Linear Algebra, EE 10810/EECS 205004

Note 1.4 – 1.5

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- **Office Hours:**
  1. RK: Every Wednesday, 1:00-5:00 PM at R 911, Delta Hall, or by appointment.
  2. Raul Robles-Robles (last year PhD student); email: raulamauryrobles@hotmail.com.
  3. TA time, Every Wednesday, 6:30-8:30 PM at Delta 217, or by appointment.

- **Integrity**: Next Quiz on September 30th, Wednesday, 10:10 AM - 10:30 AM.

- **Assignment**: for the Quiz on Sep. 30th
  1. Solve the system of linear equations by Gaussian elimination method,

$$\begin{cases} 2x_1 & -2x_2 & -3x_3 & & = -2 \\ 3x_1 & -3x_2 & -2x_3 & +5x_4 & = 7 \\ x_1 & -x_2 & -2x_3 & -x_4 & = -3 \end{cases} \tag{1}$$

  2. Show that if

$$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad M_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad M_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2}$$

  then the *span* of $\{M_1, M_2, M_3\}$ is the set of all symmetric 2×2 matrices.

  3. Show that a **subset** W of a vector space $\mathcal{V}$ is a **subspace** of $\mathcal{V}$ if and only if $\text{span}(W) = \mathcal{W}$.

  4. Show that if S_1 and S_2 are subsets of a vector space $\mathcal{V}$ such that $S_1 \subseteq S_2$, then $\text{span}(S_1) \subseteq \text{span}(S_2)$.

From Scratch !!

- Show that if S_1 and S_2 are arbitrary subsets of a vector space $\mathcal{V}$, then $\text{span}(S_1 \cup S_2) = \text{span}(S_1) + \text{span}(S_2)$.

- Definition: A *subset* $\mathcal{W}$ of a vector space $\mathcal{V}$ over a field $\mathcal{F}$ is called **subspace** of $\mathcal{V}$ if $\mathcal{W}$ is a vector space over $\mathcal{F}$ under the operation of addition and scalar multiplication defined on $\mathcal{V}$.
- Theorem 1.3: Subspace $\mathcal{W}$:
  1. $\vec{0} \in \mathcal{W}$.
  2. $\vec{x} + \vec{y} \in \mathcal{W}$ whenever $\vec{x} \in \mathcal{W}$ and $\vec{y} \in \mathcal{W}$.
  3. $a\,\vec{x} \in \mathcal{W}$ whenever $a \in \mathcal{F}$ and $\vec{x} \in \mathcal{W}$.
- Theorem 1.4: Any intersection of subspaces of a vector space $\mathcal{V}$ is a subspace of $\mathcal{V}$.
- Question: Is the Union of subspaces of a vector space $\mathcal{V}$ also a subspace of $\mathcal{V}$?

- **Systems of Linear Equations**:

$$\begin{array}{rrrrrcl} a_1 & -2a_2 & & +2a_4 & -3a_5 & = & 2 \\ 2a_1 & -4a_2 & +2a_3 & & +8a_5 & = & 6 \\ a_1 & -2a_2 & +3a_3 & -3a_4 & +16a_5 & = & 8 \end{array} \tag{3}$$

- Alternative: Geometric interpretation of Systems of Linear Equations: 2D lines intersecting at a point, 3D (intersecting at a plane)

$$\begin{array}{rrcl} x & -2y & = & 1, \\ 3x & +2y & = & 11. \end{array} \tag{4}$$

- Rules for systems of linear equations: $\overline{\overline{P}}\,\overline{\overline{D}}\,\overline{\overline{E}}$
  1. Scalar Multiplication, Diagonalized Matrix, ex, $\overline{\overline{D}} = \begin{bmatrix} a & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,
  2. Elimination, Elimination Matrix, ex, $\overline{\overline{E}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -l & 0 & 1 \end{bmatrix}$
  3. Row Exchange, Permutation Matrix, ex, $\overline{\overline{P}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

- Gauss Elimination

- Pivot

- Augmented Matrix, $\overline{\overline{M}}\vec{a} = \vec{b} \quad \Rightarrow \quad \left[\overline{\overline{M}}\,|\,\vec{b}\right]$.

- Breakdown of Elimination

- Inverse Matrix, $\overline{\overline{M}}^{-1}$

- Span: span(S)

- Linearly Dependent and Linearly Independent

- Basis