Linear Algebra, EE 10810/EECS 205004

Quiz .21 − 2.2

Student ID: ....; Your Name: ....

(Dated: October 21st, 2020)

Integrity: There is NO space to cross the **Red Line** !!

1. Prove that $\mathcal{T}$ is a linear transformation, find bases for both $N(\mathcal{T})$ and $R(\mathcal{T})$, and calculate the nullity and rank of $\mathcal{T}$.
   (a) $\mathcal{T}$: $\mathcal{R}^3 \to \mathcal{R}^2$ defined by $\mathcal{T}(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$.
   (b) $\mathcal{T}$: $P_2(\mathcal{R}) \to P_3(\mathcal{R})$ defined by $\mathcal{T}(f(x)) = x\, f(x) + f'(x)$.

2. Let $\mathcal{V}$ and $\mathcal{W}$ be finite-dimensional vector spaces and $\mathcal{T} : \mathcal{V} \to \mathcal{W}$ be linear.
   (a) Prove that if $\dim(\mathcal{V}) < \dim(\mathcal{W})$, then $\mathcal{T}$ cannot be *onto*.
   (b) Prove that if $\dim(\mathcal{V}) > \dim(\mathcal{W})$, then $\mathcal{T}$ cannot be *one-to-one*.

3. Let $\hat{\mathcal{T}} : \mathcal{R}^2 \to \mathcal{R}^3$ be defined by $\hat{\mathcal{T}}(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for $\mathcal{R}^2$ and $\gamma = \{(1, 1, 0), (0, 1, 1), (2, 2, 3)\}$.
   (a) Computer $\left[\hat{\mathcal{T}}\right]_\beta^\gamma$.
   (b) If $\alpha = \{(1, 2), (2, 3)\}$, compute $\left[\hat{\mathcal{T}}\right]_\alpha^\gamma$.