Linear Algebra, EE 10810/EECS 205004

Quiz 5.1 – 5.3

Student ID: ....; Your Name: ....

(Dated: December 2nd, 2020)

Integrity: There is NO space to cross the **Red Line** !!

1. Use Cramer's rule with ratios $det(\overline{\overline{B}}_i)/det(\overline{\overline{A}})$ to solve $\overline{\overline{A}}\,\vec{x} = \vec{b}$. Also find the inverse matrix $(\overline{\overline{A}})^{-1} = \overline{\overline{C}}^t/det(\overline{\overline{A}})$.

$$\overline{\overline{A}}\,\vec{x} = \vec{b} \qquad \text{is} \qquad \begin{pmatrix} 2 & 6 & 2 \\ 1 & 4 & 2 \\ 5 & 9 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1}$$

2. Prove Theorem 5.5: Let $\hat{T}$ be. a linear operator on a vector space $\mathcal{V}$, and let $\{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ be distinct eigenvalues of $\hat{T}$. If $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ are the corresponding eigenvectors of $\hat{T}$, then $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is linearly independent.

3. For the following linear operators $\hat{T}$ on a vector space $\mathcal{V}$, test $\hat{T}$ for diagonalizability, and if $\hat{T}$ is diagonalizable, find a basis β for $\mathcal{V}$ such that $[\hat{T}]_\beta$ is a diagonal matrix:
   (a) $\mathcal{V} = P_3(\mathcal{R})$ and $\hat{T}$ is defined by $\hat{T}(f(x)) = f'(x) + f''(x)$, respectively.
   (b) $\mathcal{V} = \mathcal{R}^3$ and $\hat{T}$ is defined by

$$\hat{T}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_2 \\ -a_2 \\ 2a_3 \end{pmatrix} \tag{2}$$