Linear Algebra, EE 10810/EECS 205004

Quiz 6.3 – 6.4

Student ID: .....; Your Name: .....

(Dated: December 30th, 2020)

Integrity: There is NO space to cross the **Red Line** !!

1. Find the minimal solution to the following system of linear equations

$$\begin{array}{rcrcrcl} x & + & y & - & z & = & 0 \\ 2x & - & y & + & z & = & 3 \\ x & - & y & + & z & = & 2 \end{array} \tag{1}$$

2. Let $\mathcal{V}$ be a complex inner product space, and let $\hat{T}$ be a linear operator on $\mathcal{V}$. Define

$$\hat{T}_1 \equiv \frac{1}{2}(\hat{T} + \hat{T}^*), \quad \text{and} \quad \hat{T}_2 = \frac{1}{2i}(\hat{T} - \hat{T}^*) \tag{2}$$

   (a) Prove that $\hat{T}_1$ and $\hat{T}_2$ are self-adjoint.

   (b) Suppose also that $\hat{T} = \hat{U}_1 + i\,\hat{U}_2$, where $\hat{U}_1$ and $\hat{U}_2$ are self-adjoint. Prove that $\hat{U}_1 = \hat{T}_1$ and $\hat{U}_2 = \hat{U}_2$.

   (c) Prove that $\hat{T}$ is normal if and only if $\hat{T}_1\hat{T}_2 = \hat{T}_2\hat{T}_1$.