

# A simple model for cavity enhanced slow lights in vertical cavity surface emission lasers

Chia-Sheng Chou<sup>1</sup>, Ray-Kuang Lee<sup>1</sup>, Peng-Chun Peng<sup>2</sup>,  
Hao-chung Kuo<sup>3</sup>, Gray Lin<sup>4</sup>, Hung-Ping Yang<sup>5</sup> and Jim Y Chi<sup>5</sup>

<sup>1</sup> Institute of Photonics Technologies, National Tsing-Hua University, Hsinchu 300, Taiwan

<sup>2</sup> Department of Applied Materials and Optoelectronic Engineering, National Chi Nan University, Nantou 545, Taiwan

<sup>3</sup> Department of Photonics and Institute of Electro-Optical Engineering, National Chiao-Tung University, Hsinchu 300, Taiwan

<sup>4</sup> Department of Electronics Engineering, National Chiao-Tung University, Hsinchu 300, Taiwan

<sup>5</sup> Opto-Electronics and System Laboratory, Industrial Technology Research Institute, Hsinchu 300, Taiwan

E-mail: [rklee@ee.nthu.edu.tw](mailto:rklee@ee.nthu.edu.tw)

Received 30 September 2007, accepted for publication 4 March 2008

Published 3 April 2008

Online at [stacks.iop.org/JOptA/10/044016](http://stacks.iop.org/JOptA/10/044016)

## Abstract

We develop a simple model for the slow lights in vertical cavity surface emission lasers (VCSELs), with the combination of cavity and population pulsation effects. The dependences of probe signal power, injection bias current and wavelength detuning for the group delays are demonstrated numerically and experimentally. Up to 65 ps group delays and up to 10 GHz modulation frequency can be achieved at room temperature at a wavelength of 1.3  $\mu\text{m}$ . The most significant feature of our VCSEL device is that the thickness of the active region is only several micrometers long. Based on the experimental parameters of quantum dot VCSEL structures, we show that the resonance effect of the laser cavity plays a significant role in enhancing the group delays.

**Keywords:** slow-light, VCSEL

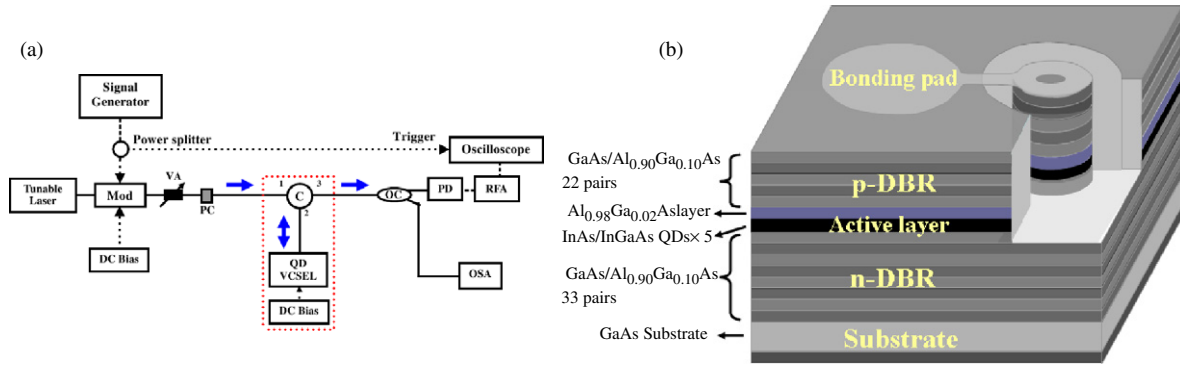
(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Slow light is believed to be a critical foundation not only for basic scientific research but also for applications in optical communication, optical memories, signal processing, and phase-array antenna systems [1]. Various systems have been demonstrated for slow lights, from electromagnetically induced transparency (EIT) [2, 3], coherent population oscillations (CPO) [4], to stimulated Brillouin [5] and Raman scatterings [6]. Unlike EIT in cryogenic systems, slow light in semiconductor optoelectronic devices based on CPO is more promising due to its inherent compactness, direct electrical controllability, and room temperature operation. CPO is the effect that the ground state population of the material will

oscillate in time at the beat frequency of the two input waves. This involves shining two lasers—a pump beam and a weaker probe beam—at the media. The probe beam experiences reduced absorption over a narrow range of wavelengths under certain conditions. The refractive index also increases rapidly in this *spectral hole*, which leads to a much reduced group velocity for the probe beam.

With state of the art fabrication technologies, quantum well and quantum dot semiconductor optical amplifiers (SOAs) have been demonstrated as a flexible platform for studying slow light phenomenon based on CPO as well as its applications at room temperature [7, 8]. Because a quantum dot (QD) can provide a better carrier confinement than a quantum well and offer reduced thermal ionization,



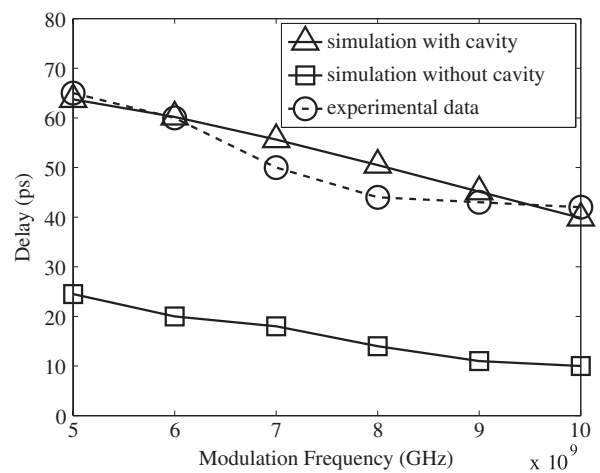
**Figure 1.** (a) Experimental setup for measuring the optical group delays in VCSELs. The arrows show the traveling directions of light. (Mod: Mach-Zehnder modulator, VA: variable optical attenuator, C: optical circulator, OC: optical coupler, PC: polarization controller, RFA: RF amplifier, PD: photodetector, OSA: optical spectrum analyzer). (b) Schematic diagram of our quantum dot VCSEL.

semiconductor lasers with quantum dot gain media have been studied intensively to improve the laser characteristics. Recently we demonstrated a tunable optical group delay in a monolithically single mode quantum dot vertical cavity surface emitting laser (VCSEL) at 10 GHz experimentally [9]. Tunable slow light with optical group delay up to several tens of picoseconds can be achieved by adjusting the bias current and wavelength detuning.

The main difference between an SOA device and a VCSEL device is that the latter has a cavity induced by two Bragg gratings. Compared to SOA devices with the active region for the gain medium being about several millimeters, the optical wave passes orthogonally over VCSELs within a typical thickness of the active region only several micrometers long. In this scenario, the commonly adopted population pulsation model of a traveling wave induced dynamic carrier index grating [10] cannot be directly applied to semiconductor lasers. Followed by the two-wave model for the pump and probe beams in the presence of coherent population oscillation, in this work we develop a simple model for the slow lights in VCSELs with the combination of a cavity effect and the rate equation for carrier undulation. A simple formulation based on a Fabry-Perot filter with gain medium, used in [11], is adopted to model real distributed Bragg reflectors in our VCSELs. Experimental data of up to 65 ps group delays and up to 10 GHz modulation frequency operating at room temperature at a wavelength of 1.3 μm are in agreement with the proposed theoretical results. Based on the experimental parameters of quantum dot VCSEL structures, we show that the resonance effect of the laser cavity plays a significant role in enhancing the group delays.

## 2. Fabrication and measurement of the slow light in VCSELs

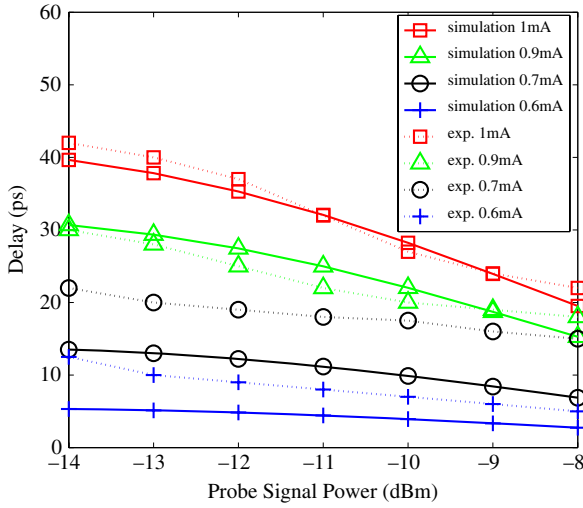
The experimental setup for the slow light in VCSELs is illustrated in figure 1(a). The key component in our experiment is a monolithically single mode GaAs-based QD VCSEL, grown by molecular beam epitaxy with fully doped n- and p-doped AlGaAs distributed Bragg reflectors (DBRs), as show in figure 1(b). The characteristics of this QD VCSEL were



**Figure 2.** Comparison of the group delays in our VCSEL with different modulation frequency detunings. Dashed line: experimental data; solid lines: simulation results based on a traveling wave CPO model without (square marks) and with (triangle marks) the consideration of the cavity effect. A big discrepancy is shown without consideration of the cavity effect.

reported in [12, 13]. For slow light measurement, an optical signal is generated by a tunable laser and modulated via a Mach-Zehnder modulator (Mod), as shown in figure 1(a). The signal power is controlled by a variable attenuator (VA) at the output of the Mach-Zehnder modulator. When the VCSEL turns on, we detect the lasing wavelength of the cavity mode first. Then the wavelength of injected optical signal is tuned to the resonance of the QD VCSEL cavity at 1.3 μm. By this method, we only have two optical fields in the system, an optical signal and its modulation component. Strong gain competition occurs when the injected optical signal tunes off-resonance to the VCSEL lasing wavelength, which shows complicated phenomena and goes beyond the scope of this work.

Then an optical circulator (C) is used to couple the probe signal into the QD VCSEL. The time delay of the reflected probe signal is measured by a digital oscilloscope. The relationship between the time delays and modulation frequencies of the probe signal are shown with the dashed line



**Figure 3.** The group delay of our VCSELs is shown as a function of the optical power of the probe signal beam at different bias currents. Solid lines are simulation results while the dashed lines are experimental data.

in figure 2, where the bias current of the QD VCSEL and the probe signal power are fixed at 1 mA and  $-14$  dB m, respectively. The time delay in the QD VCSEL increases as the modulation frequency decreases. Moreover, the time delays as functions of bias currents of the QD VCSEL and the optical power of the probe signal are shown with dashed lines in figure 3, where the modulation frequency is fixed at 10 GHz. The time delay increases as the signal power decreases. The experimental details have been reported in our previous works [12]. In this experiment, the threshold current is 0.7 mA and the thickness of the cavity is estimated to be as short as about  $1.13 \mu\text{m}$ .

### 3. Modeling and simulation results

To model the population oscillation in semiconductor lasers, our theoretical starting point is based on the carrier undulation induced by the frequency beating between two optical waves [8, 10]. The probe signal experiences gain and refractive index changes by the pump wave through the carrier index and gain grating. The dynamics of the carrier density,  $N$ , at an injected current,  $I$ , can be derived from the carrier rate equation, i.e.,

$$\frac{d}{dt}N = \frac{I}{qV} - \frac{N}{\tau_s} - \frac{g(N)}{\hbar\omega} |E|^2 + D\nabla^2 N(t, z), \quad (1)$$

where  $q$  is the unit electron charge,  $V$  is the active region volume,  $g(N)$  is the model gain,  $\tau_s$  is the carrier lifetime,  $D$  is the diffusion coefficient, and  $\hbar\omega$  is the photon energy.  $E$  is the field amplitude of total incident waves, including the pump wave  $E_p \exp(-i\omega_p t)$  and probe signal  $E_s \exp(-i\omega_s t)$ , i.e.,

$$|E|^2 \approx |E_p|^2 + |E_s|^2 + E_p E_s^* \exp(-i\Omega)t + \text{c.c.},$$

with the detuning modulation frequency, the pump wave and probe signal carrier frequencies  $\Omega$ ,  $\omega_p$  and  $\omega_s$ , respectively.

The pump wave and probe signal are assumed to be in phase since both of them come from the same tunable laser modulated by a Mach–Zehnder modulator. A linear model gain is assumed in the model so that our VCSEL is operated not far from the threshold condition,

$$g(N) = \alpha(N - N_0),$$

where  $\alpha$  is the gain coefficient and  $N_0$  is the transparent carrier density. Next we assume that the carrier density can be described by a dc term and modulated at the same detuning frequency with small ac terms,

$$N \approx \bar{N} + [\Delta N \exp(-i\Omega)t + \text{c.c.}],$$

where  $\bar{N}$  is the static carrier density and  $\Delta N$  is the amplitude of the carrier population oscillation. The index and gain changes of the probe signal beam can be derived from equation (1); then one can calculate the corresponding optical group delays caused by the population oscillation effect, with the definition

$$\Delta n_g = \Delta n + \omega_s \frac{d\Delta n}{d\Omega},$$

$$\tau_{\text{delay}} = \frac{L}{c} \Delta n_g,$$

where  $\Delta n$  is refractive index change,  $L$  is the length of the media, and  $c$  is the speed of light in free space. With equation (1), we assume that the probe signal is much weaker than the pump wave as in the experimental measurement, and obtain the index change of the probe beam by

$$\Delta n = \gamma g(\bar{N}) \frac{c}{2\omega_s} \left[ 1 - \frac{P_0(1 + P_0 - \frac{\Omega t_s}{\gamma})}{(1 + P_0)^2 + (\Omega t_s)^2} \right], \quad (2)$$

where  $\gamma$  is the line-width enhancement factor, and  $P_0 \equiv \frac{P}{P_s}$  is the normalized pump power with respect to the saturation power  $P_s \equiv \frac{\hbar\omega}{\alpha t_s}$ .

Figure 2 shows the comparison of the experimental data of slow lights in our VCSELs with a commonly adopted CPO model for SOA based on equation (2). We use following parameters in the simulations, with the order of magnitude chosen in typical semiconductor devices. The overlap factor between the nonplane optical mode and the cavity mode is assumed to be 1 (perfect mode matching). Then we follow typical values of QD device parameters by choosing the line-width enhancement factor  $\gamma = 0.5$ , the diffusion coefficient  $D = 0.8 \text{ cm}^2 \text{ s}^{-1}$ , the effective carrier lifetime  $t_s = 5 \text{ ns}$ , and the transparent carrier density  $N_0 = 1 \times 10^{18} \text{ cm}^{-3}$  [7–10]. The active region of our VCSEL is approximated by 10 nm in length, 10 nm in width, and a thickness of  $1.13 \mu\text{m}$  (estimated by a  $3\text{-}\lambda$  layer, with  $\lambda$  is the optical lasing wavelength). We also assume that the gain coefficient is  $\alpha = 2 \times 10^{-16} \text{ cm}^2$  and use  $n = 3.2$  as the effective refractive index for GaAs-based devices. Experimental operation of the VCSEL shows that the threshold current is  $I_{\text{th}} = 0.7 \text{ mA}$ , the small signal gain is  $g_0 = 5.53 \times 10^6 \text{ (m}^{-1}\text{)}$ , and the saturation power is  $P_s = 1.45 \times 10^{-8} \text{ W}$ . It can be seen clearly that there is a big discrepancy between the experimental data (dashed line)

and the simulations based on the traveling wave CPO model (solid line with square marks). Even though the CPO model can predict the tendency of the slow light effect for different modulation detunings, the thickness of the active region in our VCSEL is too short to provide enough gain to induce large delays.

The main difference between SOA and VCSEL devices is the cavity effect. In addition to the carrier rate equation in equation (1), we simplify the DBR cavity in the VCSELs by an effective Fabry–Perot filter with the response of the cavity gain described by [11],

$$G_r = \frac{(\sqrt{R_t} - \sqrt{R_b}g_s)^2 + 4\sqrt{R_t}\sqrt{R_b}g_s \sin^2 \phi}{(1 - \sqrt{R_t R_b}g_s)^2 + 4\sqrt{R_t}\sqrt{R_b}g_s \sin^2 \phi}, \quad (3)$$

where  $R_t$  is the top mirror reflectance,  $R_b$  is the bottom mirror reflectance,  $g_s$  is the single-pass gain, and  $\phi_s$  is the single-pass phase detuning.

With the same parameters as those listed above, the simulation results of the group delays with comparisons to the experimental data for different modulation frequency detunings are shown again in figure 2, as the solid line with triangle marks. Here the reflectances of the top and bottom mirrors are assumed to be  $R_t = 0.997$  and  $R_b = 0.99$ , respectively. We can see that by including the cavity effect, not only the tendency but also the values of the optical delay for different modulation frequencies at fixed probe signal power and bias current are both in agreement with the real experimental data. Moreover, the simulation results of the optical delay for different signal powers at different bias currents can fit the experimental observation well without adjusting any parameters, as shown in figure 3. The most important signature of our modeling is that the thickness of the active region is only  $1.13 \mu\text{m}$ . Without including the resonance effect through the cavity in our modeling, there is no possibility to have large group delays up to 65 ps in such a short semiconductor device.

#### 4. Discussion and conclusion

Based on a simple two-wave model and the carrier rate equation, we have consistent group delay behaviors in VCSELs as in the experimental data. We use a population pulsation modal for the SOA with additional introduction of the cavity effect by applying a Fabry–Perot filter in the theory. The simulation results of our proposed model agree well with experimental data for different operations of signal power, bias current and modulation frequency detuning with reasonable values of parameters. We also compare the simulation differences between the coherent population oscillation model with and without the cavity effect. Based on the experimental parameters of quantum dot VCSEL structures, we show that it is possible to have a 65 ps optical group delay within a compact active region as short as  $1.13 \mu\text{m}$ . Moreover, in this work the starting point of our theoretical model is based on the coherent population oscillation for SOA devices, and there is no lasing threshold condition here. From figure 3, one can see that

the agreement between theory and experiment is poor for the operation conditions at or below the threshold current, i.e.  $I = 0.7$  and  $0.6$  mA. Furthermore, in our experiments the injected optical signal is operated with the on-resonance condition to the VCSEL lasing wavelength. However in this case, one can adopt such a simple two-wave model as the standard one used for the SOA as the input signal is amplified by the VCSEL above the lasing threshold current. In general, when operated below the threshold current or in the off-resonance condition, the oscillator nature of the laser cavity is expected to make the measurement dramatically different. A more rigorous theoretical model for slow light in a semiconductor laser cavity is studied for further investigations.

It is also well known that an effective Fabry–Perot filter is not enough to describe the DBR cavity in VCSELs. Moreover, instead of the two traveling waves used in the population oscillation a standing cavity wave model should be adopted for VCSELs. And the significant difference between quantum well and quantum dot materials should be classified too. A complicated model is under investigation for a deep understanding of slow lights in QD VCSELs. But as a first step, we show that such a simple model can be used for such a compact optical slow light device at room temperature. We expect an increasing number of applications based on VCSELs for applications in light information storage as well as optics buffers to take place in the near future.

#### Acknowledgments

The authors are indebted to Professor Shun Lien Chuang for useful discussions. C-S Chou and R-K Lee are supported by the National Science Council of Taiwan with contracts NSC 95-2112-M-007-058-MY3 and NSC 95-2120-M-001-006.

#### References

- [1] Boyd R W, Gauthier D J and Gaeta A L 2006 *Opt. Photon. News* **19** 18
- [2] Harris S E 1997 *Phys. Today* **50** (7) 36
- [3] Hau L V, Harris S E, Dutton Z and Behroozi C H 1999 *Nature* **397** 594
- [4] Bigelow M S, Lepeshkin N N and Boyd R W 2003 *Science* **301** 200
- [5] Okawachi Y, Bigelow M S, Sharping J E, Zhu Z M, Schweinsberg A, Gauthier D J, Boyd R W and Gaeta A L 2005 *Phys. Rev. Lett.* **94** 153902
- [6] Sharping J E, Okawachi Y and Gaeta A L 2005 *Opt. Express* **13** 6092
- [7] Ku P C, Sedgwick F, Chang-Hasnain C J, Palinginis P, Li T, Wang H, Chang S W and Chuang S L 2004 *Opt. Lett.* **29** 2291
- [8] Su H and Chuang S L 2006 *Appl. Phys. Lett.* **88** 061102
- [9] Peng P C *et al* 2006 *Opt. Express* **14** 12880
- [10] Agrawal G P 1988 *J. Opt. Soc. Am. B* **5** 147
- [11] Laurand N, Calvez S and Dawson M D 2006 *Opt. Express* **14** 6858
- [12] Peng P C *et al* 2006 *Opt. Express* **14** 2944
- [13] Peng P C, Lin C T, Kuo H C, Lin G, Tsai W K, Yang H P, Lin K F, Chi J Y, Chi S and Wang S C 2006 *Electron. Lett.* **42** 1036