Linear Algebra, EE 10810/EECS 205004

Note 1.1 – 1.2

Ray-Kuang Lee[1]

[1] *Room 911, Delta Hall, National Tsing Hua University, Hsinchu, Taiwan.*
Tel: +886-3-5742439; E-mail: rklee@ee.nthu.edu.tw

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- **RK's goals:**
  1. You should go through the whole **Textbook**.
  2. Instead of repeating what you can find in the Textbook, I will illustrate the content **from Scratch**, by *raising questions to you* first.
  3. You need to have **Quiz** (40% to the semester score) on every Wednesday.
- **Textbook**: S. H. Friedberg, A. J. Insel, and L. E. Spence, "Linear Algebra," 4th Edition (Pearson, 2014).
- **Online Materials**: All the **Assignments**, with the Scratch notes, will be uploaded to *iLMS*.
- **Evaluation:**
  1. Assignments (Weekly)
  2. In-class Quiz (≥ 12): 40%; (Weekly, on every Wednesday morning, 10:10-10:40 AM)
- **Office hours:**
  1. RK: Every Wednesday, 1:00-5:00 PM at R 911, Delta Hall, or by appointment
  2. Raul Robles-Robles (last year PhD student); email: raulamauryrobles@hotmail.com
  3. TA time, Every Monday, 6:30-8:30 PM at Delta 217.
- **Integrity**: First Quiz on September 23rd, Wednesday, 10:10 AM - 10:30 AM.
- **Assignment** 1:
  1. **Uniqueness**:
     (a) Prove that the vector $\vec{0}$ in a vector space is *unique*.
     (b) Prove that for each element $\vec{x}$ in a vector space V, there exists a *unique* vector $\vec{y}$, such that $\vec{x}+\vec{y}=\vec{0}$.
  2. **Zero Vector Space**: Let $V=\{\vec{\theta}\}$ consist of a single vector $\vec{\theta}$, and define $\vec{\theta}+\vec{\theta}=\vec{\theta}$ and $c\vec{\theta}=\vec{\theta}$ for each c in F. Prove that V is a vector space over F.
  3. **Complex numbers field**: Let $V=\{(a_1,\ldots,a_n): a_i\in\mathcal{R}$ for $i=1,2,\ldots,n\}$. Is V a vector space over the field of complex numbers with the operations of coordinatewise addition and multiplication?
  4. Consider a vector space over **binary number field**:
     – Binary number field $\mathcal{F}$ is a set with only two numbers: 0 and 1.
     – The addition (+) and multiplication ($\bullet$) for binary number field follow the table below:

+	0	1
0	0	1
1	1	0

•	0	1
0	0	0
1	0	1

(1)

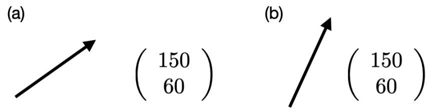
     – Let F^n be the set of all binary *n-tuples*, i.e.,

$$F^n=\{(v_1,v_2,\ldots,v_n)\,|\,v_i=\,0\,\text{or}\,1, 1\leq i\leq n\} \tag{2}$$

Show that F^n is a vector space over binary number field, by checking the 8 required conditions (VS1 - VS8) to define a vector space.

From Scratch !!

- Vector:



- Parallelogram Law for Vector Addition:

- Objects: n-tuples, matrix, polynomial function, sequence

- Rules: Linear Combination (Superposition)

- Commutativity:

- Associativity:

- Distributivity:

- Zero Vector:

- Inverse:

- Identity:

- Definition: A *vector space* (or *linear space)* V over a field F consists of a **set** on which two operations are defined, such that:
    1. addition: for each pair of elements $\vec{x}$, $\vec{y}$ in V there is a *unique* element $\vec{x} + \vec{y}$ in V
    2. scalar multiplication: for each element a in F and each element $\vec{x}$ in V there is a *unique* element $a\,\vec{x}$ in V,

  such that the conditions VS1 - VS8 are all hold.