Journal of the Optical Society of America B
OPTICAL PHYSICS

Nonseparated states from squeezed dark-state polaritons in electromagnetically induced transparency media

You-Lin Chuang,[1] Ite A. Yu,[2] and Ray-Kuang Lee[1,2,*]

[1]Institute of Photonics Technologies, National Tsing Hua University, Hsinchu 300, Taiwan
[2]Department of Physics, National Tsing Hua University, Hsinchu 300, Taiwan
*Corresponding author: rklee@ee.nthu.edu.tw

Received 17 October 2014; revised 10 April 2015; accepted 17 April 2015; posted 29 April 2015 (Doc. ID 225151); published 15 June 2015

Within the frame of quantized dark-state polaritons in electromagnetically induced transparency media, noise fluctuations in the quadrature components are studied. Squeezed state transfer, quantum correlation, and noise entanglement between probe field and atomic polarization are demonstrated in single- and double-Λ configurations, respectively. Even though a larger degree of the squeezing parameter in the continuous variable helps to establish stronger quantum correlations, the inseparability criterion is satisfied only within a finite range of the squeezing parameter. The results obtained in the present study may be useful for guiding experimental realization of quantum memory devices for applications in quantum information and computation.

OCIS codes: (270.1670) Coherent optical effects; (020.1670) Coherent optical effects.

http://dx.doi.org/10.1364/JOSAB.32.001384

1. INTRODUCTION

It is believed that the quantum network has a greater potential than the classical one in providing many powerful applications for quantum information science [1,2]. Not only theoretical schemes [3–10], but also experimental implementations [11–14] are demonstrated in various systems, intending to manipulate and control the quantum objects. Among the candidates as quantum bits, photon, the quanta of light, is the fastest and robust carrier in the quantum network. For the storage and retrieval of optical information, an electromagnetically induced transparency (EIT) system serves as an ideal quantum interface between photon and atoms [15,16]. Based on quantum coherent interference, the profile as well as the phase of optical information are well controllable and perfectly preserved in the adiabatic condition [17,18]. Moreover, instead of using a classical light source, nonclassical states are also investigated in the EIT system, in order to map the quantum state of light onto atomic ensembles as a quantum memory device [19–23].

Recently, experimental progress includes the slowing down of squeezed vacuum pulses [24,25] and the storage of squeezed states for several microseconds [26–28]. Since the photon statistics of squeezed light differs from the Poisson distribution, a full quantum theory for the storage and retrieval of nonclassical light is needed. Based on the perturbed quantum fluctuations, for quasi-continuous wave inputs, EIT media become opaque for squeezed states, with an oscillatory transfer of the initial quantum properties between the probe and pump fields [29]. The entanglement in quantum fluctuation of electromagnetic fields is possible to be preserved or to be produced through an EIT medium [30]. Furthermore, through the picture of dark-state polaritons, quantum state transfer between optical pulse and atomic polarization is clearly illustrated during the storage and retrieval process [31–33].

In addition to the quantum state transfer, in this work, we introduce squeezed dark-state polaritons by the corresponding squeezed operator, and study the quantum correlation and entanglement of noise fluctuations in the quadrature components during the storage and retrieval process. As one may expect, when the squeezing parameter $r = 0$ (a coherent state), there is no quantum correlation between probe field and atomic polarization, while with a larger degree of the squeezing parameter, the stronger quantum correlation is established. In contrast, an inseparability criterion to guarantee an entanglement state is satisfied only with a finite range of the squeezing parameter. Extension to a double-Λ configuration is also studied, in order to reveal the conditions to have mutual entanglement among the noise correlations of two probe fields, and one common atomic polarization. With successful implementation on the storage and retrieval of light with nonclassical light sources, our results pave the way to implement the quantum interface between a photon and atomic system.

The remaining part of this paper is organized as follows. In Section 2, we start from the picture of quantized dark-state polaritons, and derive related quadrature variance in the noise fluctuations for the field and atomic operators. Quantum correlation and entanglement between field and atomic polarization operators during storage and the retrieval process is demonstrated. Especially, in Section 3, we address the inseparability condition in the continuous variables for the quadrature components of field and atomic operators in a single-Λ configuration. The generalization to a double-Λ configuration is extended in Section 4, where the quantum variances of two quantized probe fields and atomic polarization are shown. Finally, we give a brief conclusion in Section 5.

2. SQUEEZED DARK-STATE POLARITONS IN A SINGLE-Λ CONFIGURATION

We begin with the EIT system in a single-Λ configuration, as illustrated in Fig. 1. Here, two copropagating beams pass through a three-level atomic ensemble in the z direction, with the total number of atoms denoted by N. The probe field excites the transition from the state $|1\rangle$ to the state $|3\rangle$, which is treated by the quantum field operator $\hat{\mathcal{E}}(z,t)$ in the slowly varying envelope approximation. The transition between $|2\rangle$ and $|3\rangle$ is driven resonantly by a classical coupling field with the Rabi frequency denoted by $\Omega_c(t)$, which is a time-dependent function during the storage and retrieval process. In the Heisenberg picture, the interaction Hamiltonian for such a single-Λ EIT system is given as [15,16]

$$\hat{H} = -(\hbar g\hat{\sigma}_{31}\hat{\mathcal{E}} + \hbar\Omega_c\hat{\sigma}_{32} + \text{H.C.}), \tag{1}$$

where $\hat{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu|$ $(\mu,\nu = 1,2,3)$ is used as the collective atomic operator, the atom-field coupling strength for the transition $|1\rangle\leftrightarrow|3\rangle$ is denoted by the constant g, and H.C. represents the Hermitian conjugate.

It is known that with the low-intensity approximation, $\langle\hat{\sigma}_{11}\rangle \approx 1$, and adiabatic limit, $\hat{\sigma}_{12} \approx -\frac{g}{\Omega_c}\hat{\mathcal{E}}$, the propagation of quantum fields in EIT media can be described by the dark-state polariton [32,33],

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{\mathcal{E}}(z,t) - \sqrt{N}\sin\theta(t)\hat{\sigma}, \tag{2}$$

which is a linear superposition of field and atomic operators. Here, $\hat{\sigma} \equiv \hat{\sigma}_{12}$ is used for the atomic polarization between two lower states, $|1\rangle$ and $|2\rangle$. In general, the rules of commutation relation for bosonic fields $\hat{\mathcal{E}}$ and atomic polarization $\hat{\sigma}$ are different, i.e.,

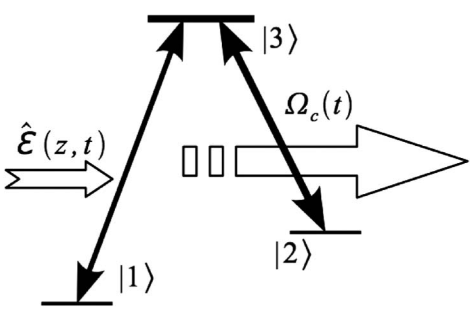

Fig. 1. EIT system considered in a single-Λ configuration, where the transitions $|1\rangle\leftrightarrow|3\rangle$ and $|2\rangle\leftrightarrow|3\rangle$ are driven resonantly by a quantized probe field, $\hat{\mathcal{E}}$, and a classical coupling field, denoted by its Rabi frequency Ω_c, separately.

$$[\hat{\mathcal{E}}(z,t), \hat{\mathcal{E}}^\dagger(z,t_1)] = T\delta(t-t_1), \tag{3}$$

$$[\hat{\sigma}(z,t), \hat{\sigma}^\dagger(z,t_1)] = -\frac{\hat{\sigma}_{22}-\hat{\sigma}_{11}}{N}T\delta(t-t_1), \tag{4}$$

where T is the characteristic time scale. Here, we have applied the *equal space* commutation relations and a single longitudinal mode for the field and atomic systems is used, too. However, if we assume that the atomic system is originally in the ground state, $\langle\hat{\sigma}_{22}-\hat{\sigma}_{11}\rangle \approx -1$ [34], this dark-state polariton $\hat{\Psi}(z,t)$ is a quasi-particle satisfying the Bosonic commutation relation

$$[\hat{\Psi}(z,t), \hat{\Psi}^\dagger(z,t_1)] \approx T\delta(t-t_1), \tag{5}$$

where the characteristic time scale T can be obtained by requiring $T^{-1}\int_0^T[\hat{\Psi}(z,t),\hat{\Psi}^\dagger(z,t_1)]\mathrm{d}t = 1$. However, the commutation relations between the dark-state polariton and field (atomic) operators are

$$[\hat{\Psi}(z,t), \hat{\mathcal{E}}^\dagger(z',t_1)] = \cos\theta(t)T\delta(t-t_1), \tag{6}$$

$$[\hat{\Psi}(z,t), \hat{\sigma}^\dagger(z',t_1)] = -\frac{\sin\theta(t)}{\sqrt{N}}T\delta(t-t_1). \tag{7}$$

Under the picture of dark-state polaritons, the governing equation of motion during the storage and retrieval process is

$$\left[\frac{\partial}{\partial t} + v_g(t)\frac{\partial}{\partial z}\right]\hat{\Psi}(z,t) = 0, \tag{8}$$

where the group velocity of dark-state polartion is given by $v_g(t) = c\cos^2\theta(t)$, with the speed of light in the vacuum c, and $\theta(t) = \tan^{-1}[g\sqrt{N}/\Omega_c(t)]$ accounts the mixing angle as a function of time. Without any decay mechanism, the evolution of a dark-state polariton is described by changing the value of $\Omega_c(t)$ with respect to the time. When $\theta(t) = 0$, or $\Omega_c(t)/g \to \infty$, the dark-state polariton is said to be a *photon-like* state, i.e., $\hat{\Psi} = \hat{\mathcal{E}}$; while $\theta(t) = \pi/2$, or $\Omega_c(t)/g = 0$, the dark-state polariton is an *atom-like* state, i.e., $\hat{\Psi} = -\sqrt{N}\hat{\sigma}$.

Based on the quantized polariton field operator, $\hat{\Psi}(z,t)$, in the following we introduce the squeezed state for dark-state polaritons by defining a squeezing operator $\hat{S}(\xi)$:

$$\hat{S}(\xi) = \exp\left(\frac{\xi^*}{2}\int_0^T\hat{\Psi}^2\mathrm{d}t - \frac{\xi}{2}\int_0^T\hat{\Psi}^{\dagger 2}\mathrm{d}t\right), \tag{9}$$

where $\xi = re^{i\delta}$ denotes the degree of noise squeezing, with the squeezing parameter $r = |\xi|$ and the related squeezing angle δ. The corresponding squeezed vacuum state for a dark-state polariton is represented in the basis of $|\psi\rangle_{\text{in}} = \hat{S}(\xi)|0\rangle$, which is composited by the vacuum state of fields and ground state of atomic polarization, i.e., $|0\rangle = |0\rangle_{\text{field}} \otimes |1\rangle_{\text{atom}}$. If $\theta = 0$, the squeezing operator is $\hat{S}(\xi) = \exp(\frac{\xi^*}{2}\int_0^T \mathrm{d}t\hat{\mathcal{E}}^2 - \frac{\xi}{2}\int_0^T \mathrm{d}t\hat{\mathcal{E}}^{\dagger 2})$ and our initial state is $|\psi\rangle_{\text{in}} = |\xi\rangle_{\text{field}}|1\rangle_{\text{atom}}$, which consists of an initial squeezed field and all the atoms being in ground state $|1\rangle$. On the other hand, if $\theta = \pi/2$, the squeezing operator is $\hat{S}(\xi) = \exp(\frac{N\xi^*}{2}\int_0^T \mathrm{d}t\hat{\sigma}^2 - \frac{N\xi}{2}\int_0^T \mathrm{d}t\hat{\sigma}^{\dagger 2})$ and our initial state becomes $|\psi\rangle_{\text{in}} = |0\rangle_{\text{field}}|\xi\rangle_{\text{atom}}$, which means that the field state is just the vacuum state and the atomic polarization state becomes the spin-squeezed state. With this squeezing operator $\hat{S}(\xi)$ working on the bare state $|0\rangle$, we can consider all the physical situations in our cases.

In practice, it would be much easier to generate the squeezed dark-state polariton by preparing a squeezed field with all the atoms in the ground state, i.e., by setting $\theta = 0$. As for the question on how to prepare the squeezed dark-state polariton in general, it is an interesting and important issue [35–41]. Even though the squeezed operator $\hat{S}$ introduced in Eq. (9) is quadratic, however, it is related to the generation of squeezed light in the nonlinear process. In our scenario, we do not take this generation process into consideration, since the main purpose of this work is on the transfer between field and atomic polarization.

For the quantum noises in continuous variables, we have the related quadrature operator as $\hat{X}_\Psi = \hat{\Psi} + \hat{\Psi}^\dagger$ for the amplitude (in-phase) fluctuations. With above definitions, the quadrature variance of dark-state polaritons is found to be

$$\begin{aligned}\Delta X_\Psi^2 &\equiv {}_{\rm in}\langle\psi|\Delta\hat{X}_\Psi^2|\psi\rangle_{\rm in}, \\ &= \cos^2\theta(t)\Delta X_\mathcal{E}^2 + N\sin^2\theta(t)\Delta X_\sigma^2 \\ &\quad - \sqrt{N}\sin\theta(t)\cos\theta(t)[\langle\hat{X}_\mathcal{E}\hat{X}_\sigma\rangle + \langle\hat{X}_\sigma\hat{X}_\mathcal{E}\rangle]. \end{aligned} \tag{10}$$

Here, the first and second terms, i.e., $\Delta X_\mathcal{E}^2 \equiv {}_{\rm in}\langle\psi|\Delta\hat{X}_\mathcal{E}^2|\psi\rangle_{\rm in}$ and $\Delta X_\sigma^2 \equiv {}_{\rm in}\langle\psi|\Delta\hat{X}_\sigma^2|\psi\rangle_{\rm in}$, are the corresponding quadrature variances of field and atomic parts, with the in-phase quadrature components $\hat{X}_\mathcal{E} = \hat{\mathcal{E}} + \hat{\mathcal{E}}^\dagger$ and $\hat{X}_\sigma = \hat{\sigma} + \hat{\sigma}^\dagger$ defined for the field and atomic operators, respectively. It can be seen that for a dark-state polariton, the quadrature variances of field and atomic operators are added together by the time-dependent coefficient $\theta(t)$, or $\Omega_c(t)$, during the storage and retrieval process. Furthermore, we also have contributions from the correlation between the field and atomic ensemble, i.e., $\langle\hat{X}_\mathcal{E}\hat{X}_\sigma\rangle$ and $\langle\hat{X}_\sigma\hat{X}_\mathcal{E}\rangle$.

Consider the possible experimental demonstration, where one can use a squeezed light source for the probe field. For a given initial quadrature variance, $\Delta X_\Psi^2(t=0) \equiv \Delta X_{\rm in}^2$, we can manipulate the distribution of quantum noise fluctuations between the field and atomic parts. That is,

$$\Delta X_\mathcal{E}^2 = \frac{\left(\frac{\Omega_c}{g}\right)^2(\Delta X_{\rm in}^2) + N}{\left(\frac{\Omega_c}{g}\right)^2 + N}, \tag{11}$$

$$\Delta X_\sigma^2 = \frac{1}{N}\left[\frac{\left(\frac{\Omega_c}{g}\right)^2 + N(\Delta X_{\rm in}^2)}{\left(\frac{\Omega_c}{g}\right)^2 + N}\right], \tag{12}$$

which can be obtained by substituting Eq. (2) into Eq. (10) along with the commutation relations shown in Eqs. (6) and (7). One can see that in the limit $\Omega_c/g \to \infty$, we have $\Delta X_\mathcal{E}^2 = \Delta X_{\rm in}^2$ and $\Delta X_\sigma^2 = 1/N$ for a photon-like dark-state polariton. In the other limit, $\Omega_c/g = 0$, the noise fluctuations for an atom-like dark-state polariton are $\Delta X_\mathcal{E}^2 = 1$ and $\Delta X_\sigma^2 = \Delta X_{\rm in}^2/N$.

Consider typical experimental conditions in the realization of EIT phenomena, such as the systems of cold ^{87}Rb atoms, we have $(\Omega_c/g)^2 \ll N$. In this scenario, since the dark-state polartion is in the atom-like state, the quantum fluctuation of a dark-state polariton is dominated by the atomic quadrature variance. With this condition, the quadrature variances can be approximated by

$$\Delta X_\mathcal{E}^2 \simeq 1 - \frac{1}{N}\left(\frac{\Omega_c}{g}\right)^2(1 - \Delta X_{\rm in}^2), \tag{13}$$

$$\Delta X_\sigma^2 \simeq \frac{1}{N}\left[\Delta X_{\rm in}^2 + \frac{1}{N}\left(\frac{\Omega_c}{g}\right)^2(1 - \Delta X_{\rm in}^2)\right]. \tag{14}$$

We want to remark that when the initial quadrature of the input state is a coherent state, i.e., $\Delta X_{\rm in}^2 = 1$, the noise variance in the field component remains the same as that of vacuum states, while the quantum fluctuation in the atomic component corresponds to that of a spin coherent state. That is, when a coherent state is used as the input (a classical light source), both the quantum noise variance in the field and atomic components are independent from the value of control field, $\Omega_c(t)$.

Naively, one may take an EIT media as a linear system and expect a complete transfer for the nonclassical properties from input field to the atomic system under the picture of dark-state polaritons. Nevertheless, due to the last term in Eq. (10), nontrivial quantum correlations between the field and atomic operators will be shown through the quantum noise squeezing. Here, the quantum correlation between field and atomic components of a dark-state polartion has the form

$$\langle\hat{X}_\mathcal{E}\hat{X}_\sigma\rangle = \langle\hat{X}_\sigma\hat{X}_\mathcal{E}\rangle = \frac{\Omega_c/g}{(\Omega_c/g)^2 + N}(1 - \Delta X_{\rm in}^2), \tag{15}$$

which can be obtained through the commutation relations between the dark-state polariton and field (atomic) operators, shown in Eqs. (6) and (7). Again, for an initial coherent state, $\Delta X_{\rm in}^2 = 1$, the quantum correlation between field and atomic operators is zero. In Fig. 2, we show the quantum correlation between the field and atomic polarization, $\langle\hat{X}_\mathcal{E}\hat{X}_\sigma\rangle$, as a function of the normalized control field, Ω_c/g, for different values of the squeezing parameter, r. As expected, the more nonclassical properties there are, with a larger value of the squeezing parameter r, the stronger quantum correlation is. Moreover, when the dark-state polariton can be approximated by a photon-like state ($\Omega_c/g \to \infty$) or an atom-like state ($\Omega_c/g = 0$), the quantum correlation becomes zero as well. That is, the quantum correlation exists only with a superposition of partial-photon and partial-atom states.

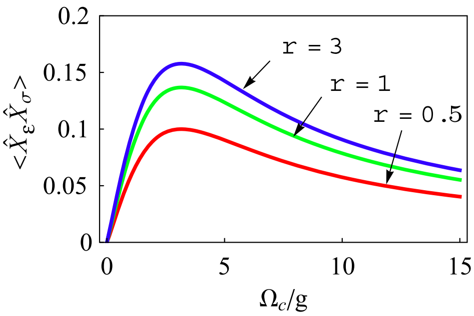

Fig. 2. Quantum correlation between the field and atomic polarization, $\langle\hat{X}_\mathcal{E}\hat{X}_\sigma\rangle$, shown as a function of the normalized control field, Ω_c/g, for different values of the squeezing parameter, r. Here, the number of atoms is fixed at $N = 10$.

3. INSEPARABILITY CONDITION FOR SQUEEZED DARK-STATE POLARITONS

Next, we study the entanglement between the quadrature components of a field and atomic ensemble, by using the inseparability criterion for bipartite continuous variables [42,43]. Only when the following inequality is satisfied, a bipartite system is said to be entangled,

$$I_c \equiv \Delta(\hat{X}_{\mathcal{E}} - \hat{X}_\sigma)^2 + \Delta(\hat{Y}_{\mathcal{E}} + \hat{Y}_\sigma)^2 < 2 + \frac{2}{N}, \tag{16}$$

where $\hat{X}_{\mathcal{E}}$ and $\hat{X}_\sigma$ correspond to the in-phase quadrature components, while $\hat{Y}_{\mathcal{E}} \equiv -i(\hat{\mathcal{E}} - \hat{\mathcal{E}}^\dagger)$ and $\hat{Y}_\sigma \equiv -i(\hat{\sigma} - \hat{\sigma}^\dagger)$ are the out-of-phase quadrature operators for field and atomic operators, respectively. To have a clear comparison, we normalize this inseparability criterion, i.e.,

$$\mathcal{I}_c \equiv \frac{I_c}{2 + \frac{2}{N}} \equiv 1 + \frac{2}{1 + \frac{1}{N}} \left[\frac{\mathcal{F}(\Omega_c/g, r, \delta)}{\frac{\Omega_c^2}{g^2} + N} \right] < 1, \tag{17}$$

where the numerator in the brackets in Eq. (17) is defined as

$$\mathcal{F}(\Omega_c/g, r, \delta) \equiv \left(\frac{\Omega_c^2}{g^2} + 1 \right) \sinh^2 r - 2 \frac{\Omega_c}{g} \sinh r \cosh r \cos \delta. \tag{18}$$

The nonseparation condition is guaranteed only when $\mathcal{F}(\Omega_c/g, r, \delta)$ is smaller than 0. According to the inseparability condition shown in Eq. (18), it is obvious that for a coherent state at the input, $r = 0$, we do not have nonseparated states, i.e., $\mathcal{F} = 0$, no matter what the value of the control field $\Omega_c(t)$ is. Moreover, the existence of entanglement is independent from the number of atoms, N, despite that the value of $\mathcal{I}_c$ (I_c) changes with the number of atoms.

In order to demonstrate the inseparability condition, in Fig. 3(a), we show the surface obtained by requiring the function $\mathcal{F}(\Omega_c/g, r, \delta)$ in Eq. (18) to be zero, which gives the border between separated and nonseparated states. Here, the parameter space is expanded by the normalized control field Ω_c/g, the degree of the squeezing parameter r, and the related squeezing angle δ. Only the colored region, beneath the surface but above the plane $r = 0$, supports the nonseparated states from squeezed dark-state polaritons during the storage and retrieval process.

To give a clear illustration, we project the parameter space satisfying the inseparability condition into the planes of $(\Omega_c/g, \delta)$, (δ, r), and $(\Omega_c/g, r)$ in Figs. 3(b), 3(c), and 3(d), respectively. As the same scenario in the quantum correlation between the field and atomic polarization shown in Fig. 2, it can be seen in Fig. 3(b) that the nonseparated states (the colored regions) are also not supported when $\Omega_c/g = 0$ or $\Omega_c/g \to \infty$. Moreover, these nonseparated states are measured dominantly along the angle $\delta = 0$, as shown in Fig. 3(c), due to the reason that we have assumed the phase difference between the field and atomic operators is zero. However, as shown in Fig. 3(d), the nonseparated states are only supported within a finite range of the squeezing parameter, r. Counterintuitively, for a larger degree of the squeezing parameter, which is believed to possess more nonclassical properties, the corresponding inseparability criterion happens to be invalid.

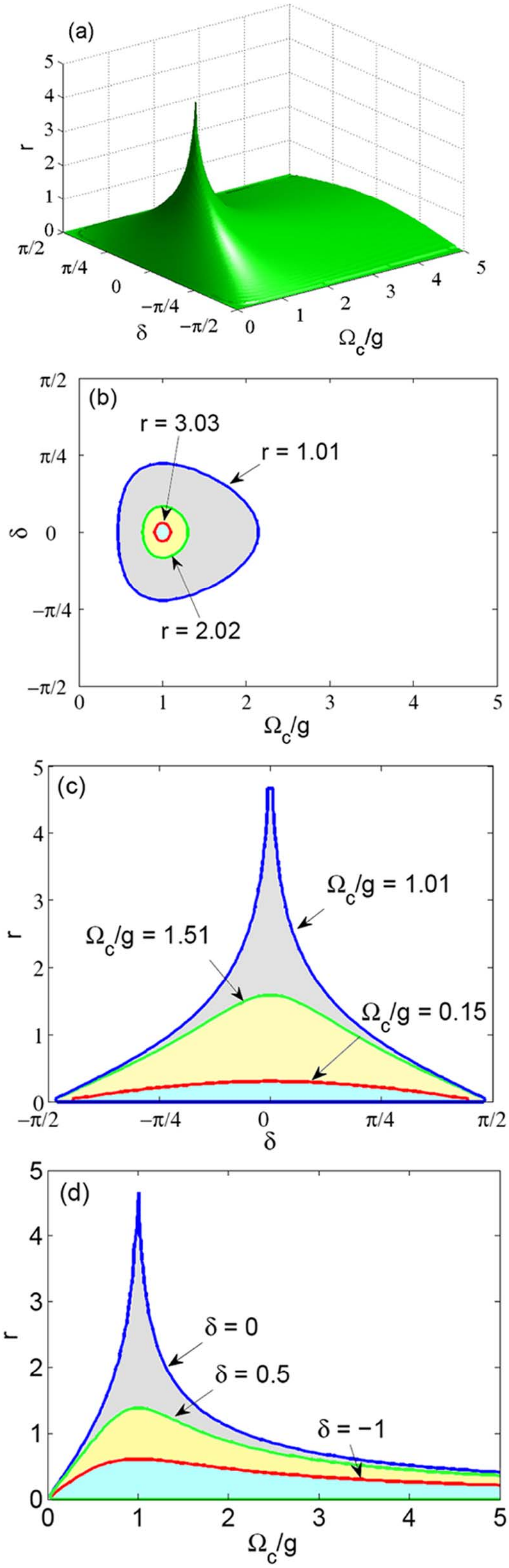

Fig. 3. (a) Surface for the inseparability condition, defined by requiring the function $\mathcal{F}(\Omega_c/g, r, \delta) = 0$, shown in Eq. (18). The parameter space is expanded by the normalized control field Ω_c/g, the degree of the squeezing parameter r, and the squeezing angle δ. The contour plots are obtained by projecting the surface into the plane of (b) $(\Omega_c/g, \delta)$, (c) (δ, r), and (d) $(\Omega_c/g, r)$, respectively, with the other parameter shown in the markers. The colored regions indicate the parameter space that $\mathcal{F}(\Omega_c/g, r, \delta) < 0$.

The reason why only a finite range of the squeezing parameter supports nonseparated states can be illustrated in the following way. In terms of the quadrature variances, the inseparability criterion in Eq. (16) can be rewritten as

$$
\begin{aligned}
I_c - \left(2 + \frac{2}{N}\right) &= [\Delta X_{\mathcal{E}}^2 + \Delta Y_{\mathcal{E}}^2 + \Delta X_{\sigma}^2 + \Delta Y_{\sigma}^2] - \left[\left(2 + \frac{2}{N}\right)\right] \\
&\quad -2[\langle \hat{X}_{\mathcal{E}} \hat{X}_{\sigma}\rangle - \langle \hat{Y}_{\mathcal{E}} \hat{Y}_{\sigma}\rangle] \\
&\equiv V - V_{\mathrm{CS}} - C < 0. \qquad (19)
\end{aligned}
$$

From the above expansion, it can be seen that to satisfy the inseparability criterion, we have competing terms in Eq. (19). They correspond to the sum of total variances of field and atomic fluctuations both in the in-phase and out-of-phase quadratures, $V \equiv \Delta X_{\mathcal{E}}^2 + \Delta Y_{\mathcal{E}}^2 + \Delta X_{\sigma}^2 + \Delta Y_{\sigma}^2$, the sum of variance for the coherent photon and coherent atomic states, $V_{\mathrm{CS}} \equiv 2 + 2/N$, and the difference in the quantum correlations between them in two orthogonal quadratures, $C \equiv 2\langle \hat{X}_{\mathcal{E}} \hat{X}_{\sigma}\rangle - 2\langle \hat{Y}_{\mathcal{E}} \hat{Y}_{\sigma}\rangle$. For a coherent state, the last term is zero, $C = 0$, for there is no quantum correlation existed. As a result, we do not have nonseparated states with an input of coherent states. Nevertheless, a nonclassical state cannot always ensure the inseparability. In Fig. 4, we plot the curves for $V - V_{\mathrm{CS}}$ and C, as a function of the squeezing parameter, r. From Fig. 4, we can see that the entanglement can only happen when the quantum correlations between field and atomic fluctuations are stronger than the total sum of quadrature variances, i.e., $C > (V - V_{\mathrm{CS}})$.

4. SQUEEZED DARK-STATE POLARITONS IN A DOUBLE-Λ CONFIGURATION

In the single-Λ configuration discussed above, quadrature fluctuations between the field and atomic parts can be entangled within some parameter space. However, in the practical experimental setup, one may need to measure both the quantum noise fluctuations of the probe field as well as the variance of the atomic ensemble, via homodyne detection schemes. Due to the difficulties in measuring the collective atomic operators, here, we extend the concept of dark-state polaritons from a single-Λ configuration to a double-Λ one, in order to have possible experimental realizations with the output fields arriving at a detection apparatus. As illustrated in Fig. 5, now we have two quantized probe fields, $\hat{\mathcal{E}}_1$ and $\hat{\mathcal{E}}_2$, driving resonantly to the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$, with the corresponding coupling strengths g_1 and g_2, respectively. At the same time, two classical coupling fields, denoted by its Rabi frequency $\Omega_1(t)$ and $\Omega_2(t)$, drive the transitions $|2\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, simultaneously.

Due to the share of a common atomic polarization, $\hat{\sigma}_{12}$, we can extend the concept of dark-state polaritons to describe the storage and retrieval process in such a double-Λ configuration [44,45]. In this picture, the corresponding quantized dark-state polariton, $\hat{\Psi}$, is composited by two probe field operators, $\hat{\mathcal{E}}_1$ and $\hat{\mathcal{E}}_2$, and the atomic polarization operator, $\hat{\sigma} \equiv \hat{\sigma}_{12}$, i.e.,

$$\hat{\Psi} = \cos\theta \cos\phi \hat{\mathcal{E}}_1 + \cos\theta \sin\phi \hat{\mathcal{E}}_2 - \sqrt{N} \sin\theta \hat{\sigma}, \qquad (20)$$

with the mixing angles $\theta(t)$, between the field and atomic polarization, and $\phi(t)$, between two probe fields defined as

$$
\begin{aligned}
\theta(t) &\equiv \tan^{-1}\left[\frac{g_1\sqrt{N}}{\Omega_1}\left(1 + \frac{g_1^2 \Omega_2^2}{g_2^2 \Omega_1^2}\right)^{-1/2}\right], \\
\phi(t) &\equiv \tan^{-1}(g_1 \Omega_2 / g_2 \Omega_1).
\end{aligned}
$$

With the same concept for the squeezed operator introduced in Eq. (9), the corresponding squeezed state of dark-state polaritons in a double-Λ configuration is defined as $|\xi\rangle \equiv \hat{S}|0\rangle \equiv |0\rangle_{\mathcal{E}_1} \otimes |0\rangle_{\mathcal{E}_2} \otimes |1\rangle_{\mathrm{atom}}$. The quadrature variance of this dark-state polariton is found to be

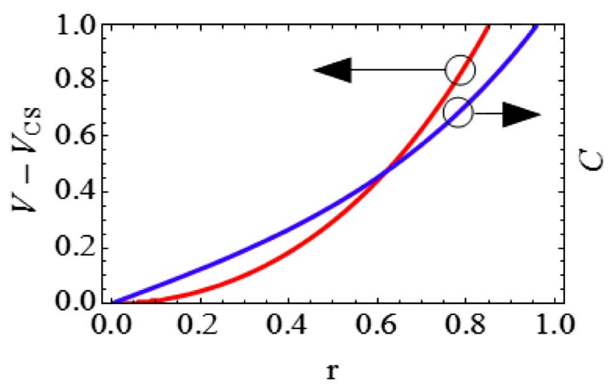

Fig. 4. Competition between the terms $(V - V_{\mathrm{SC}})$ and C in the inseparability criterion, shown in Eq. (19). Here, $V = \Delta X_{\mathcal{E}}^2 + \Delta Y_{\mathcal{E}}^2 + \Delta X_{\sigma}^2 + \Delta Y_{\sigma}^2$ and $V_{\mathrm{CS}} = 2 + 2/N$ are the sum of quadrature variances in the bipartite system and coherent states, respectively, while $C = 2\langle \hat{X}_{\mathcal{E}} \hat{X}_{\sigma}\rangle - 2\langle \hat{Y}_{\mathcal{E}} \hat{Y}_{\sigma}\rangle$ denotes the difference in the quantum correlations between two orthogonal quadratures. Other parameters used are $\Omega_c/g = 0.3$ and $\delta = 0$.

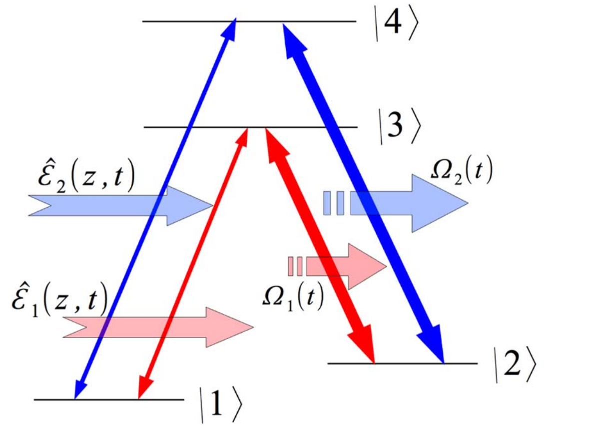

Fig. 5. EIT system considered in a double-Λ configuration, where the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$ are driven resonantly by two quantized probe fields, $\hat{\mathcal{E}}_1$ and $\hat{\mathcal{E}}_2$, while two classical coupling fields, denoted by its Rabi frequency Ω_1 and Ω_2 drive the transitions $|2\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$, simultaneously.

$$\Delta X_\Psi^2 = N\sin^2\theta(t)\Delta X_\sigma^2 + \cos^2\theta(t)\cos^2\phi(t)\Delta X_1^2 + \cos^2\theta(t)\sin^2\phi(t)\Delta X_2^2 + \cos\theta(t)\cos\phi(t)\sin\phi(t)[\langle\hat{X}_1\hat{X}_2\rangle + \langle\hat{X}_2\hat{X}_1\rangle] - \sqrt{N}\sin\theta(t)\cos\theta(t)\cos\phi(t)[\langle\hat{X}_1\hat{X}_\sigma\rangle + \langle\hat{X}_\sigma\hat{X}_1\rangle] - \sqrt{N}\sin\theta(t)\cos\theta(t)\sin\phi(t)[\langle\hat{X}_2\hat{X}_\sigma\rangle + \langle\hat{X}_\sigma\hat{X}_2\rangle], \quad (21)$$

where $\hat{X}_i \equiv \hat{\mathcal{E}}_i + \hat{\mathcal{E}}_i^\dagger$, $i = 1, 2$, denotes the in-phase quadrature component of the probe field, $\hat{\mathcal{E}}_i$. Again, for a given initial noise variance in the in-phase quadrature component, $\Delta X_\Psi^2(t=0) \equiv \Delta X_{\text{in}}^2$, the corresponding partition of noise variances in the quadrature components for two probe fields and atomic polarization operators are

$$\Delta\hat{X}_1^2 = \frac{\left(\frac{\Omega_1}{g_1}\right)^2\Delta X_{\text{in}}^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}, \quad (22)$$

$$\Delta\hat{X}_2^2 = \frac{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2\Delta X_{\text{in}}^2 + N}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}, \quad (23)$$

$$\Delta\hat{X}_\sigma^2 = \frac{1}{N}\left[\frac{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N\Delta X_{\text{in}}^2}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}\right]. \quad (24)$$

The quantum correlation between each probe field, $\hat{\mathcal{E}}_i$, and the atomic components has the form

$$\langle\hat{X}_i\hat{X}_\sigma\rangle = \frac{\Omega_i/g_i}{(\Omega_1/g)^2 + (\Omega_2/g)^2 + N}[1 - \Delta X_{\text{in}}^2], \quad (25)$$

which shares a similar formula as that in a single-Λ configuration shown in Eq. (15), except for the addition terms from two prob fields in the denominator. The quantum correlation between the quadrature components of two fields is found to have the form

$$\langle\hat{X}_1\hat{X}_2\rangle = \frac{\left(\frac{\Omega_1}{g_1}\right)\left(\frac{\Omega_2}{g_2}\right)}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}[\Delta X_{\text{in}}^2 - 1]. \quad (26)$$

In Fig. 5, we show the quantum correlation between two probe fields, $\langle\hat{X}_1\hat{X}_2\rangle$, as a function of the normalized control fields, Ω_i/g_i, $i = 1, 2$, for different values of the squeezing parameter, r. When the second probe field is fixed as a constant, for example $\Omega_2/g_2 = 1$, the quantum correlation between two probe-field vanishes as $\Omega_1/g_1 = 0$ or $\Omega_1/g_1 \to \infty$. Moreover, due to the phase shift, π, defined for the dark-state polariton in Eq. (20), the correlation between two probe fields is negative (anticorrelated).

Below, we show the normalized inseparability criterion for the mutual entanglement among the two probe fields and atomic polarization, denoted as $\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2)$, $\mathcal{I}_c(\mathcal{E}_1, \sigma)$, and $\mathcal{I}_c(\mathcal{E}_2, \sigma)$, respectively,

$$\mathcal{I}_c(\mathcal{E}_i, \sigma) = 1 + \left(\frac{2}{1 + 1/N}\right)\left[\frac{\mathcal{G}(\Omega_i/g_i, r, \delta)}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}\right] < 1, \quad (27)$$

$$\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2) = 1 + \left(\frac{2}{1 + 1}\right)\left[\frac{\mathcal{H}(\Omega_1/g_1, \Omega_2/g_2, r, \delta)}{\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2 + N}\right] < 1, \quad (28)$$

where

$$\mathcal{G}(\Omega_i/g_i, r, \delta) \equiv \left[\left(\frac{\Omega_i}{g_i}\right)^2 + 1\right]\sinh^2 r - 2\frac{\Omega_i}{g_i}\sinh r\cosh r\cos\delta, \quad (29)$$

$$\mathcal{H}(\Omega_1/g_1, \Omega_2/g_2, r, \delta) \equiv \left[\left(\frac{\Omega_1}{g_1}\right)^2 + \left(\frac{\Omega_2}{g_2}\right)^2\right]\sinh^2 r + 2\frac{\Omega_1}{g_1}\frac{\Omega_2}{g_2}\sinh r\cosh r\cos\delta. \quad (30)$$

For a given squeezing degree r, we can immediately find the parameter space to satisfy the inseparability condition to ensure entanglement.

To access these nonclassical properties at the output of atomic ensembles, one can measure the correlations between two probe fields through a homodyne detection. In this scenario, we show in Fig. 6 the conditions to generate entanglement in the two probe fields, while only one of the input probe fields needs to have nonclassical properties. The entanglement is achieved through the collective atoms. Moreover, in Fig. 7, the condition to have entanglement between two probe fields is revealed as a function of two control fields, Ω_1 and Ω_2. By requiring $\mathcal{H} < 0$ in Eq. (30), we have the following inseparability condition for two probe fields that are bounded by two curves:

$$\left(\frac{\Omega_2}{g_2}\right) - (A_-)^{-1}\left(\frac{\Omega_1}{g_1}\right) = 0, \quad (31)$$

$$\left(\frac{\Omega_2}{g_2}\right) - (A_+)^{-1}\left(\frac{\Omega_1}{g_1}\right) = 0, \quad (32)$$

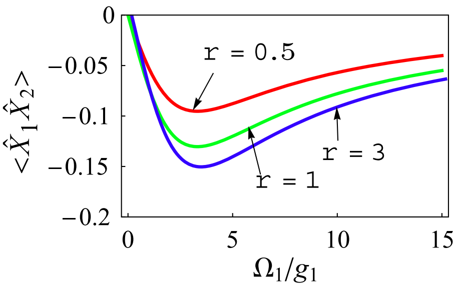

Fig. 6. Quantum correlation between two probe fields in a double-Λ configuration, $\langle\hat{X}_1\hat{X}_2\rangle$, shown as a function of the normalized control field, Ω_1/g_1, for different values of the squeezing parameter, r. Here, the other parameters used are $\Omega_2/g_2 = 1$ and $N = 10$.

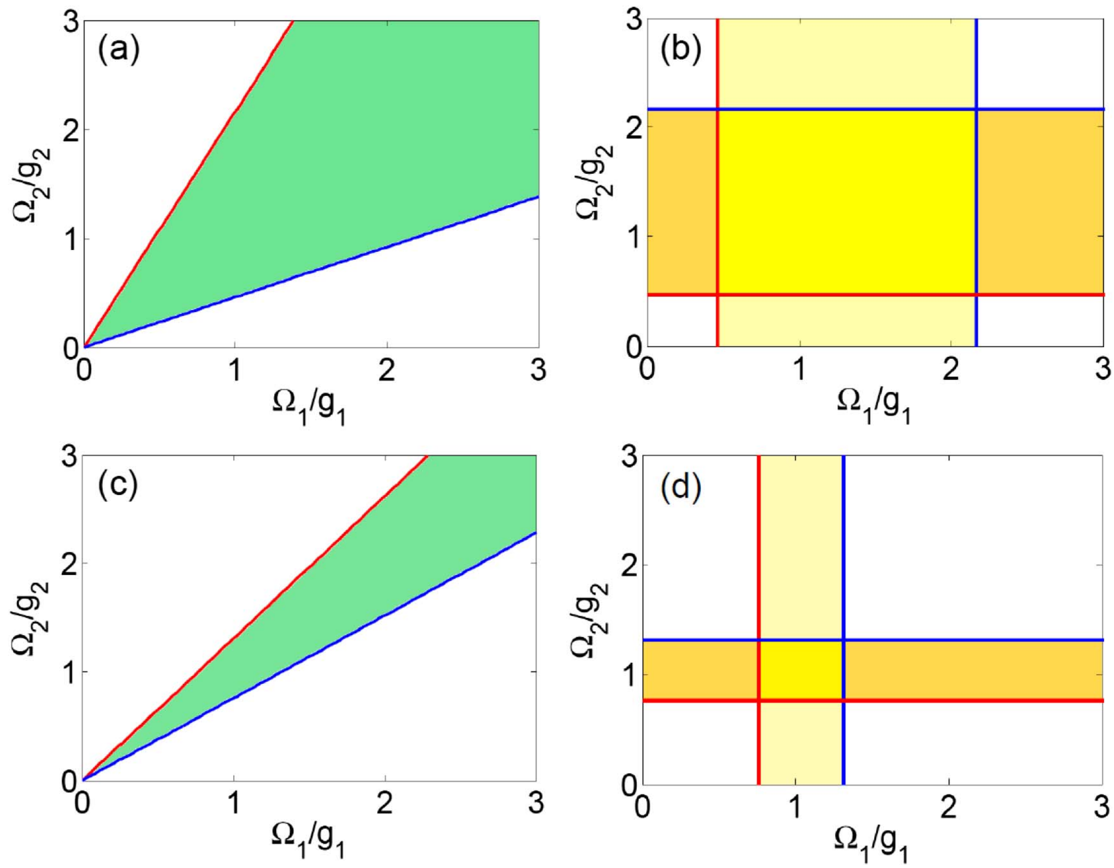

Fig. 7. Regions to support nonseparated states between field–field and field–atom quadrature components, i.e., $\mathcal{I}_c(\mathcal{E}_i, \sigma)$ and $\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2)$, as a function of the normalized control fields. (a) $\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2) < 0$ at $r = 1$, $\delta = \pi$. (b) $\mathcal{I}_c(\mathcal{E}_i, \sigma) < 0$ at $r = 1$, $\delta = 0$. (c) $\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2) < 0$ at $r = 2$, $\delta = \pi$. (d) $\mathcal{I}_c(\mathcal{E}_i, \sigma) < 0$ at $r = 2$, $\delta = 0$.

with $A_\pm \equiv -\coth(r)\cos\delta \pm \sqrt{\coth^2 r \cos^2\delta - 1}$. These two curves are plotted in red and blue colors, shown in Fig. 7(a). The colored region within these two curves is the parameter space to have entangled probe fields, i.e., $\mathcal{I}_c(\mathcal{E}_1, \mathcal{E}_2) < 0$. In addition to $r = 1$, we also show the region to support field–field entanglement for the squeezing parameter $r = 2$ in Fig. 7(c). As the same scenario in a single-Λ configuration, only a finite range of the squeezing parameter supports nonseparable states.

Beside direct measurement on the quantum fluctuations in two probe fields in the output, in this double-Λ scheme, we can also infer the nonseparability between one of the probe fields and collective atomic excitations indirectly. In Fig. 7(b), we demonstrate the entanglement regions for these two probe fields and field–atomic ensembles. By requiring $\mathcal{G} < 0$ in Eq. (29), the criterion to have entanglement between the output probe field, $\mathcal{E}_1$ or $\mathcal{E}_2$, and the atomic ensemble, i.e., $\mathcal{I}_c(\mathcal{E}_i, \sigma) < 0$, can be achieved when the Rabi frequencies of coupling fields fall in between

$$B_- < \Omega_i/g_i < B_+, \quad (33)$$

where $B_\pm \equiv \coth(r)\cos\delta \pm \sqrt{\coth^2 r \cos^2\delta - 1}$. In this way, one can measure the output probe fields through a state-of-the-art quantum detection scheme, which is readily and reliably realized in presently available systems.

However, in terms of the quantized operators, $\hat{\mathcal{E}}_1$, $\hat{\mathcal{E}}_2$, and $\hat{\sigma}$, we can take such a double-Λ configuration as a tripartite system. From the inseparability criterion for field–atom and field–field quadrature components given in Eqs. (27) and (28), it requires that both $\mathcal{G}(\Omega_i/g_i, r, \delta)$ and $\mathcal{H}(\Omega_i/g_i, r, \delta)$ must be negative values simultaneously, in order to have a tripartite entanglement. It is the phase difference between field and atomic components in the definition of a dark-state polariton shown in Eq. (20), which automatically results in a π phase shift in the squeezing angle. In such a double-Λ configuration, it is impossible to support the coexistence of mutual entanglements among field–field and field–atom simultaneously for this tripartite system.

5. CONCLUSION

In summary, we have introduced the squeezed operator for dark-state polaritons in EIT media, including single- and double-Λ configurations. We show that quantum squeezed state transfer from a field to atomic ensemble can be achieved by a time-dependent coupling field, and reveal the quantum correlation and noise entanglement between probe field and atomic polarization. Even though a larger degree of the squeezing parameter in the quadrature components helps to establish stronger quantum correlations, the inseparability criterion is satisfied only within a finite range of the squeezing parameter. The results in our work provide the possible condition to implement the quantum interface between a photon and atomic system.

Ministry of Science and Technology, Taiwan; National Center of Theoretical Science, Taiwan.

We thank Michael Fleischhauer, Gediminas Juzeliunas, Yinchieh Lai, and Yong-Fan Chen for useful discussions.

REFERENCES

1. L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, "Long-distance quantum communication with atomic ensembles and linear optics," Nature **414**, 413–418 (2001).
2. A. Kuzmich, W. P. Bowen, A. D. Boozer, A. Boca, C. W. Chou, L.-M. Duan, and H. J. Kimble, "Generation of nonclassical photon pairs for scalable quantum communication with atomic ensembles," Nature **423**, 731–734 (2003).
3. S. J. van Enk, J. I. Cirac, and P. Zoller, "Photonic channels for quantum communication," Science **279**, 205–208 (1998).
4. H.-J. Briegel, W. Dur, J. I. Cirac, and P. Zoller, "Quantum repeaters: the role of imperfect local operations in quantum communication," Phys. Rev. Lett. **81**, 5932–5935 (1998).
5. Z. Kurucz and M. Fleischhauer, "Continuous-variable versus electromagnetically-induced-transparency-based quantum memories," Phys. Rev. A **78**, 023805 (2008).
6. T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, "Decoherence, continuous observation, and quantum computing: a cavity QED model," Phys. Rev. Lett. **75**, 3788–3791 (1995).
7. J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, "Quantum state transfer and entanglement distribution among distant nodes in a quantum network," Phys. Rev. Lett. **78**, 3221–3224 (1997).
8. A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, "Quantum information processing using quantum dot spins and cavity QED," Phys. Rev. Lett. **83**, 4204–4207 (1999).
9. A. I. Lvovsky, B. C. Sanders, and W. Tittel, "Optical quantum memory," Nat. Photonics **3**, 706–714 (2009).
10. W.-X. Yang, J.-M. Hou, and R.-K. Lee, "Ultraslow bright and dark solitons in semiconductor quantum wells," Phys. Rev. A **77**, 033838 (2008).
11. T. Chaneliere, D. N. Matsukevich, S. D. Jenkins, S.-Y. Lan, T. A. B. Kennedy, and A. Kuzmich, "Storage and retrieval of single photons transmitted between remote quantum memories," Nature **438**, 833–836 (2005).
12. M. D. Eisaman, A. Andre, F. Massou, M. Fleischhauer, A. S. Zibrov, and M. D. Lukin, "Electromagnetically induced transparency with tunable single-photon pulses," Nature **438**, 837–841 (2005).
13. M. Mucke, E. Figueroa, J. Bochmann, C. Hahn, K. Murr, S. Ritter, C. J. Villas-Boas, and G. Rempe, "Electromagnetically induced transparency with single atoms in a cavity," Nature **465**, 755–758 (2010).
14. D. N. Matsukevich and A. Kuzmich, "Quantum state transfer between matter and light," Science **306**, 663–666 (2004).
15. S. E. Harris, "Electromagnetically induced transparency," Phys. Today **50** (7), 36 (1997).
16. M. Fleischhauer, A. Imamoglu, and J. P. Marangos, "Electromagnetically induced transparency: optics in coherent media," Rev. Mod. Phys. **77**, 633–673 (2005).
17. M. D. Lukin, "Trapping and manipulating photon states in atomic ensembles," Rev. Mod. Phys. **75**, 457–472 (2003).
18. M. D. Lukin, S. F. Yelin, and M. Fleischhauer, "Entanglement of atomic ensembles by trapping correlated photon states," Phys. Rev. Lett. **84**, 4232–4235 (2000).
19. Y.-F. Chen, S.-H. Wang, C.-Y. Wang, and I. A. Yu, "Manipulating the retrieved width of stored light pulses," Phys. Rev. A **72**, 053803 (2005).
20. Y.-F. Chen, P.-C. Kuan, S.-H. Wang, C.-Y. Wang, and I. A. Yu, "Manipulating the retrieved frequency and polarization of stored light pulses," Opt. Lett. **31**, 3511–3513 (2006).
21. C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, "Observation of coherent optical information storage in an atomic medium using halted light pulses," Nature **409**, 490–493 (2001).
22. D. F. Phillips, A. Fleischhauer, A. Mair, and R. L. Walsworth, "Storage of light in atomic vapor," Phys. Rev. Lett. **86**, 783–786 (2001).
23. A. Peng, M. Johnsson, W. P. Bowen, P. K. Lam, H.-A. Bachor, and J. J. Hope, "Squeezing and entanglement delay using slow light," Phys. Rev. A **71**, 033809 (2005).
24. D. Akamatsu, K. Akiba, and M. Kozuma, "Electromagnetically induced transparency with squeezed vacuum," Phys. Rev. Lett. **92**, 203602 (2004).
25. D. Akamatsu, Y. Yokoi, M. Arikawa, S. Nagatsuka, T. Tanimura, A. Furusawa, and M. Kozuma, "Ultraslow propagation of squeezed vacuum pulses with electromagnetically induced transparency," Phys. Rev. Lett. **99**, 153602 (2007).
26. K. Honda, D. Akamatsu, M. Arikawa, Y. Yokoi, K. Akiba, S. Nagatsuka, T. Tanimura, A. Furusawa, and M. Kozuma, "Storage and retrieval of a squeezed vacuum," Phys. Rev. Lett. **100**, 093601 (2008).
27. M. Arikawa, K. Honda, D. Akamatsu, S. Nagatsuka, K. Akiba, A. Furusawa, and M. Kozuma, "Quantum memory of a squeezed vacuum for arbitrary frequency sidebands," Phys. Rev. A **81**, 021605(R) (2010).
28. J. Appel, E. Figueroa, D. Korystov, M. Lobino, and A. I. Lvovsky, "Quantum memory for squeezed light," Phys. Rev. Lett. **100**, 093602 (2008).
29. P. Barberis-Blostein and M. Bienert, "Opacity of electromagnetically induced transparency for quantum fluctuations," Phys. Rev. Lett. **98**, 033602 (2007).
30. Y.-L. Chuang and R.-K. Lee, "Conditions to preserve quantum entanglement of quadrature fluctuation fields in electromagnetically induced transparency media," Opt. Lett. **34**, 1537–1539 (2009).
31. A. Dantan and M. Pinard, "Quantum-state transfer between fields and atoms in electromagnetically induced transparency," Phys. Rev. A **69**, 043810 (2004).
32. M. Fleischhauer and M. D. Lukin, "Dark-state polaritons in electromagnetically induced transparency," Phys. Rev. Lett. **84**, 5094–5097 (2000).
33. M. Fleischhauer and M. D. Lukin, "Quantum memory for photons: dark-state polaritons," Phys. Rev. A **65**, 022314 (2002).
34. R.-K. Lee and Y. Lai, "Quantum squeezing and correlation of self-induced transparency solitons," Phys. Rev. A **80**, 033839 (2009).
35. J. J. Hopfield, "Theory of the contribution of excitons to the complex dielectric constant of crystals," Phys. Rev. **112**, 1555–1567 (1958).
36. M. Artoni and J. L. Birman, "Quantum-optical properties of polariton waves," Phys. Rev. B **44**, 3736–3756 (1991).
37. M. Artoni and J. L. Birman, "Non-classical states in solids and detection," Opt. Commun. **104**, 319–324 (1994).
38. G. A. Garrett, A. G. Rojo, A. K. Sood, J. F. Whitaker, and R. Merlin, "Vacuum squeezing of solids: macroscopic quantum states driven by light pulses," Science **275**, 1638–1640 (1997).
39. V. V. Dodonov, "Nonclassical states in quantum optics: a squeezed review of the first 75 years," J. Opt. B **4**, R1–R33 (2002).
40. V. I. Yukalov and E. P. Yukalova, "Atomic squeezing under collective emission," Phys. Rev. A **70**, 053828 (2004).
41. A. Quattropani and P. Schwendimann, "Polariton squeezing in microcavities," Phys. Status Solidi B **242**, 2302–2314 (2005).
42. L. M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, "Inseparability criterion for continuous variable systems," Phys. Rev. Lett. **84**, 2722–2725 (2000).
43. Y. Lai and R.-K. Lee, "Entangled quantum nonlinear Schrödinger solitons," Phys. Rev. Lett. **103**, 013902 (2009).
44. X.-J. Liu, H. Jing, and M.-L. Ge, "Quantum memory process with a four-level atomic ensemble," Eur. Phys. J. D **40**, 297–303 (2006).
45. Z. Li, L. Xu, and K. Wang, "The dark-state polaritons of a double-Λ atomic ensemble," Phys. Lett. A **346**, 269–274 (2005).