Linear Algebra, EE 10810/EECS 205004

Note 4.1

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- Next Quiz on Nov. 13th, Friday.

- **Assignment**:

1. Compute the determinants of the following matrix in $\overline{\overline{M}}_{2\times 2}(C)$:

$$\begin{pmatrix} -1 + i & 1 - 4i \\ 3 + 2i & 2 - 3i \end{pmatrix} \tag{1}$$

2. Prove that $det(\overline{\overline{A}}\,\overline{\overline{B}}) = det(\overline{\overline{A}}) \cdot det(\overline{\overline{B}})$ for any $\overline{\overline{A}}, \overline{\overline{B}} \in \overline{\overline{M}}_{2\times 2}(F)$.

3. The **classical adjoint** of a 2×2 matrix $\overline{\overline{A}} = \begin{pmatrix} a_{11} & 12 \\ a_{21} & a_{22} \end{pmatrix} \in \overline{\overline{M}}_{2\times 2}(F)$ is the matrix

$$\overline{\overline{C}} \equiv \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}. \tag{2}$$

Prove that

(a) $\overline{\overline{C}}\,\overline{\overline{A}} = \overline{\overline{A}}\,\overline{\overline{C}} = [det(\overline{\overline{A}}]\,\overline{\overline{I}}$

(b) $det(\overline{\overline{C}}) = det(\overline{\overline{A}})$

(c) The classical adjoint of $\overline{\overline{A}}^t$ is $\overline{\overline{C}}^t$

(d) If $\overline{\overline{A}}$ is invertible, then $\overline{\overline{A}}^{-1} = [det(\overline{\overline{A}})]^{-1}\overline{\overline{C}}$

From Scratch !!

- **Section 4.1: Determinants of Order** 2
- $\overline{\overline{A}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- Reduced row echelon form:

$$\overline{\overline{D}} = \begin{pmatrix} a & b \\ 0 & \frac{ad-bc}{a} \end{pmatrix} \tag{3}$$

- Inverse:

$$\overline{\overline{A}}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{4}$$

- Determinant: $det(\overline{\overline{A}}) \equiv |\overline{\overline{A}}| = ad - bc$
- Product of Pivots: $p_1 \times p_2$

- Theorem 4.1:

$$\begin{vmatrix} t\,a & t\,b \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix} \tag{5}$$

$$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix} \tag{6}$$

- Theorem 4.2: $\overline{\overline{A}}$ is invertible iff $det(\overline{\overline{A}}) \neq 0$

- Area of a Parallelogram

- Properties of Determinant
  - The determinant changes sign when two rows (or two columns) are exchanged.
  - $det(\overline{\overline{A}}\,\overline{\overline{B}}) = det(\overline{\overline{A}}) \cdot det(\overline{\overline{B}})$
  - $det(\overline{\overline{A}}) = det(\overline{\overline{A}}^t)$
  - $det(\overline{\overline{I}}) = 1$
  - The determinants equal *Volumes*.
  - If two rows of $\overline{\overline{A}}$ are equal, then $det(\overline{\overline{A}}) = 0$.
  - Subtracting a multiple of one row from another row leaves $det(\overline{\overline{A}})$ unchanged.

$$\begin{vmatrix} a & b \\ c - m\,a & d - m\,b \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \tag{7}$$

  - A matrix with a row of zeros has $det(\overline{\overline{A}}) = 0$.
  - If $\overline{\overline{A}}$ is *singular* then $det(\overline{\overline{A}}) = 0$.