity and period of entanglement can be enhanced significantly with the increasing of the atomic relaxation due to the existence of the perfect spontaneously generated interference between two atomic decay channels. Moreover, we also show that the entanglement can be controlled efficiently by tuning the intensity of spontaneously generated interference and the detuning of the cavity field.

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1. Introduction

Entanglement is one of the most counter-intuitive aspects of the quantum world. The concept has come to the fore in recent years with the advent of quantum information science. The generation and manipulation of entanglement has attracted a great deal of interest owing to their wide applications in quantum computation and quantum communication [1–13]. In particular, due to the simplicity and high efficiency in their generation, manipulation, and detection, optical continuous-variable (CV) states provide an alternative but powerful source in quantum information processing [14–18]. In order to check whether a state is entangled or not, several inseparability criteria for continuous-variable systems were proposed by Duan, Giedke, Cirac, and Zoller (DGCZ) [19], Simon [20], and Hillery and Zubairy (HZ) [21] in the form of inequalities. A violation of these inequalities provides evidence of entanglement for a continuous-variable system. Recently, based on these entanglement conditions, more and more

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theoretical and experimental efforts have been devoted to the generation and measurement of CV entanglement [22–29].

Conventionally, CV entanglement has been produced by nondegenerate parametric down-conversion (NPD) in nonlinear crystals [14, 30]. In order to improve the strength of the NPD, engineering the NPD Hamiltonian within cavity QED has also attracted much attention [28, 29, 31–39]. For example, Xiong *et al.* [28] proposed a scheme for an entanglement amplifier based on two-mode correlated spontaneous emission lasers in a nondegenerate three-level cascade atomic system, whereby the atom coherence is induced by pumping the atom from the lower level to the upper level. Li *et al.* [31] considered the generation of two-mode entangled states in a cavity field via the four-wave mixing process, by means of the interaction of properly driven V-type three-level atoms with two cavity modes. Subsequently, Tan *et al.* [32] extended above concept and studied the generation and evolution of entangled light by taking into account the spontaneously generated interference between two atomic decay channels. In an earlier study, Qamar *et al.* [33] proposed a scheme to generate two-mode entangled states in a quantum beat laser [34], which consists of a V-type three-level atom interacting with two modes of the cavity field in a doubly resonant cavity. Related properties of entanglement for different values of Rabi frequencies in the presence of cavity losses are also investigated. Fang *et al.* [35] added into this scheme the effects of phase shift and Rabi frequency in the classical driving field, cavity loss, and the purity and nonclassicality of the initial state of the cavity field [34].

In the present paper, we propose an efficient scheme for the generation of CV entanglement in a V-type three-level atom trapped in a doubly resonant cavity. In our scheme, two transitions between the ground state and two excited states of the V-type atom are driven by the two cavity modes independently, and the two excited states of the atom are driven by a strong microwave field. Using the entanglement criterion proposed by DGCZ [19], we show that CV entanglement with large mean number of photons can be obtained in the present system. By numerically simulating the evolution of the CV entanglement, we also show that the intensity and period of entanglement can be amplified significantly by increasing the atomic relaxation due to the existence of the perfect spontaneously generated interference between two atomic decay channels. We also analyze the effect of the strength of the spontaneously generated interference and the detuning of the cavity field on the evolution of entanglement.

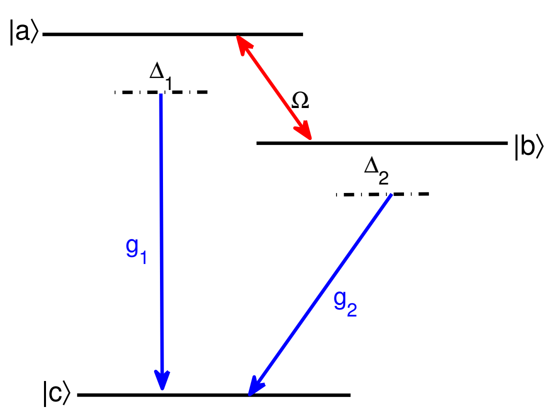

Figure 1. Schematic diagram of the three-level atom system in a V-type configuration. Two (nondegenerate) cavity modes with the coupling constant g_1 and g_2 interact with the transition $|a\rangle \leftrightarrow |c\rangle$ and $|b\rangle \leftrightarrow |c\rangle$, respectively, while the atom transition $|a\rangle \leftrightarrow |b\rangle$ is driven by an assisting microwave driving field with Lamor frequency Ω. Here, Δ_1 and Δ_2 correspond to the frequency detunings.

2. Model and master equation

Let us consider a three-level V-type atom trapped in a doubly resonant cavity. The atom level configuration is depicted in Fig. 1. Assume the atom is pumped at a rate γ_a into the level $|a\rangle$. Two nondegenerate cavity modes with frequencies ν_1 and ν_2 independently interact with the transitions $|a\rangle \leftrightarrow |c\rangle$ with the resonant frequency ω_{ac} and $|b\rangle \leftrightarrow |c\rangle$ with the resonant frequency ω_{bc}, respectively. An assisting microwave driving field with Lamor frequency Ω is used to couple Zeeman sublevels $|a\rangle$ and $|b\rangle$ through an allowed magnetic dipole transition and to further form a closed-loop configuration. The detuning of two cavity modes from the atomic transitions $|a\rangle \leftrightarrow |c\rangle$ and $|b\rangle \leftrightarrow |c\rangle$ are denoted as $\Delta_1 = \omega_{ac} - \nu_1$ and $\Delta_2 = \omega_{bc} - \nu_2$, respectively. For simplicity, in the following calculations we will note $\Delta_1 = \Delta_2 = \Delta$.

In the interaction picture, with the rotating-wave approximation and the electric-dipole approximation, the Hamiltonian describing the atom-field interaction for the system under investigation can be written as (taking $\hbar = 1$) [40–43]

$$V_I = \Delta |a\rangle\langle a| + \Delta |b\rangle\langle b| + (-\Omega |a\rangle\langle b| + g_1 a_1 |a\rangle\langle c| + g_2 a_2 |b\rangle\langle c| + H.c.), \quad (1)$$

where the symbol $H.c.$ denotes the Hermitian conjugate. In obtaining the Hamiltonian (1), we have taken the ground state $|c\rangle$ as the energy origin. $\Omega = |\Omega| \exp(i\phi)$ describes the Lamor frequency of the microwave driving field with ϕ the phase of the field. The parameters g_1 and g_2 are used for describing the corresponding atom—field coupling intensities. a_j ($a_j^\dagger$) is the annihilation (creation) operator of the corresponding cavity modes.

Considering the vacuum damping of the atom and the cavity modes, the reduced density equations of the cavity fields can be obtained from the Hamiltonian (1) by taking a trace over the atom degrees of freedom [44],

$$\begin{aligned}\dot{\rho}_f &= -iTr_{\text{atom}}[V_I, \rho] + Tr_{\text{atom}}(L_f\rho + L_a\rho)\\ &= -ig_1[a_1^\dagger, \rho_{ac}] - ig_2[a_2^\dagger, \rho_{bc}] + \sum_{j=1}^{2}\kappa_j[a_j, \rho_f a_j^\dagger] + H.c.,\end{aligned} \tag{2}$$

with

$$L_f\rho = \sum_{j=1}^{2}\kappa_j[a_j, \rho a_j^\dagger] + H.c., \tag{3}$$

$$\begin{aligned}L_a\rho =& \gamma_1[s_{ca}, \rho s_{ac}] + \gamma_2[s_{cb}, \rho s_{bc}] + \gamma_{12}([s_{ca}, \rho s_{bc}]\\ &+ [s_{cb}, \rho s_{ac}]) + H.c.,\end{aligned} \tag{4}$$

where $L_f\rho$, $L_a\rho$ are the damping of cavity modes and atomic excitation, respectively. In the above equations, γ_1 and γ_2 are used to describe the decay rates from the excited states $|a\rangle$ and $|b\rangle$ to the ground state $|c\rangle$, respectively. The parameter κ_j ($j = 1, 2$) denotes the damping constants of two cavity modes. It should be noted that $\gamma_{12} = P\sqrt{\gamma_1\gamma_2}$ describes the spontaneously generated interference effect, which comes from the cross coupling between the transitions $|a\rangle \leftrightarrow |c\rangle$ and $|b\rangle \leftrightarrow |c\rangle$. The parameter $P = \overrightarrow{b_{ac}} \cdot \overrightarrow{b_{bc}}/(\left|\overrightarrow{b_{ac}}\right| \cdot \left|\overrightarrow{b_{bc}}\right|) = \cos\theta$ represents the strength of spontaneously generated interference, where the values $P = 0$ and $P = 1$ correspond to no interference and perfect interference, respectively. Here, $\overrightarrow{b_{ac}}$ and $\overrightarrow{b_{bc}}$ represent the atomic dipole polarizations and θ is the angle between the two dipole moments. From the above expressions for the parameter P and γ_{12}, one can find that the spontaneously generated interference is dependent on the angle between the two dipole moments. The interference effect disappears ($P = 0$) when the two dipole moments are perpendicular to each other, and reaches its maximum ($P = 1$) when they are parallel to each other.

Based on the standard methods of laser theory [44], and considering the spontaneous decay of the atom, ρ_{ac} and ρ_{bc} can be evaluated to the first order in the coupling constants g_1 and g_2 as

$$\begin{aligned}i\dot{\rho}_{ac} =& -i\gamma_1\rho_{ac} - i\gamma_{12}\rho_{bc} + \Delta\rho_{ac} - \Omega\rho_{bc}\\ &- g_1\rho_{aa}^{(0)}a_1 - g_2\rho_{ab}^{(0)}a_2 + g_1a_1\rho_{cc}^{(0)},\end{aligned} \tag{5}$$

$$\begin{aligned}i\dot{\rho}_{bc} =& -i\gamma_2\rho_{bc} - i\gamma_{12}\rho_{ac} + \Delta\rho_{bc} - \Omega^*\rho_{ac}\\ &- g_1\rho_{ba}^{(0)}a_1 - g_2\rho_{bb}^{(0)}a_2 + g_2a_2\rho_{cc}^{(0)},\end{aligned} \tag{6}$$

where the density matrix elements $\rho_{ij}^{(0)}$ can be obtained by the corresponding zeroth-order equations

$$i\dot{\rho}_{aa}^{(0)} = -2i\gamma_1\rho_{aa}^{(0)} - i\gamma_{12}(\rho_{ba}^{(0)} + \rho_{ab}^{(0)}) - \Omega\rho_{ba}^{(0)} + \Omega^*\rho_{ab}^{(0)} + \Omega_a\rho_f, \tag{7}$$

$$i\dot{\rho}_{bb}^{(0)} = -2i\gamma_2\rho_{bb}^{(0)} - i\gamma_{12}(\rho_{ba}^{(0)} + \rho_{ab}^{(0)}) - \Omega^*\rho_{ab}^{(0)} + \Omega\rho_{ba}^{(0)}, \tag{8}$$

$$i\dot{\rho}_{ab}^{(0)} = -i(\gamma_1 + \gamma_2)\rho_{ab}^{(0)} - i\gamma_{12}(\rho_{aa}^{(0)} + \rho_{bb}^{(0)}) - \Omega\rho_{bb}^{(0)} + \Omega\rho_{aa}^{(0)}, \tag{9}$$

$$i\dot{\rho}_{ba}^{(0)} = -i(\gamma_1 + \gamma_2)\rho_{ba}^{(0)} - i\gamma_{12}(\rho_{aa}^{(0)} + \rho_{bb}^{(0)}) + \Omega^*\rho_{bb}^{(0)} - \Omega^*\rho_{aa}^{(0)}, \tag{10}$$

$$i\dot{\rho}_{cc}^{(0)} = 0. \tag{11}$$

By inserting the steady-state solution of $\rho_{ij}^{(0)}$ into Eqs. (5-6), we can obtain the steady-state solutions for ρ_{ac} and ρ_{bc}, which have the form:

$$ig_1\rho_{ac} = \alpha_{11}\rho_f a_1 + \alpha_{12}\rho_f a_2, \tag{12}$$

$$ig_2\rho_{bc} = \alpha_{21}\rho_f a_1 + \alpha_{22}\rho_f a_2, \tag{13}$$

with the explicit expressions for the coefficients α_{ij} given in Appendix A. By substituting Eqs.(12-13) into Eq. (2), the corresponding reduced master equation governing the evolution of the cavity field can be obtained as

$$\begin{aligned}\dot{\rho}_f =& -\alpha_{11}(a_1^\dagger\rho_f a_1 - \rho_f a_1 a_1^\dagger) - \alpha_{12}(a_1^\dagger\rho_f a_2 - \rho_f a_2 a_1^\dagger)\\ &- \alpha_{22}(a_2^\dagger\rho_f a_2 - \rho_f a_2 a_2^\dagger) - \alpha_{21}(a_2^\dagger\rho_f a_1 - \rho_f a_1 a_2^\dagger)\\ &+ \kappa_1(a_1\rho_f a_1^\dagger - \rho_f a_1^\dagger a_1) + \kappa_2(a_2\rho_f a_2^\dagger - \rho_f a_2^\dagger a_2) + H.c..\end{aligned} \tag{14}$$

Here we should note that we only consider the second order in the coupling constants g_1, g_2 and retain all orders in the Lamor frequency of the microwave driving field Ω, because the coupling constants of the two cavity modes are smaller than other system parameters in our scheme. Thus we can ignore the saturation effects and operate in the regime of linear amplification [29].

3. Entanglement of the cavity fields

According to Ref. [19], an inseparability criterion is proposed for continuous variable systems with the aid of the total variance of a pair of Einstein-Podolsky-Rosen (EPR) type operators, and the criterion provides a sufficient condition for entanglement of any two-party continuous-variable states. In this section, we use the sufficient inseparability criterion [19] to study the entanglement properties of the two cavity modes. Based on the spirit of this criterion, the system is said to be in an entangled state if the sum of the quantum fluctuations of the two EPR type operators $\hat{u} = \hat{x}_1 + \hat{x}_2$ and $\hat{v} = \hat{p}_1 + \hat{p}_2$ of the two modes satisfies the following inequality

$$\langle (\Delta\hat{u})^2 + (\Delta\hat{v})^2 \rangle < 2, \tag{15}$$

with $\hat{x}_j = (a_j + a_j^\dagger)/\sqrt{2}$ and $\hat{p}_j = -i(a_j - a_j^\dagger)/\sqrt{2}$ $(j = 1, 2)$ the quadrature operators of the two cavity modes 1 and 2. For a general state, this is a sufficient criterion for entanglement and furthermore, as demonstrated in Ref. [19], for all Gaussian states, this criterion turns out to be a necessary and sufficient condition for inseparability. By substituting $\hat{x}_j$ and $\hat{p}_j$ into Eq. (15), we can obtain the total variance for the operators $\hat{u}$ and $\hat{v}$ in terms of the operators a_j and $a_j^\dagger$. That is

$$\begin{aligned}\langle (\Delta\hat{u})^2 + (\Delta\hat{v})^2 \rangle =& 2[1 + \langle a_1^\dagger a_1\rangle + \langle a_2^\dagger a_2\rangle + \langle a_1 a_2\rangle + \langle a_1^\dagger a_2^\dagger\rangle \\ &- \langle a_1\rangle\langle a_1^\dagger\rangle - \langle a_2\rangle\langle a_2^\dagger\rangle - \langle a_1\rangle\langle a_2\rangle \\ &- \langle a_1^\dagger\rangle\langle a_2^\dagger\rangle]. \end{aligned}\tag{16}$$

With the help of Eq. (14), we can obtain the equations of motion for the expectation values of the field operators shown in Eq. (16) as

$$\frac{\partial}{\partial t}\langle a_1\rangle = -(\alpha_{11} + \kappa_1)\langle a_1\rangle - \alpha_{12}\langle a_2\rangle, \tag{17}$$

$$\frac{\partial}{\partial t}\langle a_2\rangle = -(\alpha_{22} + \kappa_2)\langle a_2\rangle - \alpha_{21}\langle a_1\rangle, \tag{18}$$

$$\frac{\partial}{\partial t}\langle a_1^\dagger\rangle = -(\alpha_{11}^* + \kappa_1)\langle a_1^\dagger\rangle - \alpha_{12}^*\langle a_2^\dagger\rangle, \tag{19}$$

$$\frac{\partial}{\partial t}\langle a_2^\dagger\rangle = -(\alpha_{22}^* + \kappa_2)\langle a_2^\dagger\rangle - \alpha_{21}^*\langle a_1^\dagger\rangle, \tag{20}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_1^\dagger a_1\rangle =& -(\alpha_{11} + \alpha_{11}^* + 2\kappa_1)\langle a_1^\dagger a_1\rangle - \alpha_{12}\langle a_1^\dagger a_2\rangle \\ &- \alpha_{12}^*\langle a_1 a_2^\dagger\rangle - (\alpha_{11} + \alpha_{11}^*),\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_2^\dagger a_2\rangle =& -(\alpha_{22} + \alpha_{22}^* + 2\kappa_2)\langle a_2^\dagger a_2\rangle - \alpha_{21}^*\langle a_1^\dagger a_2\rangle \\ &- \alpha_{21}\langle a_1 a_2^\dagger\rangle - (\alpha_{22} + \alpha_{22}^*),\end{aligned} \tag{22}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_1^\dagger a_2\rangle =& -\alpha_{21}\langle a_1^\dagger a_1\rangle - \alpha_{12}^*\langle a_2^\dagger a_2\rangle \\ &- (\alpha_{11}^* + \alpha_{22} + \kappa_1 + \kappa_2)\langle a_1^\dagger a_2\rangle - (\alpha_{12}^* + \alpha_{21}),\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_1 a_2^\dagger\rangle =& -\alpha_{21}^*\langle a_1^\dagger a_1\rangle - \alpha_{12}\langle a_2^\dagger a_2\rangle \\ &- (\alpha_{11} + \alpha_{22}^* + \kappa_1 + \kappa_2)\langle a_1 a_2^\dagger\rangle - (\alpha_{12} + \alpha_{21}^*),\end{aligned} \tag{24}$$

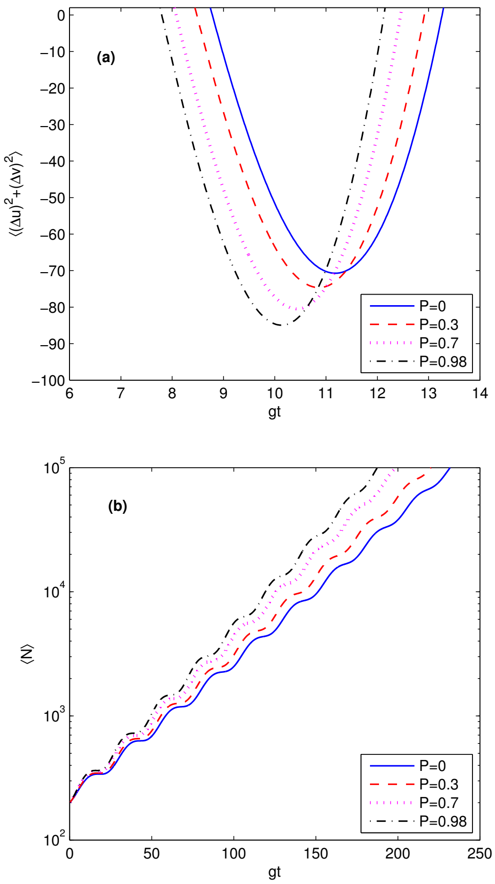

Figure 2. (a) The time evolution of $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$, and (b) the total mean photon numbers $\langle\hat{N}\rangle$, for different values of P. Here, the cavity field is initially in the coherent state $|10, -10\rangle$. Other parameters used are $\kappa_1 = \kappa_2 = 0.001g$, $\gamma_1 = \gamma_2 = \gamma = g$, $|\Omega| = 10g$, $\Delta_1 = \Delta_2 = \Delta = g$, $\gamma_a = 5g$, and $\phi = \pi/2$.

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_1 a_2\rangle =& -\alpha_{21}\langle a_1 a_1\rangle - \alpha_{12}\langle a_2 a_2\rangle \\ &- (\alpha_{11} + \alpha_{22} + \kappa_1 + \kappa_2)\langle a_1 a_2\rangle,\end{aligned} \tag{25}$$

$$\frac{\partial}{\partial t}\langle a_1 a_1\rangle = -2(\alpha_{11} + \kappa_1)\langle a_1 a_1\rangle - 2\alpha_{12}\langle a_1 a_2\rangle, \tag{26}$$

$$\frac{\partial}{\partial t}\langle a_2 a_2\rangle = -2(\alpha_{22} + \kappa_2)\langle a_2 a_2\rangle - 2\alpha_{21}\langle a_1 a_2\rangle, \tag{27}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle a_1^\dagger a_2^\dagger\rangle =& -\alpha_{21}^*\langle a_1^\dagger a_1^\dagger\rangle - \alpha_{12}^*\langle a_2^\dagger a_2^\dagger\rangle \\ &- (\alpha_{11}^* + \alpha_{22}^* + \kappa_1 + \kappa_2)\langle a_1^\dagger a_2^\dagger\rangle,\end{aligned} \tag{28}$$

$$\frac{\partial}{\partial t}\langle a_1^\dagger a_1^\dagger\rangle = -2(\alpha_{11}^* + \kappa_1)\langle a_1^\dagger a_1^\dagger\rangle - 2\alpha_{12}^*\langle a_1^\dagger a_2^\dagger\rangle, \tag{29}$$

$$\frac{\partial}{\partial t}\langle a_2^\dagger a_2^\dagger\rangle = -2(\alpha_{22}^* + \kappa_2)\langle a_2^\dagger a_2^\dagger\rangle - 2\alpha_{21}^*\langle a_1^\dagger a_2^\dagger\rangle. \tag{30}$$

In order to make clear whether entanglement between two modes of the field can be generated, we calculate the time evolution of various moments $\langle\hat{N}\rangle = \langle a_1^\dagger a_1\rangle + \langle a_2^\dagger a_2\rangle$ and $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ by use of the equations of motion given in Eqs. (17-30). However, the exact analytical results are rather complicated in this situation. Since we mainly focus on examining the influence of the spontaneously generated interference on the property of the CV entanglement, we show the typical numerical results for the time evolution of various moments $\langle\hat{N}\rangle = \langle a_1^\dagger a_1\rangle + \langle a_2^\dagger a_2\rangle$ and $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ for different values of parameters P, γ and Δ. Note that the two cavity modes are assumed to be in the coherent state $|10,-10\rangle$. For simplicity, all the parameters used here are scaled with g, and we let $\phi = \pi/2$ during our numerical calculations.

We first analyze the influences of the spontaneously generated interference strength on the property of the CV entanglement when the cavity field is initially in the coherent state $|10,-10\rangle$. Here the cavity loss rates, atomic relaxation rates, intensity of the microwave driving field, the detuning, and the incoherent pumping are chosen as $\kappa_1 = \kappa_2 = 0.001g$, $\gamma_1 = \gamma_2 = \gamma = g$, $|\Omega| = 10g$, $\Delta = g$, and $\gamma_a = 5g$. We plot in Fig. 2, the variable moment $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ and the mean photon number $\langle\hat{N}\rangle$ as a function of the time gt for different values of the spontaneously generated interference strength P, i.e., $P = 0$ (solid curve), $P = 0.3$ (dashed curve), $P = 0.7$ (dotted curve), and $P = 0.98$ (dash-dotted curve). From Fig. 2(a), one can see that the intensity and period of entanglement increase slightly with the increase of the value of P when the cavity field is initially in the coherent state. However, as shown in Fig. 2(b), the maximum mean photon number $\langle\hat{N}\rangle$ can be enlarged by increasing the spontaneously generated interference strength P with the same set of the parameters. As a result, a perfect spontaneously generated interference is needed for obtaining a long entanglement time interval, entanglement intensity, and large mean number of photons.

Choosing a certain strength of the spontaneously generated interference (i.e., $P = 0.5$), we now analyze the effects of different atomic relaxation rates on the properties of the CV entanglement with the cavity field initially in the same coherent state as Fig. 2. As shown in Fig. 3, we plot the time evolution of the variable moment $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$ and the mean photon number $\langle\hat{N}\rangle$ for different values of the atomic relaxation rates γ, i.e., $\gamma = g$ (solid curve), $\gamma = 1.5g$ (dashed curve), and $\gamma = 2g$ (dotted curve). From Fig. 3, one finds that the intensity and period of entanglement can be enhanced significantly without decreasing the maximum mean photon numbers $\langle\hat{N}\rangle$ as the spontaneous emission decay rate of the atom increases. The physical interpretation of these results is rather clear. The spontaneous emission rates of the atom play a constructive role in the spontaneously generated

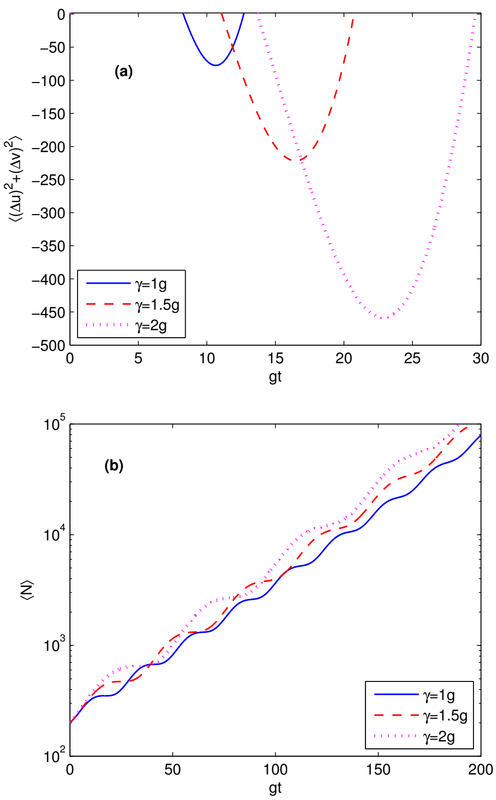

Figure 3. (a) The time evolution of $\langle(\Delta\hat{u})^2 + (\Delta\hat{v})^2\rangle$, and (b) the total mean photon numbers $\langle\hat{N}\rangle$, with different atomic decay rates γ ($\gamma_1 = \gamma_2 = \gamma$). Here the cavity field is initially in the coherent state $|10,-10\rangle$. Other parameters used are $\kappa_1 = \kappa_2 = 0.001g$, $P = 0.5$, $|\Omega| = 10g$, $\Delta_1 = \Delta_2 = \Delta = g$, $\gamma_a = 5g$, and $\phi = \pi/2$.

interference γ_{12}. For a given value of the parameter P, γ_{12} increases with increasing spontaneous emission rates γ_j. As illustrated in Fig. 2, the entanglement intensity and time interval can be enhanced by the spontaneously generated interference; as a result, the intensity and period of entanglement can be increased accordingly when the atom spontaneous emission rates increase. In other words, we can achieve the optimal combination of a long entanglement time interval and enhanced entanglement intensity by properly controlling the atom's spontaneous emission rates.

Up to now, we have investigated the effect of atomic spontaneous emission and spontaneously generated interference on the properties of CV entanglement by considering the cavity field initially in coherent state. We now examine the effect of the frequency detuning between the cavity modes and the corresponding atomic transition on the time evolution of entanglement. By considering that the initial state of the cavity field is the coherent state $|10,-10\rangle$, we

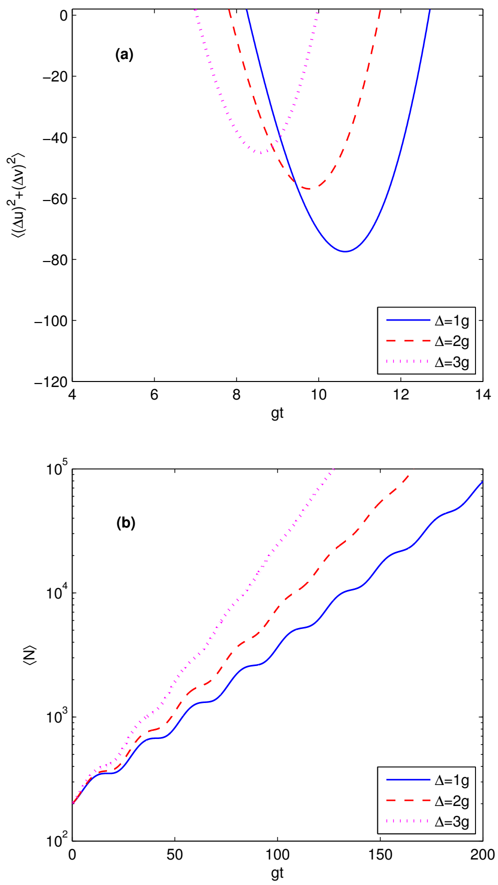

Figure 4. (a) The time evolution of $\langle(\Delta\hat{u})^2+(\Delta\hat{v})^2\rangle$, and (b) the total mean photon numbers $\langle\hat{N}\rangle$, for different detuning frequencies $\Delta(\Delta_1=\Delta_2=\Delta)$. Here, the cavity field is initially in the coherent state $|10,-10\rangle$. Other other parameters used are $\gamma_1=\gamma_2=g$, $\kappa_1=\kappa_2=0.001g$, $P=0.5$, $|\Omega|=10g$, $\gamma_a=5g$, and $\phi=\pi/2$.

plot in Fig. 4 the time evolution of the variable moment $\langle(\Delta\hat{u})^2+(\Delta\hat{v})^2\rangle$ and the mean photon number $\langle\hat{N}\rangle$ for different values of the detunings Δ, i.e., $\Delta=g$ (solid curve), $\Delta=2g$ (dashed curve), and $\Delta=3g$ (dotted curve). It can be easily seen from Fig. 4(a) that the intensity and period of entanglement between the two cavity modes can be decreased markedly by augmenting the frequency detuning Δ. On the other hand, it can be found from Fig. 4(b) that the total mean photon numbers $\langle\hat{N}\rangle$ increase significantly with the increasing of the frequency detuning Δ and the maximum mean number of photons for which the entanglement criterion (15) is still satisfied. These results indicate that one can obtain the entanglement of cavity modes with a greater magnitude and a longer period by choosing the near-resonant condition. Based on this interesting result, it is possible to obtain the entanglement of cavity modes with a low-Q cavity.

4. Conclusion

In conclusion, we have demonstrated the generation of two-mode continuous-variable (CV) entanglement and analyzed the properties of the entanglement in a V-type three-level atom trapped in a doubly resonant cavity using a microwave field driving a hyperfine transition between two upper excited states. Our numerical results have indicated that the CV entanglement with large mean number of photons can be generated even in the presence of atomic relaxation and cavity losses. Importantly, we have shown that the intensity and period of entanglement can be enhanced significantly with increasing atomic relaxation due to the existence of the perfect spontaneously generated interference between two atomic decay channels. Moreover, we have also shown that CV entanglement of cavity modes with large mean photon numbers can be generated even under near-resonant conditions, which implies that it is possible to obtain the entanglement of cavity modes even with a low-Q cavity.

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Appendix A: COEFFICIENTS

Here, we give the explicit expressions for the coefficients A_{ij} and B_{ij} $(i,j=1,2)$ in Eqs. (12-14),

$$\alpha_{11}=-g_1^2[(R+i\Delta)\beta_{aa}+(i\Omega-\gamma_{12})\beta_{ba}]/D_1, \tag{A1}$$

$$\alpha_{12}=-g_1g_2[(R+i\Delta)\beta_{ab}+(i\Omega-\gamma_{12})\beta_{bb}]/D_1, \tag{A2}$$

$$\alpha_{22}=-g_2^2[(R+i\Delta)\beta_{bb}+(i\Omega^*-\gamma_{12})\beta_{ab}]/D_1, \tag{A3}$$

$$\alpha_{21}=-g_1g_2[(R+i\Delta)\beta_{ba}+(i\Omega^*-\gamma_{12})\beta_{aa}]/D_1, \tag{A4}$$

$$\beta_{aa}=(2R^2-\gamma_{12}^2+|\Omega|^2)R\gamma_a/D_2, \tag{A5}$$

$$\beta_{bb}=(\gamma_{12}+i\Omega)(\gamma_{12}-i\Omega^*)R\gamma_a/D_2, \tag{A6}$$

$$\beta_{ab}=-(\gamma_{12}+i\Omega)(2R^2-i\gamma_{12}\Omega-i\gamma_{12}\Omega^*)R\gamma_a/(2D_2), \tag{A7}$$

$$\beta_{ba} = -(\gamma_{12} - i\Omega^*)(2R^2 + i\gamma_{12}\Omega + i\gamma_{12}\Omega^*)R\gamma_a/(2D_2), \tag{A8}$$

with

$$D_1 = (R + i\Delta)^2 - (i\Omega - \gamma_{12})(i\Omega^* - \gamma_{12}), \tag{A9}$$

$$D_2 = 4R^4 + 4R^2|\Omega|^2 - 4R^2\gamma_{12}^2 - \gamma_{12}^2(\Omega + \Omega^*)^2, \tag{A10}$$

where we have assumed $\gamma_1 = \gamma_2 = \gamma = R$.

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