Linear Algebra, EE 10810/EECS 205004

Quiz .21 − 2.2

Student ID: . . . . ; Your Name: . . . .
(Dated: October 28th, 2020)

Integrity: There is NO space to cross the **Red Line** !!

1. Let $\overline{\overline{A}}$ and $\overline{\overline{B}}$ be $n \times n$ matrices. Recall the trace of $\overline{\overline{A}}$ is defined by

$$\mathrm{tr}(\overline{\overline{A}}) = \sum_{i=1}^{n} A_{ii}. \tag{1}$$

Prove that $\mathrm{tr}(\overline{\overline{AB}}) = \mathrm{tr}(\overline{\overline{BA}})$ and $\mathrm{tr}(\overline{\overline{A}}) = \mathrm{tr}(\overline{\overline{A^t}})$.

2. Let $\overline{\overline{A}}$ and $\overline{\overline{B}}$ be $n \times n$ *invertible* matrices. Prove that
   (a) $\overline{\overline{AB}}$ is invertible.
   (b) $(\overline{\overline{AB}})^{-1} = \overline{\overline{B^{-1}A^{-1}}}$.

3. For each matrix $\overline{\overline{A}}$ and ordered basis β, find $[\hat{L}_A]_\beta$ and an invertible matrix $\overline{\overline{Q}}$ such that $[\hat{L}_A]_\beta = \overline{\overline{Q^{-1}AQ}}$.

$$\overline{\overline{A}} = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}, \quad \text{and} \quad \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} \tag{2}$$