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Optical bistability in nonlinear periodical structures with $\mathcal{PT}$-symmetric potential

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Abstract

The interaction between forward and backward waves in a periodical structure with $\mathcal{PT}$-symmetric potential is investigated. The results demonstrate that the $\mathcal{PT}$-term can change the bandgap of the periodical structure and affect the effective feedback mechanism. The linear solution, reflectivity, dispersion relation, and a generalized analytical solution of this periodical structure are obtained. The influences of the $\mathcal{PT}$-term and detunings on the characteristic of bistability (or multistability) are also discussed.

Keywords: optical bistability, periodical structures, PT-symmetric

(Some figures may appear in colour only in the online journal)

Optical bistability[5] [2–6] has been found to be important, both for providing a useful tool to design all-optical switches, and for its potential applications in ultracompact optical storage and memory components. Optical bistability, which uses an absorber inside a Fabry–Perot cavity (or optical ring cavity), has been the subject of intense theoretical and experimental studies for atomic and semiconductor quantum well system [3–8]. Importantly, bistability can occur in a distributed feedback structure without a Fabry–Perot-type device [9]. Recently, it has been shown that opposite directionality of the phase velocity and the Poynting vector can offer effective feedback mechanism for bistability in a nonlinear optical coupling with a negative index channel [10].

Parity-time ($\mathcal{PT}$) symmetry has attracted considerable attention in different physical systems because such a class of non-Hermitian Hamiltonians exhibit entirely real and positive eigenvalue spectra [11, 12]. It should be noted that non-Hermitian Hamiltonians without $\mathcal{PT}$ symmetry may also have real eigenvalues. The Hamiltonian of a $\mathcal{PT}$ symmetric system requires a necessary (but not sufficient) condition for which the potential $V(x)$ must satisfy $V(x) = V^*(-x)$. It was suggested that complex $\mathcal{PT}$-symmetric potentials can be realized in an optical system. In optics, $\mathcal{PT}$ symmetry requires that the real part and imaginary part of the refractive index should be even and odd functions of position, respectively, so the complex refractive index obeys the condition $n(x) = n^*(-x)$. An experimental scheme of $\mathcal{PT}$ Hamiltonians has been proposed in a planar slab waveguide with a complex refractive index [13]. A $\mathcal{PT}$ symmetric optical system possesses several unique features, which include nonreciprocal propagation of light [14], spontaneous $\mathcal{PT}$ symmetry breaking and power oscillation [15, 16], left–right symmetric oscillation [17], Bloch oscillation in complex crystal with $\mathcal{PT}$-symmetry [18], and many kinds of solitary-wave-like solutions [19–24] in dual-core optical systems with Kerr nonlinearity and $\mathcal{PT}$-balanced gain and loss.

An interesting characteristic of the $\mathcal{PT}$-symmetric system is the existence of spontaneous $\mathcal{PT}$ symmetry-breaking threshold

[5] See, for example, a review by [1] and references therein.

[12, 25], which is related to an abrupt phase transition. The result of the transition corresponds to a completely real spectrum changing to a non-strictly real spectrum. Above this critical threshold, some of the eigenvalues become complex, which is related to $\mathcal{PT}$-symmetry breaking. Passive $\mathcal{PT}$-symmetry breaking has recently been experimentally achieved in the realm of optics with non-Hermitian optical potentials [26]. The result of this abrupt phase transition leads to a loss-induced optical transparency. Power oscillations violating left–right symmetry and $\mathcal{PT}$-symmetry breaking have been observed in a $\mathcal{PT}$ optical coupled system with a complex index potential [27]. In addition, the unidirectional invisibility and unconventional reflection have been achieved in $\mathcal{PT}$-synthetic photonic lattices around the exceptional point [28], and the non-reciprocity of the transmission has been examined in a ladder system with the $\mathcal{PT}$-balanced combination of gain and loss [29].

A lot of interesting physical phenomena have been found in periodic structures in optics [30–36]. Slow Bragg solitons [30], for which carrier frequencies are closed to Bragg resonance and power spectra fall within frequency band gap, can exist in nonlinear periodic structure. The nonstationary soliton-like solutions in a periodic Kerr medium were obtained by Aceves and Wabnitz [31] and, employing the averaged Lagrangian variational technique, the stability of the Bragg soliton was first investigated by Malomed and Tasgal [32]. The applications [33] of periodic structures in optics include not only frequency filtering, but also all optical switches which are produced through nonlinear effects by applying optical signals to periodic structure. Due to the striking features of complex periodic structure, reflection and transmission spectra in a complex nonreciprocal Bragg grating were analyzed and a strong amplification was found to occur at the resonance wavelength [35]. Recently, the interplay of Bragg scattering and $\mathcal{PT}$ symmetry in periodic structure with a $\mathcal{PT}$-symmetric potential has been studied, and unidirectional invisibility has been shown to occur around the Bragg point with a broad range of frequencies [36]. The linear and nonlinear $\mathcal{PT}$-symmetric Bragg gratings have been studied by Sewell *et al* [37, 38]. They found that there is a different response when the signal incident from the left and right sides of the grating is in a linear $\mathcal{PT}$-symmetric Bragg grating. For a nonlinear $\mathcal{PT}$-symmetric Bragg grating, the bistability will occur for high gain/loss saturation intensity. In addition, a new family of slow Bragg soliton solutions has been obtained in nonlinear $\mathcal{PT}$-symmetric periodic structures [39]. Analysis implies that the grating band structure and effective linear coupling can be modified by the $\mathcal{PT}$-symmetric component of the periodic optical refractive index. Following the physical model in [39], we discuss the steady-state solutions and the behavior of the steady-state solutions in nonconservative environments, especially in the presence of linear gain or loss.

In this paper, the interplay between forward and backward waves in periodic structure with a complex $\mathcal{PT}$ potential is investigated. Starting from the nonlinear coupling wave equation, the property of dispersion is analyzed and the solutions for the linear coupling case are obtained. Then, a generalized analytical solution for forward and backward waves with nonlinear coupling is obtained. Furthermore, the effect of $\mathcal{PT}$-symmetric component of the periodic optical refractive index on the optical bistability (or multistability) is also discussed. A concluding remark is given in the last section.

We consider an N-period $\mathcal{PT}$ Bragg grating which is embedded in a background material with a refractive index n_0. The total length of the N-period $\mathcal{PT}$ Bragg grating is $L = N\Lambda$, where Λ is the spatial period of the grating, and N is the total number of periods. The real part of the refractive index is an even function of propagation z for a single period, but the imaginary part of the refractive index is an odd function of propagation z. So the linear refractive index variation can be expressed as

$$n = n_0 + n_{1R}\cos\left(2\beta_\Lambda z\right) + \mathrm{i}n_{1I}\sin\left(2\beta_\Lambda z\right) \qquad (1)$$

where $\beta_\Lambda = \pi/\Lambda$, n_{1R} and n_{1I} are small, e.g. $n_0 >> n_{1R,\,I}$, the second term in equation (1) stands for the periodic index variations inside the grating, and the third term accounts for the superimposed complex $\mathcal{PT}$ potential. Furthermore, this includes an intensity-dependent refractive index term which can be described by the nonlinear polarization $P^{\mathrm{NL}} = n_0 n_2 |E|^2 E/4\pi$ (n_2 is the nonlinear nonlinear Kerr coefficient of the material, and E is the electric field). Electric field E can be described by

$$E(z,t) = \left[E_{\mathrm{F}}(z)\,\exp\left(\mathrm{i}\beta_0 z\right) + E_{\mathrm{B}}(z)\,\exp\left(-\mathrm{i}\beta_0 z\right)\right]\exp\left(\mathrm{i}\omega_0 t\right) \qquad (2)$$

where $E_{\mathrm{F}}(z)$ and $E_{\mathrm{B}}(z)$ are the envelopes of forward and backward waves in the material, respectively. $\beta_0 = n_0\omega_0/c$, $\omega_0 = 2\,\pi\,c/\lambda_0$ with the free space wavelength of electric field λ_0. Under slowly varying envelope approximation and in the steady state case ($\partial/\partial t = 0$), which corresponds to optical beam propagation. We can obtain the following two coupling equations as [39]

$$-\mathrm{i}\frac{\partial E_{\mathrm{F}}}{\partial z} = (\kappa + g)\,\mathrm{e}^{-2\mathrm{i}\delta z}E_{\mathrm{B}} + \gamma\left(2|E_{\mathrm{B}}|^2 + |E_{\mathrm{F}}|^2\right)E_{\mathrm{F}} \qquad (3a)$$

$$\mathrm{i}\frac{\partial E_{\mathrm{B}}}{\partial z} = (\kappa - g)\,\mathrm{e}^{2\mathrm{i}\delta z}E_{\mathrm{F}} + \gamma\left(|E_{\mathrm{B}}|^2 + 2|E_{\mathrm{F}}|^2\right)E_{\mathrm{B}} \qquad (3b)$$

where $\delta = \beta_0 - \beta_\Lambda$ is the mismatch between the propagation constant, $\kappa = n_{1R}\pi/\lambda_0$, and $g = n_{1I}\pi/\Lambda$ is the linear coupling coefficient, κ comes from the real part of the linear refractive index, and g arises from the complex $\mathcal{PT}$ term. $\gamma = \pi\,n_2/\lambda_0$ is the nonlinear coupling coefficient. Assuming the solutions of equation (3) are of the forms $E_{\mathrm{F,B}} = E_{1,2}\exp[\mathrm{i}\,q\,z]\exp[\mp\mathrm{i}\,\delta z]$, in the linear regime, one can obtain the $q - \delta$ relation

$$q^2 = \delta^2 - (\kappa + g)(\kappa - g)\,. \qquad (4)$$

When $g = 0$, the $\mathcal{PT}$-symmetric periodical structure returns to the ordinary periodical structure and equation (4) reduces [40–42] $q^2 = \delta^2 - \kappa^2$. The changes of the dispersion relations, which are caused by the imaginary part of the $\mathcal{PT}$-symmetric potential, is shown in equation (4). One also can obtain the nonlinear dispersion relations with the forms

$$q = \frac{(\kappa + g)f^2 + g - f}{2f} + \frac{(f^2 - 1)}{2\left(1 + f^2\right)}\gamma a^2 \qquad (5a)$$

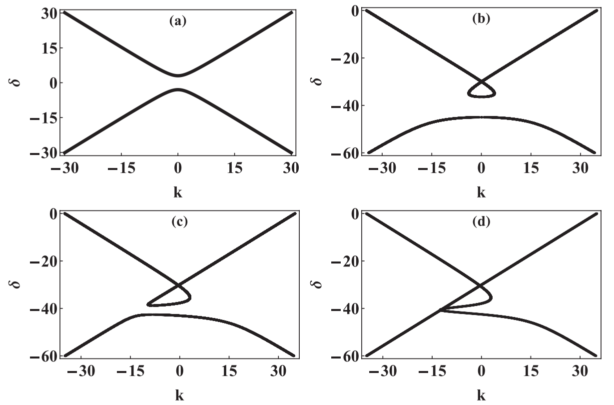

Figure 1. Linear and nonlinear dispersion of a $\mathcal{PT}$-symmetric periodic grating for different ratios of g/κ. (*a*) $\gamma = 0$ and $g/\kappa = 0.8$, which corresponds to the linear case. (*b*) $\kappa = 5$, $\gamma\, a^2 = 25$, $g/\kappa = 0$; (*c*) $\kappa = 3$, $\gamma\, a^2 = 25$, $g/\kappa = 2/3$; (*d*) $\kappa = 2.5$, $\gamma\, a^2 = 25$, $g/\kappa = 1$. There is nonlinear dispersion in (*b*)–(*d*).

$$\delta = -\frac{f^2\,(\kappa+g)+\kappa-g}{2f} - \frac{3}{2}\gamma a^2. \tag{5b}$$

In deriving equation (5), we have introduced the relations $a^2 = E_1^2 + E_2^2$ and $f = E_2/E_1$, which are related to the total power and the ratio of the backward to forward propagating wave's amplitude, respectively.

From equation (3), we note that the effect of the $\mathcal{PT}$ term comes from g and the spontaneous $\mathcal{PT}$-symmetry point is $\kappa = g$ [36]. In the next section, we only consider the case $g < \kappa$ which ensures that all eigenvalues of the non-Hermitian system, and which is described by equation (3), are completely real. Figure 1 [40–42] demonstrates linear (*a*) and nonlinear dispersion curves (*b*)–(*d*) for the same nonlinear coupling coefficient γ. For the linear case in figure 1(*a*), the shape of the band structure is similar to the ordinary periodical structure, with the difference that the band gap reduces. The effect of the $\mathcal{PT}$ term on nonlinear dispersion curves is illustrated in figures 1(*b*)–(*d*). Figure 1(*b*) shows that the higher branch of the dispersion relation's curve forms a loop. Figure 1(*b*) also shows that for $g/\kappa = 2/3$, the band gap reduces and the shape of loop becomes irregular. At the $\mathcal{PT}$-symmetry breaking point in figure 1(*d*), the band gap is closed. This implies that there are more choices of design freedom in the case with $\mathcal{PT}$-symmetric potential in comparison with the traditional periodical structures.

Assuming the propagation of light between 0 and L, in the linear case ($\gamma = 0$), the solutions of equation (3) with boundary conditions $E_F(0) = 1$ and $E_B(L) = 0$ are

$$E_F(z) = \frac{F\cosh(F(z-L))}{F\cosh(FL) - i\Delta\sinh(FL)}e^{-i\delta z} + \frac{i\delta\sinh(F(z-L))}{F\cosh(FL) - i\delta\sinh(FL)}e^{-i\delta z} \tag{6a}$$

$$E_B(z) = \frac{(\kappa-g)\sinh(F(z-L))}{iF\cosh(FL) - \delta\sinh(FL)}e^{i\Delta z} \tag{6b}$$

where $F = \sqrt{\kappa^2 - g^2 - \delta^2}$. The ratio of the power in the backward to the power in forward is the definition of reflectivity value ($R = |E_B(0)|^2/|E_F(0)|^2$). Under the phase-matched conditions ($\delta = 0$), the reflectivity as a function of linear coupling coefficient is found from equation (6) to be

$$R = \frac{\kappa-g}{\kappa+g}\tanh^2((\kappa^2-g^2)L). \tag{7}$$

For the nonlinear case ($\gamma \neq 0$), the analytical solutions of equation (3) are complicated. Taking the methods in [43], and assuming $E_F = A_f\exp(i\phi_F)$ and $E_B = A_b\exp(i\phi_B)$, where A_f, A_b, ϕ_F and ϕ_B are real functions of z, one can obtain the following equation

$$\frac{\partial A_f}{\partial z} = (\kappa+g)\,A_b\sin(\Phi) \tag{8a}$$

$$\frac{\partial A_b}{\partial z} = (\kappa-g)\,A_f\sin(\Phi) \tag{8b}$$

$$\frac{\partial \Phi}{\partial z} = \left[(\kappa+g)\,\frac{A_b}{A_f} + (\kappa-g)\,\frac{A_f}{A_g}\right]\cos(\Phi)\,. \tag{8c}$$

In deriving equation (8), we have used the relation $\Phi = \phi_F - \phi_B + 2\,\delta z$. After introducing the two new parameters $P_f = A_f^2, P_b = A_b^2$, we obtain two constraint equations for equation (8) as

$$C = \frac{P_f}{\kappa+g} - \frac{P_b}{\kappa-g} \tag{9a}$$

$$\Gamma = \sqrt{P_fP_b}\cos(\Phi) + \frac{3\gamma P_f^2}{4(\kappa+g)} + \frac{3\gamma P_b^2 + 4\Delta P_b}{4(\kappa-g)} \tag{9b}$$

where C is the effective transmitted flux in the periodical structure with the $\mathcal{PT}$-symmetric potential. By introducing a 'critical intensity' $P_c = 4\lambda_0/3\pi\, n_2L$ and adopting the following normalized variables $P_B = P_b/P_c$, and $P_T = C/P_c$, the equation for backward power in the structure can be written as

$$\left(\frac{P'_B}{2P_B}\right)^2 = \left(\kappa_1^2 - g_1^2\right)\left(1 + \frac{P_T(L)}{P_B}\right) - \left(\frac{2P_B\kappa_1}{\kappa_1 - g_1} + \Re\right)^2 \tag{10}$$

where P'_B stands for $dB/d\xi$ with $\xi = Lz$, and $\kappa_1 = \kappa L$, $g_1 = gL$ and $\Re = 2P_T(L)(\kappa_1 + g_1)(\kappa_1 - g_1)^{-1} + \delta L$. At the input of the structure, we assume that $P_F(0) = I_0$ and $P_B(0) = 0$, $P_T(L) = P_F(L)(\kappa+g)^{-1}P_c^{-1}$, where I_0 has been normalized to P_c. Combining equations (8)–(10) and applying the Weierstrass elliptic function's properties, the analytical solutions for $P_B(\xi)$ and $P_F(\xi)$ are found in terms of the Weierstrass elliptic function $\wp(\xi, g_2, g_3)$:

$$P_F(\xi) = \frac{\kappa_1+g_1}{\kappa_1-g_1}P_T(L)\left(1 + \frac{\left(\kappa_1^2-g_1^2\right)}{\wp\left(1-\xi; g_2, g_3\right) + \mathbb{B}}\right) \tag{11}$$

$$P_B(\xi) = \frac{(\kappa_1-g_1)\left(\kappa_1^2-g_1^2\right)C}{\wp\left(1-\xi; g_2, g_3\right) + \mathbb{B}} \tag{12}$$

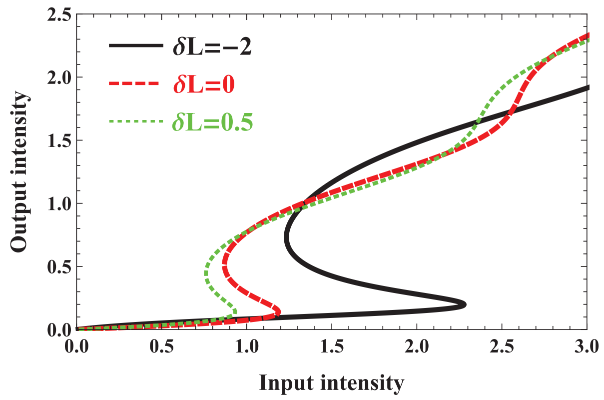

Figure 2. Output versus input intensity of nonlinear distributed-feedback structure for three different values of detuning $\delta L = -2$, 0, and 0.5 and the fixed linear coupling coefficients $gL = 0.25$, and $\kappa L = 2$.

where $\mathbb{B} = \left[(\Delta + \delta L)^2 - \kappa_1^2 + g_1^2\right]/3$, and $\Delta = 2(\kappa_1 + g_1)(\kappa_1 + g_1)^{-1}P_T(L)$, $\wp(z;g_2, g_3)$ is the Weierstrass elliptic function and the invariants g_2 and g_3 are defined as

$$g_2 = 16\kappa_1(\Delta + \delta L)(\kappa_1 + g_1)P_T(L) + 12\mathbb{B}^2 \quad (13a)$$

$$g_3 = 16\kappa_1^2(\kappa_1 + g_1)^2 P_T^2(L) + 8\mathrm{B}^3 - 16\kappa_1(\kappa_1 + g_1)P_T(L)(\Delta + \delta L)\mathrm{B}. \quad (13b)$$

Adopting the boundary condition, the output intensity $P_T(L)$ can be determined by solving the following equation

$$P_F(0) = \frac{\kappa_1 + g_1}{\kappa_1 - g_1}\left(1 + \frac{(\kappa^2 - g^2)}{\wp(1; g_2, g_3) + \mathrm{B}}\right)P_T(L). \quad (14)$$

Figure 2 shows a relation between input $P_F(0)$ and output intensities $P_F(L)$ for three values of the detuning $\delta L = -2$ (black solid line), $\delta L = 0$ (red dashed line), $\delta L = 0.5$ (green dotted line) and two fixed coupling coefficients $\kappa L = 2$ and $gL = 0.25$. From figure 2, we clearly see that the shape of the hysteresis loop and the bistable threshold depend on the detuning δL. The effect of the detunings δL on the characteristic of bistability is very similar to those in ordinary periodical structure [9], which is not considered the $\mathcal{PT}$ term. The threshold of bistability is bigger than that in [9]. The dependence of the properties for bistability (or multistability) on the imaginary parts of the complex potential are shown in figure 3. The multistability can also be formed in this periodical structure with a $\mathcal{PT}$-symmetric potential in figure 3(*a*). From figure 3(*a*), the threshold becomes bigger when the parameter gL change from 0 to 0.4. The reason may be that, with the increase of the imaginary part coefficient gL, the absorption for the forward field in the periodical structure with the $\mathcal{PT}$-symmetric potential increases, which makes it harder for the forward field to reach saturation. From figures 3(*a*) and (*b*), as the values of gL change from 0.4 to 2.5, the multistability becomes bistability, which is caused by the absorption. As a result, one can achieve optimally the desired bistable curve via properly designing the imaginary part of the $\mathcal{PT}$-symmetric potential and adjusting the phase matching parameter δL.

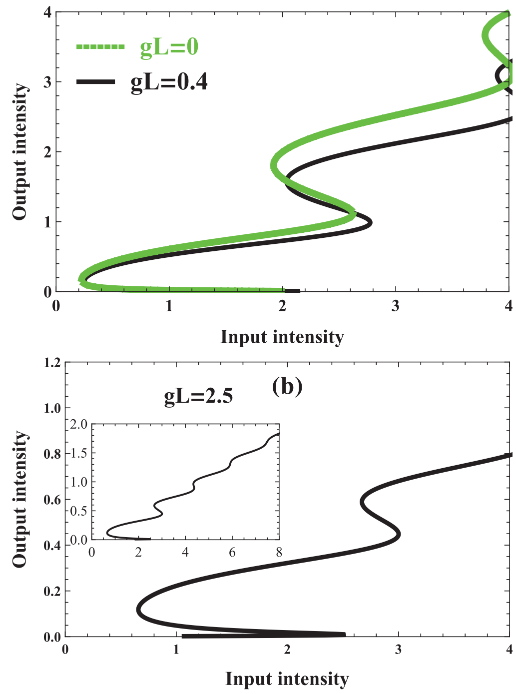

Figure 3. Output intensity $P_F(L)$ as a function of input intensity $P_F(0)$ with a fixed $\delta L = 0$ and $\kappa L = 4.0$, and three values of gL, (*a*) $gL = 0.4$ (solid line) and $gL = 0$ (dotted line) (*b*) $gL = 2.5$. The inset figure in (*b*) shows a larger region for input and output intensity.

In conclusion, the interaction of forward and backward waves in a Kerr nonlinear $\mathcal{PT}$-symmetric periodic structure was investigated. The linear and nonlinear dispersion relations and linear solutions for forward and backward waves were first discussed. Then, a generalized analytical solution was found in this nonlinear $\mathcal{PT}$-symmetric periodic structure. Furthermore, the behavior of the bistability and multistability has also been illustrated. The results showed that the $\mathcal{PT}$-symmetric complex potential and the frequency detuning can dramatically affect the bistability (or multistability) behavior, which can be used to manipulate the bistable threshold intensity and the hysteresis loop. These results offer an alternative proposal for the optimal design of nonlinear periodic systems to achieve very fast all-optical switches, and the effective feedback mechanism of bistability (or multistability) comes essentially from the nonlinear coupling between forward and backward waves in the periodic structure which is related to the notion of Bragg solitons [39].

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References

[1] Lugiato L A 1984 *Progr. Opt.* **21** 69

[2] Gibbs H M, Mccall S L, Venkatesan T N C 1976 *Phys. Rev. Lett.* **36** 1135
Rosenberger A T, Orozco L A, Kimble H J 1983 *Phys. Rev.* A **28** 2569
Orozco L A, Kimble H J, Rosenberger A T, Lugiato L A, Asquini M L, Brambilla M and Narducci L M 1989 *Phys. Rev.* A **39** 1235

[3] Bonifacio R, Lugatio L 1976 *Opt. Commun.* **19** 172
Gibbs H M, McCall S L, Venkatesan T N C, Gossard A C, Passner A, Wiegmann W 1979 *Appl. Phys. Lett.* **35** 451
Wall D F and Zoller P 1980 *Opt. Commun.* **34** 260

[4] Joshi A, Xiao M 2003 *Phys. Rev. Lett.* **91** 143904
Chang H, Wu H, Xie C and Wang H 2004 *Phys. Rev. Lett.* **93** 213901

[5] Harshawardhan W and Agarwal G S 1996 *Phys. Rev.* A **53** 1812

[6] Hu X and Xu Z 2001 *J. Opt. B: Quantum Semiclass. Opt.* **3** 35

[7] Joshi A and Xiao M 2004 *Appl. Phys. B: Lasers Opt.* **79** 65

[8] Li J-H 2007 *Phys. Rev.* B **75** 155329

[9] Winful H G, Marburger J H and Garmire E 1979 *Appl. Phys. Lett.* **35** 379

[10] Litchinitser N M, Gabitov I R and Maimistov A I 2007 *Phys. Rev. Lett.* **99** 113902

[11] Bender C M and Boettcher S 1998 *Phys. Rev. Lett.* **80** 5243

[12] Bender C M 2007 *Rep. Prog. Phys.* **70** 947

[13] Ruschhaupt A, Delgado F and Muga J G 2005 *J. Phys. A: Math. Gen.* **38** L171

[14] Feng L, Ayache M, Huang J, Xu Y-L, Lu M-H, Chen Y-F, Fainman Y and Scherer A 2011 *Science* **333** 729

[15] Makris K G, El-Ganainy R, Christodoulides D N and Musslimani Z H 2008 *Phys. Rev. Lett.* **100** 103904

[16] Wu J and Xie X-T 2012 *Phys. Rev.* A **86** 032112

[17] Chen L, Li R, Yang N, Chen D and Li L 2012 *Proc. Rom. Acad.* A **13** 46

[18] Longhi S 2009 *Phys. Rev. Lett.* **103** 123601

[19] Driben R, Malomed B A 2011 *Opt. Lett.* **36** 4323
Nixon S, Ge L and Yang J 2012 *Phys. Rev.* A **85** 023822

[20] Driben R and Malomed B A 2012 *Europhys. Lett.* **99** 54001

[21] Zhu X, Wang H, Zheng L-X, Li H, He Y-J 2011 *Opt. Lett.* **36** 2680
Li C, Huang C, Liu H and Dong L 2012 *Opt. Lett.* **37** 4543

[22] Alexeeva N V, Barashenkov I V, Sukhorukov A A and Kivshar Y S 2012 *Phys. Rev.* A **85** 063837

[23] Bludov Y V, Konotop V V and Malomed B A 2013 *Phys. Rev.* A **87** 013816

[24] Bludov Y V, Driben R, Konotop V V and Malomed B A 2013 *J. Opt.* **15** 064010

[25] Klaiman S, Günther U and Moiseyev N 2008 *Phys. Rev. Lett.* **101** 080402

[26] Guo A, Salamo G J, Duchesne D, Morandotti R, Volatier-Ravat M, Aimez V, Siviloglou G A and Christodoulides D N 2009 *Phys. Rev. Lett.* **103** 093902

[27] Rüter C E, Makris K G, El-Ganainy R, Christodoulides D N, Segev M and Kip D 2010 *Nat. Phys.* **6** 192

[28] Regensburger A, Bersch C, Miri M A, Onishchukov G, Christodoulides D N and Peschel U 2012 *Nature* **488** 167

[29] D'Ambroise J, Lepri S, Malomed B A and Kevrekidis P G 2014 *Phys. Lett.* A **378** 2824

[30] Christodoulides D N and Joseph R I 1989 *Phys. Rev. Lett.* **62** 1746

[31] Aceves A B and Wabnitz S 1989 *Phys. Lett.* A **141** 37

[32] Malomed B A and Tasgal R S 1994 *Phys. Rev.* E **49** 5787

[33] Winful H G, Stegeman G I 1985 *Proc. SPIE* **517** 214
Elachi C and Yeh C 1973 *J. Appl. Phys.* **44** 3146

[34] Wu Y and Yang X 2007 *Phys. Rev. Lett.* **98** 013601

[35] Kulishov M, Laniel J M, Bélanger N, Azaña J and Plant D V 2005 *Opt. Express* **13** 3068

[36] Lin Z, Ramezani H, Eichelkraut T, Kottos T, Cao H and Christodoulides D N 2011 *Phys. Rev. Lett.* **106** 213901

[37] Phang S, Vukovic A, Susanto H, Benson T M and Sewell P 2013 *J. Opt. Soc. Am.* B **30** 2984

[38] Phang S, Vukovic A, Susanto H, Benson T M and Sewell P 2014 *Opt. Lett.* **39** 2603

[39] Miri M-A, Aceves A B, Kottos T, Kovanis V and Christodoulides D N 2012 *Phys. Rev.* A **86** 033801

[40] de Sterke C M 1998 *J. Opt. Soc. Am.* B **15** 2660

[41] Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (New York: Academic)

[42] Diakonov D, Jensen L M, Pethick C J and Smith H 2002 *Phys. Rev.* A **66** 013604

[43] Armstrong J A, Bloembergen N, Ducuing J and Pershan P S 1962 *Phys. Rev.* **127** 1918