Optics Communications 379 (2016) 68–73

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Control of Goos–Hänchen shift via input probe field intensity

Ziauddin [a,b,*], Ray-Kuang Lee [b], Sajid Qamar [a]

[a] Quantum optics Lab. Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan
[b] Institute of Photonics and Technologies, National Tsing-Hua University, Hsinchu 300, Taiwan

ARTICLE INFO

Article history:
Received 14 April 2016
Received in revised form
16 May 2016
Accepted 18 May 2016

PACS:
42.50.Gy
32.80.Qk
42.65.-k

Keywords:
GH shift
Rydberg atoms
Blockade mechanism

ABSTRACT

We suggest a scheme to control Goos–Hänchen (GH) shift in an ensemble of strongly interacting Rydberg atoms, which act as super-atoms due to the dipole blockade mechanism. The ensemble of three-level cold Rydberg-dressed (^{87}Rb) atoms follows a cascade configurations where two fields, *i.e*, a strong control and a weak field are employed [D. Petrosyan, J. Otterbach, and M. Fleischhauer, Phys. Rev. Lett. 107, 213601 (2011)]. The propagation of probe field is influenced by two-photon correlation within the blockade distance, which are damped due to the saturation of super-atoms. The amplitude of GH shift in the reflected light depends on the intensity of probe field. We observe large negative GH shift in the reflected light for small values of the probe field intensities.

1. Introduction

The Goos–Hänchen (GH) shift means the lateral displacement of a light beam from its expected geometrical optics path. The existence of it was first confirmed by Goos and Hänchen in 1947 [1] in the phenomenon of total internal reflection from the interface of two different media. Since then, a lot of attention has been given to studied GH shift using different systems [2–4]. Furthermore, due to the fundamental nature of the lateral shift, there are also interesting applications to measure various quantities such as beam angle, refractive index, displacement, temperature, and film thickness [5]. The phenomenon of GH shift can also be used for the characterization of permeability and permittivity of the materials [6] and in the development of near-field optical microscopy and lithography [7]. Recently, control of positive and negative GH shifts in the reflected light has been studied [8–13]. In these proposals, different atomic systems have been considered in a cavity and studied the control of positive and negative GH shifts in the reflected light by modifying the susceptibility of the atomic medium via external parameters. It has been predicted that the positive as well as negative GH shifts in the reflected light are based on positive and negative group index of the medium, respectively.

In addition, Rydberg states with a high principal quantum number have been demonstrated to address EIT in atomic ensembles and have exhibited important properties. These include for example, strong dipole-dipole interactions, long radiative life time and van der waals (vdW) interactions [16]. The properties of highly excited Rydberg atoms are very constructive for quantum gates and quantum information processing [17–19], interesting many body effects [20–22] and light atom quantum interface [23–25]. In these studies, most are based on the dipole blockade mechanism, in which atom in Rydberg state suppresses the excitation of more than one Rydberg states of neighboring atoms within a certain specific volume [26,27]. Previously, it has been reported that the dipole blockade mechanism makes the Rydberg atom a strong candidate for single photon quantum devices such as single photon sources, single photon filters, single photon subtractors and single photon switches etc. [28–30]. In 2010, an experiment has been performed by considering a strong vdW interaction between the atomic Rydberg states [31]. It has been reported that by increasing the input probe field intensity led to reduction of its transmission within the EIT window. Later, this unpredictable reduction of probe field transmission has been exploited by considering a theoretical model of blockade mechanism [32]. In this model [32], Rydberg-dressed EIT systems in the three-level cascade configuration has been proposed, with the transparency window controlled by the strength of the probe field. The EIT phenomenon has been explained with super-atom (SA) in the mean field with two-photon correlation for the non-linear response to Rydberg excitation [32,33].

Further, the control of negative and positive GH shifts in the

* Corresponding author.
E-mail addresses: ziauddin@comsats.edu.pk (Ziauddin), rklee@ee.nthu.edu.tw (R.-K. Lee), sajid_qamar@comsats.edu.pk (S. Qamar).

http://dx.doi.org/10.1016/j.optcom.2016.05.055
0030-4018/

reflected light is already noticed when the light is incident on a dispersive atomic medium [8–13]. The noticeable point is that, the measurement of GH shift in the reflected light in the optical regime has a difficult task due to its small magnitude. Several proposals have been reported for the amplitude control of GH shift, these include for example, amplitude control via Kerr nonlinear field, spontaneously generated coherence and $\mathcal{PT}$-symmetry [12,14,15]. In best of our knowledge no one has considered the amplitude control of the GH shift via input probe field intensity. In this work, we consider Rydberg atomic medium with strong vdW interaction between the atoms in a cavity. The atomic medium consists of large number of SAs, where each SA contains an ensemble of three-level atoms with cascade configuration having only one atom in Rydberg state. We study GH shift in the reflected light which depends on the input probe field intensity. We expect large negative GH shift in the reflected light for weak probe field intensity.

2. Model

We consider a system consisted of three layers, i.e., 1, 2, and 3 with thickness d_1, d_2 and d_1, respectively. Layers 1 and 3 are the slabs having permittivity ϵ_1, whereas layer 2 is the intracavity medium having permittivity ϵ_2. A Gaussian-shaped beam is incident on a system making an angle θ with z-direction, see Fig. 1 (a). The electric field of the incident Gaussian light can be written as

$$E(z, y) = \frac{1}{\sqrt{2\pi}} \int E(k_y) e^{i(k_z z + k_y y)} dk_y, \tag{1}$$

where

$$E(k_y) = \frac{\sigma_y}{\sqrt{2\pi}} e^{\frac{-\sigma_y^2 (k_y - k_{y0})^2}{4}},$$

is the angular spectrum of the Gaussian beam centered at $y=0$ of the plane of $z=0$, $k_{y0} = k \ \sin\theta$, $\sigma_y = \sigma \sec\theta$ and θ is the incident angle. The reflected probe field can be written as [8,11]

$$E^r(z, y) = \frac{1}{\sqrt{2\pi}} \int r(k_y, \omega_p) E(K_y) e^{i(-k_z z + k_y y)} dk_y, \tag{2}$$

where $r(k_y, \omega_p)$ is the reflection coefficient, which can be calculated using characteristics matrix approach [9] as

$$m_j(k_y, \omega_p, d_j) = \begin{pmatrix} \cos(k_j^z d_j) & i\sin(k_j^z d_j)/q_j \\ iq_j \sin(k_j^z d_j) & \cos(k_j^z d_j) \end{pmatrix},$$

where $k_j^z = k\sqrt{\epsilon_j - \sin^2\theta}$ is the wave number of three-layer structure, $k = \omega/c$ in vacuum, c is the speed of light, $q_j = k_j^z/k$, d_j is the thickness of jth layer. Our system consists of three layers, therefore, the total transfer matrix for the system can be written as,

$$Q(k_y, \omega_p) = m_1(k_y, \omega_p, d_1) m_2(k_y, \omega_p, d_2) m_1(k_y, \omega_p, d_1),$$

and finally the reflection coefficient can therefore be calculated as,

$$r(k_y, \omega_p) = \frac{q_0(Q_{22} - Q_{11}) - (q_0^2 Q_{12} - Q_{21})}{q_0(Q_{22} + Q_{11}) - (q_0^2 Q_{12} + Q_{21})}, \tag{3}$$

where Q_{ij} be the elements of total transfer matrix $Q(k_y, \omega_p)$ and $q_0 = \sqrt{\epsilon_0 - \sin^2\theta}$.

The expression of the GH shift is defined as the normalized first moment of the electric field in the reflected light as [11]

$$S_r = \frac{\int_{-\infty}^{\infty} y |E^r(z, y)|^2 dy}{\int_{-\infty}^{\infty} |E^r(z, y)|^2 dy}. \tag{4}$$

2.1. Atom-field interaction

We consider an ensemble of cold ^{87}Rb atoms in cascade atomic configuration interacting with two optical fields. The probe and control fields interacting with atomic medium as shown in Fig. 1 (b). Each atom has energy-levels $|g\rangle$, $|e\rangle$ and $|r\rangle$. A probe field of frequency ω_p drives the transition between $|g\rangle$ and $|e\rangle$ with Rabi frequency Ω_p, whereas the control field with Rabi frequency Ω_c drives the transition between $|r\rangle$ and $|e\rangle$. The control field excite the atoms to the Rydberg state $|r\rangle$ and the atoms interact with each other via a vdW potential $|\Delta(\mathbf{r}_i - \mathbf{r}_j) = C_6/|\mathbf{r}_i - \mathbf{r}_j|^6$ [32], where $\mathbf{r}_i$ and $\mathbf{r}_j$ are the positions of atoms i and j, respectively. We can write the total Hamiltonian for our system as

$$H = H_a + H_{af} + H_{\mathrm{vdW}}, \tag{5}$$

where

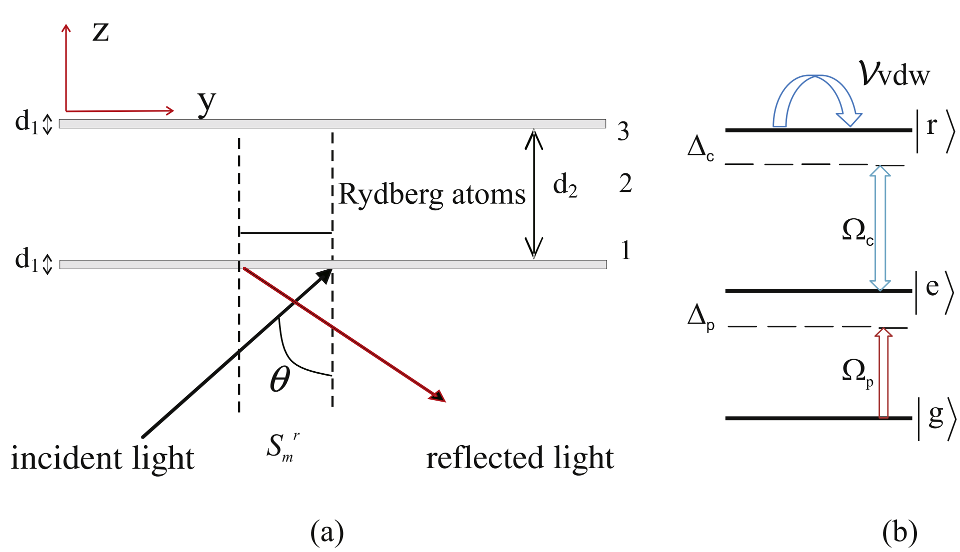

Fig. 1. (a) Schematics of coherent light field incident on a cavity; (b) the energy-level configuration of a three-level Rydberg atom.

$$H_a = -\hbar \sum_j^N [\Delta_p \sigma_{ee}^j + \Delta_2 \sigma_{rr}^j],$$
$$H_{af} = -\hbar \sum_j^N [\Omega_p \sigma_{eg}^j + \Omega_c \sigma_{re}^j + H.c.],$$
$$H_{\text{vdW}} = \hbar \sum_{i<j}^N \sigma_{rr}^i \Delta(\mathbf{r}_i - \mathbf{r}_j) \sigma_{rr}^j, \tag{6}$$

whereas $\Delta_p = \omega_p - \omega_{eg}$, $\Delta_c = \omega_c - \omega_{re}$, $\Delta_2 = \Delta_p + \Delta_c$ is the two-photon detuning and $\sigma_{\alpha\beta}^j = |\alpha\rangle_{jj}\langle\beta|$ is the transition operator for atom j at position r_j. Using the Hamiltonian (Eq. (5)) we can write down the equations of motion as

$$\dot{\sigma}_{gg}^j = i\Omega_p^* \sigma_{ge}^j - i\Omega_p \sigma_{eg}^j + \Gamma_e \sigma_{ee}^j,$$
$$\dot{\sigma}_{ee}^j = i\Omega_p \sigma_{eg}^j - i\Omega_p^* \sigma_{ge}^j - i\Omega_c \sigma_{re}^j,$$
$$\dot{\sigma}_{ge}^j = \left(i\Delta_p - \gamma_{ge}\right)\sigma_{ge}^j + i\Omega_p\left(\sigma_{gg}^j - \sigma_{ee}^j\right) + i\Omega_c^* \sigma_{gr}^j,$$
$$\dot{\sigma}_{gr}^j = \left[i\left(\Delta_2 - S\left(\mathbf{r}\right)\right) - \gamma_{gr}\right]\sigma_{gr}^j + i\Omega_c \sigma_{ge}^j - i\Omega_p \sigma_{er}^j,$$
$$\dot{\sigma}_{er}^j = i\left(\Delta_p - S\left(\mathbf{r}\right) - \gamma_{er}\right)\sigma_{er}^j + i\Omega_p^* \sigma_{gr}^j + i\Omega_c\left[\sigma_{ee}^j - \sigma_{rr}^j\right], \tag{7}$$

where Γ_e is the atomic decay rate whereas γ_{ge}, γ_{er} and γ_{gr} are dephasing rates. As the Rydberg state exhibits longer life time so $\gamma_{ge} \approx \gamma_{er} >> \gamma_{gr}$. Here, $S(\mathbf{r})$ is the total van der Waals force induced shift of Rydberg state $|r\rangle$ for an atom at position $\mathbf{r}$ and can therefore be written as

$$S(\mathbf{r}) = \sum_{i<j}^{n_{SA}} \Delta(\mathbf{r} - \mathbf{r}_j)\sigma_{rr}. \tag{8}$$

The steady state solution of Eq. (7) will be

$$\sigma_{eg} = \frac{i\left[\gamma_{rg} - i\left(-S\left(\mathbf{r}\right) + \Delta_2\right)\right]\Omega_p}{\left(\gamma_{eg} - i\Delta_p\right)\left[\gamma_{rg} - i\left(-S\left(\mathbf{r}\right) + \Delta_2\right)\right] + \Omega_c^2},$$
$$\sigma_{rg} = -\frac{\Omega_c \Omega_p}{\left(\gamma_{eg} - i\Delta_p\right)\left[\gamma_{rg} - i\left(-S\left(\mathbf{r}\right) + \Delta_2\right)\right] + \Omega_c^2}. \tag{9}$$

We follow the same method as described earlier in Ref. [32] and consider the stationary state solution of Eq. (7) without the van der Waals shift $S(\mathbf{r})$. We can write the population of Rydberg state as

$$\langle\sigma_{rr}\rangle = \sigma_{rg}.\sigma_{gr}, \tag{10}$$

By considering $\gamma_{rg} << \gamma_{eg}$ and $\Delta_p < \gamma_{eg}$, the population of Rydberg state $|r\rangle$ can be written as

$$\langle\sigma_{rr}\rangle \approx \frac{|\Omega_c|^2|\Omega_p|^2}{|\Omega_c|^4 + \Delta_2^2\gamma_{eg}^2} \tag{11}$$

Next, we discuss vdW shift and consider that an atom is in a Rydberg state which induces a vdW shift $\Delta(R)$ for another atom located at a distance R. The vdW interaction suppresses the excitation of all the atoms in a small volume (V_{SA}), which is called as a Rydberg blockade or super-atom (SA) [26]. The number of atoms in a super-atom may be defined as $n_{SA} = \rho(r)V_{SA}$, where $\rho(\mathbf{r})$ is the atomic density. There is only one Rydberg excited atom in each super-atom i.e., in V_{SA}. The total medium can then be treated as the collection of super-atoms, and the number of super-atoms in volume V will be $N_{SA} = \rho_{SA}V$. Then, the total vdW shift at position $\mathbf{r}$ can be written as

$$S(\mathbf{r}) = \sum_j^{N_{SA}} \Delta(\mathbf{r} - \mathbf{r}_j)\Sigma_{RR}(\mathbf{r}_j) = \bar{\Delta}\Sigma_{RR}(\mathbf{r}) + s(\mathbf{r}), \tag{12}$$

where the first term in right side of Eq. (12) shows the excited SA at $\mathbf{r}_j = \mathbf{r}$ i.e., $\Sigma_{RR}(\mathbf{r}) \to 1$, which induces divergent vdW shift in a volume of SA, and then $\Delta\left(0\right) \cong \frac{1}{V_{SA}}\int_{V_{SA}} \Delta\mathbf{r}' d^3\mathbf{r}' \to \infty$. The second part in the right side of Eq. (12) shows the vdW shift induces the external SAs outside the volume and can be expressed as $s\left(\mathbf{r}\right) = \sum_j^{N_{SA}} \Delta\left(\mathbf{r} - \mathbf{r}_j\right)\Sigma_{RR}\left(\mathbf{r}_j\right)$. We can calculate the expression for $s(\mathbf{r})$ by replacing the summation over integration of the total volume and then using the mean field approximation as [32]

$$\left\langle s\left(\mathbf{r}\right)\right\rangle = \frac{w}{8}\langle\Sigma_{RR}(\mathbf{r})\rangle, \tag{13}$$

where w is the half-width of Lorentzian function of population in Rydberg state. To find the analytical expression for $s(\mathbf{r})$, we should calculate $\Sigma_{RR}(\mathbf{r})$, so that the ground and single collective Rydberg excited states of a SA can be considered as

$$|G\rangle = |g_1, g_2, g_3 .., g_{n_{SA}}\rangle, \tag{14}$$

$$|R^{(1)}\rangle = \frac{1}{\sqrt{n_{SA}}} \times \sum_j^{n_{SA}} |g_1, g_2, g_3 ... r_j .., g_{n_{SA}}\rangle. \tag{15}$$

For a single atom treatment and by considering a SA in state $|G\rangle$, $\Sigma_{RR}(\mathbf{r})$ can be represented as

$$\Sigma_{RR} = \Sigma_{RG}.\Sigma_{GR}, \tag{16}$$

where

$$\Sigma_{RG} = \frac{\sqrt{n_{SA}}\,\Omega_c\Omega_p\Sigma_{GG}}{\Delta_2\left(i\gamma_{eg} + \Delta_p\right) - \left|\Omega_c\right|^2}. \tag{17}$$

Using Eqs. (16) and (17) and with considering $\Sigma_{GG} + \Sigma_{RR} = 1$, the final expression for Σ_{RR} can take the form as

$$\Sigma_{RR} = \frac{n_{SA}\left|\Omega_c\right|^2\left|\Omega_p\right|^2}{\left|\Omega_c\right|^2\left|\Omega_p\right|^2 n_{SA} + [\Delta_2\Delta_p - \left|\Omega_c\right|^2]^2 + \gamma_{eg}^2\Delta_2^2}. \tag{18}$$

Finally, the optical susceptibility of the proposed atomic medium can therefore be calculated as

$$\chi = \xi\left[\Sigma_{RR}\frac{i\gamma_{eg}}{\gamma_{eg} - i\Delta_p} + [1 - \Sigma_{RR}] \times \frac{i\gamma_{eg}}{\gamma_{eg} - i\Delta_p + |\Omega_c|^2[\gamma_{rg} - i\left(\Delta_2 - \left\langle s\left(\mathbf{r}\right)\right\rangle\right)]^{-1}1}\right], \tag{19}$$

where $\xi = \frac{2N_{SA}|\mu_{eg}|^2}{\hbar\epsilon_0}$. From Eq. (18), Σ_{RR} depends on n_{SA} which directly related to the super-atom picture. By which, we can clearly see the blockade effect from the quantity Σ_{RR} which arises from the dipole-dipole interactions between atoms. Thus, the blockade effect change the susceptibility of the system as shown in Eq. (19). For the two extremal cases, the blockade effect is so strong, when $\Sigma_{RR} \to 1$ so that the probe field sees a two-level system. On the contrary, for the non-interacting atoms, $\Sigma_{RR} \to 0$ and whole of the system is reduced to a single three-level EIT configuration.

3. Results and discussion

Previously, many experiments have been performed with the realization of EIT in the Rydberg atoms [31]. In Ref. [31], strong vdW interactions have been considered between the atomic Rydberg states and has been observed experimentally that the transmission within the EIT widows decreases with increasing the probe field intensity. Later, a theoretical model has been developed

[32] which has revealed the reduction of transmission with increasing the probe field in EIT window as has been reported in Ref. [31]. The basic approach of the theoretical model [32] was based on the treatment of atomic medium consist of super-atoms (SAs), where each SA represented by collective states of atoms in the blockade volume whereas each SA have only one Rydberg excitation. When a weak probe field propagates through the EIT medium, a small attenuation has been reported [31,32]. The attenuation of light propagation through the atomic medium increased by increasing the input probe field intensity. It is due to the fact that for a high intensity of probe field there exist more than one photon per SA, and the excess photons are subjected to enhance the absorption of two-level atoms [32].

In the following, we proceed the same concept as used in Refs. [31,32] and study GH shift in the Rydberg atoms using the blockade mechanism. We consider an ensemble of cold ^{87}Rb atoms with energy levels $|g\rangle \equiv 5S_{1/2}|F=2, m_F=2\rangle$, $|e\rangle \equiv 5P_{3/2}|F=3, m_F=3\rangle$ and $|r\rangle \equiv 60S_{1/2}$. Using Eq. (4), we plot GH shift in the reflected light versus incident angle θ ranging from 0 to 1 radian, see Fig. 2. We consider different input probe field intensities i.e., $\Omega_p = 0.5\gamma$, 0.1γ, 0.01γ and 0.001γ and investigate the effect of probe field intensity on the GH shifts in the reflected light. We observe negative GH shifts in the reflected light for different incident angle θ. The plot shows that the amplitude of the GH shift is high for small values of Ω_p and decreases for large values. The amplitude of the GH shift at $\Omega_p = 0.5\gamma$ is very small as compared to $\Omega_p = 0.001\gamma$. This is due to the fact that for small values of Ω_p the input field intensity is very low and for that the medium encounters linear EIT response and the transmission is high. It means that the penetration depth of the probe field is high for low probe field intensity. And it is also developed that the amplitude of the GH shift is directly proportional to the penetration depth of the field inside the medium. We get giant GH shift in the reflected light for small values of Ω_p. In contrary of low probe field intensity, we investigate small negative GH shift in the reflected light for large values of probe field intensities which is according to the earlier observation of reduction of transmission of probe field in EIT window [31,32]. The decreasing of amplitude of the GH shift for large values of Ω_p is due to higher possibility of two or more than two photons excitation in Rydberg states $|r\rangle$ in the same super-atom. The two or more than two photon excitation in SA suppress the linear EIT response and the medium then behaves like an absorber.

To see a more clear picture of position of resonances in Fig. 2. Next, we discuss about the reflection coefficient ($|r|$) and phase (ϕ_r) of the reflected light. We plot the reflection coefficient and phase of the reflected light versus incident angle θ ranging from 0 to 1 radian for $\Omega_p = 0.5\gamma$ and $\Omega_p = 0.001\gamma$, see Fig. 3. The dips in the reflection curves in Fig. 3a, b correspond to the resonance condition. Similarly, the plots in Fig. 3c, d reveals that the phase change at those incident angle where the dips occur in the reflected light. Now the resonances in Fig. 2 describe the GH shifts in the reflected light for different incident angles. The GH shifts occur at those incident angles where there are dips in the reflection coefficient and phase change of the reflected light. The deep dips and steep change of phases show giant GH shifts in the reflected light for $\Omega_p = 0.001\gamma$, see Fig. 3b–d.

As reported in Refs. [31,32] the transmission probe field intensity depends on the probe and the control field intensities. Following these investigations, we study the GH shift in the reflected light for different choices of probe (Ω_p) and control (Ω_c) field intensities. We consider incident angle θ=0.13 radian and plot the GH shift versus Ω_p ranging from 0.0001γ to 1γ, see Fig. 4 (a). The plot in Fig. 4(a) shows that the amplitude of GH shift in the reflected light varies with changing the probe field intensity. For small values of Ω_p, the amplitude of the GH shift is high and decreases with increasing the values of Ω_p. The inset in Fig. 4 (a) shows that the amplitude of the GH shift is very small for higher values of Ω_p. This is in accordance with the previous theoretical and experimental investigations [31,32], where the transmission of the probe field is maximum at low input intensities and decreases when the input intensity increases. We also study the role of control field Ω_c on the amplitude of the GH shift and reveal that the amplitude of the GH shift varies with different choices of control field. By considering the incident angle θ=0.13 radian, we plot GH shift in the reflected light versus the

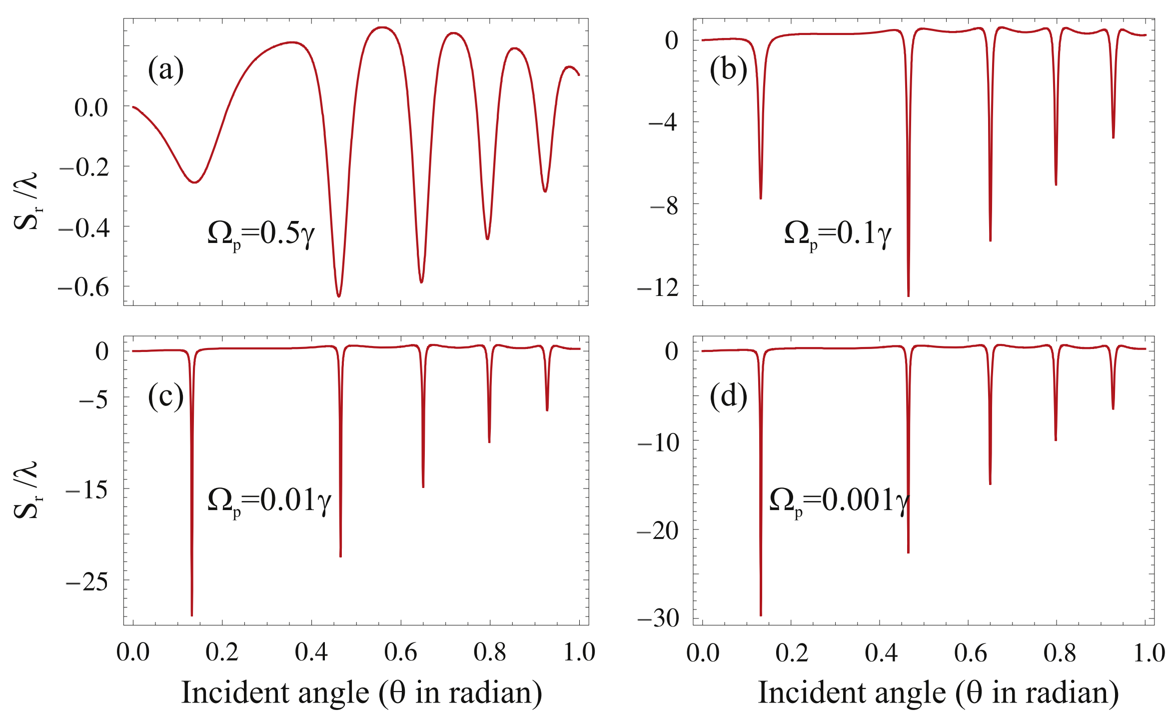

Fig. 2. The GH shift (S_r/λ) versus incident angle θ for (a) $\Omega_p = 0.5\gamma$ (b) $\Omega_p = 0.1\gamma$ (c) $\Omega_p = 0.01\gamma$ (d) $\Omega_p = 0.001\gamma$. The other parameters are $\gamma = 10^6$ MHz, $n_{SA} = 8$, $\Omega_c = 9\gamma$, $\Delta_c = \Delta_p = 0$, $\gamma_{eg} = 2\gamma$, $\gamma_{rg} = 0.01\gamma$, $\epsilon_1 = 2.22$, $d_1 = 0.2$ μm, $d_2 = 5$ μm.

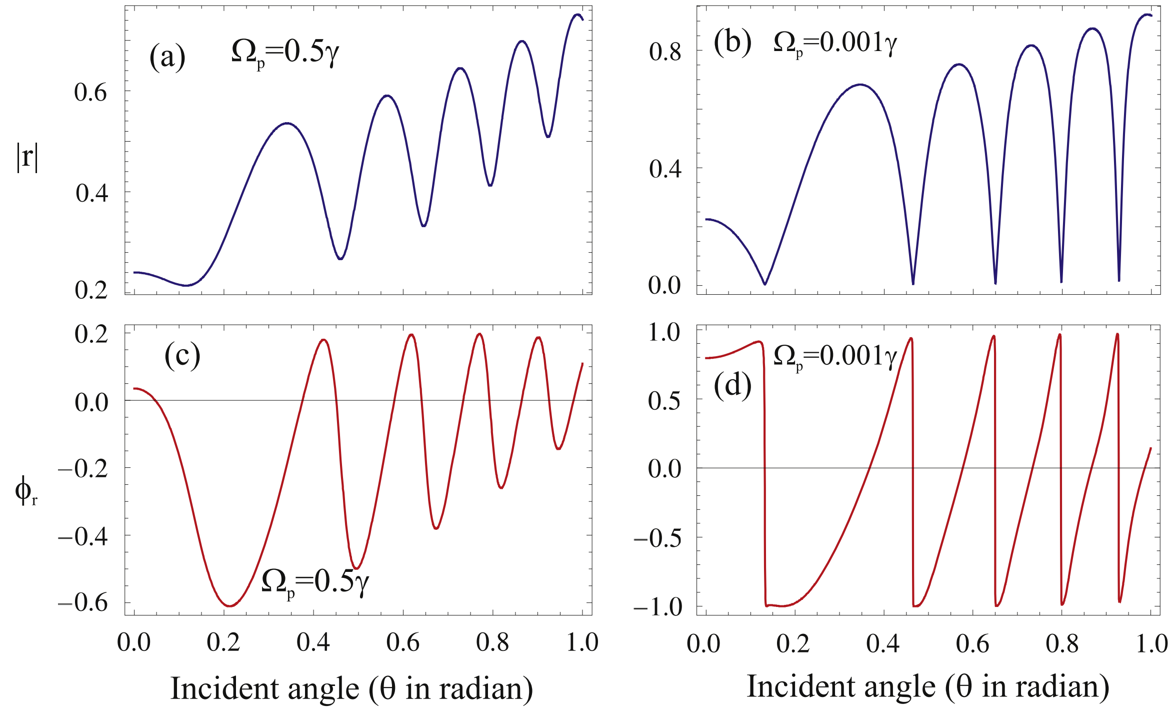

Fig. 3. The reflection coefficient ($|r|$) versus incident angle θ for (a) $\Omega_p = 0.5\gamma$ (b) $\Omega_p = 0.001\gamma$. The phase (ϕ_r) of the reflected light versus incident angle θ for (c) $\Omega_p = 0.5\gamma$ (d) $\Omega_p = 0.001\gamma$. The other parameters remain the same as those in Fig. 2.

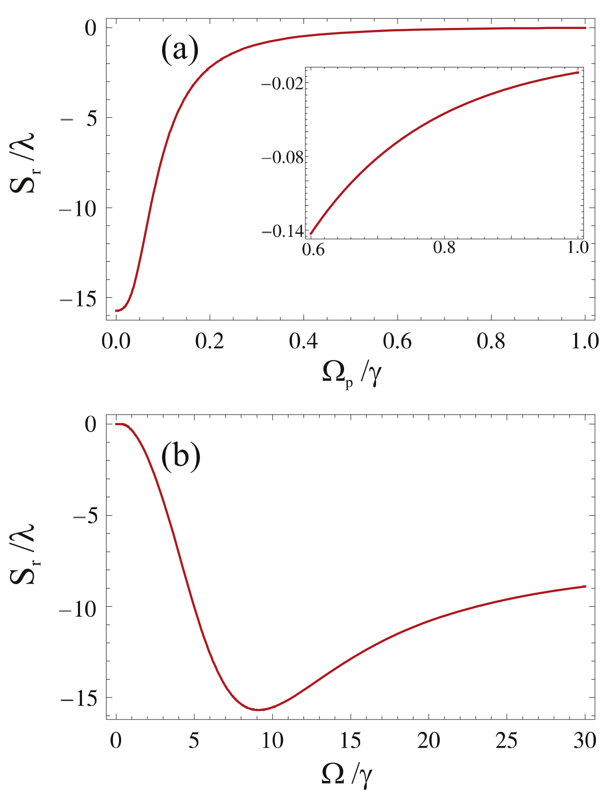

Fig. 4. The GH shift (S_r/λ) versus (a) probe field Ω_p for $\Omega_c = 9\gamma$ and (b) control field Ω_c at incident angle θ=0.13 rad by considering $\Omega_p = 0.001\gamma$. The other parameters remain the same as those in Fig. 2.

strength of control field Ω_c ranging from 0 to 30γ as shown in 4(b). In the absence of control field the atomic configuration reduces to simple two-level system and the medium then behaves like an absorber and highly absorption takes place of the input probe field. It is clearly shown in 4(b) that the amplitude of the GH shift is zero at $\Omega_c = 0$. The amplitude of the GH shift increases slightly with increasing the strength of control field and reaches to its maximum position around $\Omega_c = 9\gamma$ and then decreases for further increase of the control field.

Next, we exploit the investigation of negative GH shift in the reflected light using the concept of total group index (N_g) of the cavity. We follow the same approach as has been reported previously by considering the group index of the cavity N_g that depends on the thickness of the cavity and the derivative (with respect to probe light frequency) of the phase corresponding with the reflected probe field as $N_g \approx 1/L \times d\varphi_r/d\omega_p$ [9]. Here, $L = 2d_1 + d2$ and ϕ_r is the phase of the reflected light field. Now we show that the GH shift in the reflected light depends upon group index N_g of the total cavity. For negative GH shift in the reflected light the group index must be negative. We consider the incident angle θ=0.13 rad and plot the group index N_g versus probe (Ω_p) and control (Ω_c) field intensities, see Fig. 5. The plot in Fig. 5 (a) shows that the group index is negative for all values of probe field intensity ranging from 0.001γ to 1γ. The group index of the cavity is high for small values of probe field intensities and decreases with increasing Ω_p which is according to the earlier investigation of Fig. 4(a). In Comparison of Figs. 4(a) and 5(a), we conclude that the probe field intensity has the same effect on the GH shift S_r/λ and total group index N_g of the cavity. Similarly, we plot the group index N_g versus control field Ω_c as depicted in Fig. 5 (b). The group index of the total cavity remains negative for all values ranging from 1 to 30γ. Comparing Figs. 4(b) and 5(b), we can conclude that the control field has the same effect on GH shift S_r/λ and total group index N_g of the cavity. This is in agreement with our earlier investigation [9] that whenever the cavity group index is negative the GH shift is also negative.

Finally, we study the effect of vdW shift ($s(r)$) induces by the external *SAs* on the GH shift in the reflected light and group index of the total cavity. It is already mentioned that the vdW shift is directly related to Σ_{RR}, where Σ_{RR} play an important role in the susceptibility of the atomic system. When Σ_{RR} increases then two or multi photons per SA induce and creates absorption in the

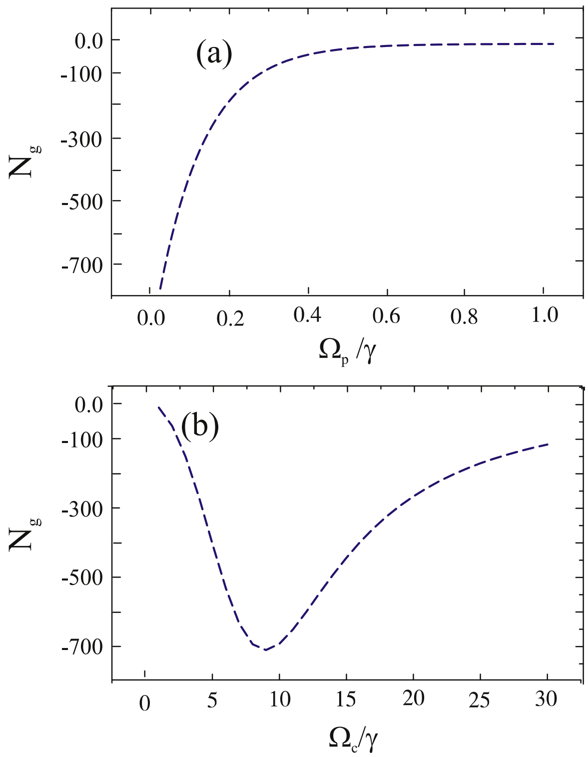

Fig. 5. The group index (N_g) versus (a) probe field Ω_p for $\Omega_c = 9\gamma$ and (b) the control field Ω_c at incident angle θ=0.13 rad by considering $\Omega_p = 0.001\gamma$. The other parameters remain the same as those in Fig. 2.

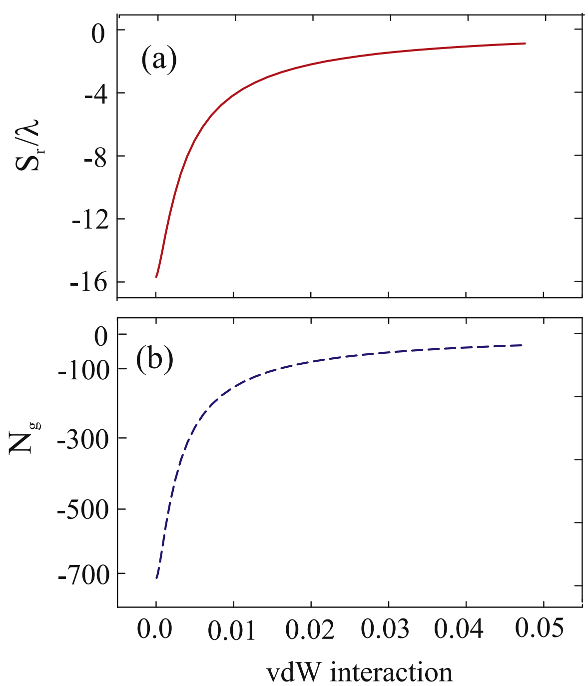

Fig. 6. (a) The GH shift (S_r/λ) versus vdW interaction $s(r)$ (b) the Group index (N_g) versus vdW interaction at incident angle θ=0.13 rad. The other parameters remain the same as those in Fig. 2.

system. Whenever, Σ_{RR} approaches to 1, then the three-level cascade atomic configuration reduces to Two-level atomic system and at that case the vdW interaction goes to its maximum value. It is due to the fact that the second term of Eq. (19) at right side vanishes and we get only two-level atomic system. It means that the strength of vdW interaction is directly dependent on the input probe field intensity. To see the influence of vdW interaction on the GH shift and group index, we consider incident angle θ=0.13 rad and plot the GH shift and group index versus $s(r)$, see 6. The plot shows that the amplitude of GH shift and group index is high for small values of the vdW shift (s(r)) and decreases exponentially when the vdW shift increases. This is due to the fact that the medium behaves like an EIT medium for small vdW shift and converts to two-level atomic system due to high absorption when the vdW shift goes to its maximum value.

4. Summary

In summary, we considered an atomic medium consist of three-level cascade configuration with Rydberg excitations in a cavity. A Gaussian shaped beam is incident on a cavity making an angle θ with the z-direction. We studied the GH shift in the reflected light using the blockade mechanism. It is revealed that the amplitude of negative GH shift in the reflected light is suppressed in atomic Rydberg states due to collective Rydberg excitations of super-atoms which depends upon the local input probe field intensity. For small values of probe field intensity, the amplitude of GH shift in the reflected light increased while decreased for strong input probe field. Furthermore, we explored the dependence of GH shift on the total group index of the cavity. In most practical implementation, SA model with Rydberg excitations have great importance and our theoretical results on the amplitude control of GH shift may provide a direction on the applications of GH shift with Rydberg excitations which provides strong long range atom-atom interactions.

References

[1] F. Goos, H. Häncehn, Ann. Phys. (Leipz.) 1 (1947) 333;
F. Goos, H. Häncehn, Ann. Phys. (Leipz.) 440 (1949) 251.
[2] R.H. Renard, J. Opt. Soc. Am. 54 (1964) 1190.
[3] D.K. Qing, G. Chen, Opt. Lett. 29 (2004) 872.
[4] H.K.V. Lotsch, J. Opt. Soc. Am. 58 (1968) 551.
[5] T. Hashimoto, T. Yoshino, Opt. Lett. 14 (1989) 913.
[6] X. Hu, Y. Huang, W. Zhang, D.-K. Qing, J. Peng, Opt. Lett. 30 (2005) 899.
[7] F. de Fornel, Evanescent Waves, Springer, 2001.
[8] L.G. Wang, M. Ikram, M.S. Zubairy, Phys. Rev. A 77 (2008) 023811.
[9] Ziauddin, S. Qamar, M.S. Zubairy, Phys. Rev. A 81 (2010) 023821.
[10] Ziauddin, S. Qamar, Phys. Rev. A 84 (2011) 053844.
[11] Ziauddin, S. Qamar, Phys. Rev. A 85 (2012) 055804.
[12] Ziauddin, S. Qamar, M. Abbas, Laser Phys. Lett. 11 (2014) 015201.
[13] Ziauddin, S. Qamar, Opt. Commun. 319 (2014) 1.
[14] Ziauddin, J. Mod. Opt. 62 (2015) 1660.
[15] Ziauddin, Y.-L. Chuang, R.-K. Lee, Phys. Rev. A 92 (2015) 013815.
[16] T.F. Gallagher, Rydberg Atoms, Cambridge University Press, Cambridge, 1994.
[17] M. Saffman, T.G. Walker, K. Molmer, Rev. Mod. Phys. 82 (2010) 2313.
[18] D. Jaksch, J.I. Cirac, P. Zoller, S.L. Rolston, R. Côté, M.D. Lukinj, Phys. Rev. Lett. 85 (2000) 2208.
[19] M. Muller, I. Lesanovsky, H. Weimer, H.P. Buchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502.
[20] H. Weimer, et al., Phys. Rev. Lett. 101 (2008) 250601.
[21] H. Weimer, H.P. Buchler, Phys. Rev. Lett. 105 (2010) 230403.
[22] T. Pohl, E. Demler, M.D. Lukin, Phys. Rev. Lett. 104 (2010) 043002.
[23] K. Hammerer, A.S. Sorensen, E.S. Polzik, Rev. Mod. Phys. 82 (2010) 1041.
[24] A.E.B. Nielsen, K. Molmer, Phys. Rev. A 81 (2010) 043822.
[25] D. Petrosyan, M. Fleischhauer, Phys. Rev. Lett. 100 (2008) 170501.
[26] M.D. Lukin, et al., Phys. Rev. Lett. 87 (2001) 037901.
[27] T. Vogt, et al., Phys. Rev. Lett. 97 (2006) 083003.
[28] J. Honer, R. Low, H. Weimer, T. Pfau, H.P. Buchler, Phys. Rev. Lett. 107 (2011) 093601.
[29] J.L. Miller, Phys. Today 65 (2012) 14.
[30] J.-F. Huang, J.-Q. Liao, C.-P. Sun, Phys. Rev. A 87 (2013) 023822.
[31] J.D. Pritchard, D. Maxwell, A. Gauguet, K.J. Weatherill, M.P.A. Jones, C.S. Adams, Phys. Rev. Lett. 105 (2010) 193603.
[32] D. Petrosyan, J. Otterbach, M. Fleischhauer, Phys. Rev. Lett. 107 (2011) 213601.
[33] Yi-Mou Liu, Dong Yan, Xue-Dong Tian, Cui-Li Cui, Jin-Hui Wu, Phys. Rev. A 89 (2014) 033839.