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Nearly complete survival of an entangled biphoton through bound states in continuum in disordered photonic lattices

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Abstract: Bound states in the continuum (BICs) of periodic lattices have been the recent focus in a variety of photonic nanostructures. Motivated by the recent results about the photons evolving in BIC structures, we investigate the quantum decay of entangled biphotons through disordered photonic lattices. We report that the persistence of bound states in disordered photonic lattices leads to an interplay between the BIC and disorder-induced Anderson localized states. We reveal a novel effect resulting from such an interplay: a nearly complete quantum survival for the entangled biphoton respecting the antisymmetric exchange symmetry. This is in contrast to the complete vanishment in a periodic photonic lattice.

1. Introduction

Photonic lattices with photons walking inside offer an attractive platform with great flexibility and controllability to implement quantum walks. By endowing the walkers, i.e., photons with quantum attributes, a variety of celebrated models or dynamical processes have been demonstrated by photonic quantum walks [1–6]. One important achievement is the bound states with energy embedded in the continuous spectrum (BICs) which have been firstly observed in photonic lattices [7], and later in other periodic optical nanostructures [8–10]. The so-called BICs are counterintuitive modes of a system that exist within a continuous spectral range where energy leaks out of the system. They are localized resonances that have no coupling to the continuum. Actually, this concept became a well-known wave phenomenon since the early work of von Neumann and Wigner [11, 12]. In recent years considerable interest has arisen due to its relevance to various applications like a novel way of light trapping and lasing action [12, 13]. Now this BIC concept has been successfully brought into quantum optics [14, 15], exploiting the BIC in photonic lattices to investigate the effect of quantum statistics on the decay of entangled biphoton. It is showed that the BIC-trapping of nonclassical excitations has a dependence on the exchange symmetries of *identical* particles. Especially, for the entangled biphoton respecting antisymmetric exchange the survival probability will evolve into zero; while the symmetric biphoton can sustain some survival probability. Based on these, the concept of BIC filter for indistinguishable particle statistics was proposed. So far, most of studies on BIC states have been limited to the periodic systems, either classical or nonclassical excitations. They all have continuous spectra with BICs' energy immersed inside.

On the other hand, the disorder physics in wave systems has been fueling research since Anderson's original work [16]. Disorder is especially of natural relevance in optics, where random fluctuations are always presented. Studies of light propagating in disordered media have revealed a range of fascinating wave phenomena [17, 18], including Anderson localization of light [19], coherent backscattering [20–22], and statistical properties of light in the disordered ensembles [23, 24]. These phenomena originate from the wave interference of multiple scattering

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paths and come to appear after averaging over all configurations of disorder samples. Different from the periodic structures, disordered optical systems have point spectrum. Correspondingly, the optical waves are exponentially arrested in space. While such a concept has motivated discussions on classical light waves over thirty years [25, 26], the role of quantum nature of photons in disordered media has only recently come into attention [27–29]. As for the photonic BICs [12], either classical or nonclassical light, the effect of disorder remains to be elucidated. Therefore, considering the ubiquitousness of disorder, several interesting questions naturally arise: What is the fate of BICs in the disordered systems which are characterized by the point spectrum? And whether or not the disorder washes out the nonclassical behaviors of entangled photons?

In this work, we primarily dedicate to the stability of the BIC in disordered photonic lattices, and the corresponding quantum decay dynamics of entangled biphoton. We firstly test the stability of BIC states against the random perturbations and show that the symmetry-protected BIC states are robust against the disorder introduced in photonic lattices. This gives birth to an interplay between the BIC and disorder-induced localized states. Such an interplay produces an intriguing effect on the nonclassical transport of the entangled biphoton. The entangled biphoton is found to survive after averaging over all configurations of disorder. Surprisingly, the survival probability of the biphoton respecting the antisymmetric exchange symmetry can be enhanced up to one by the disorder. While for the biphoton with symmetric exchange the survival probability will saturate to a certain value with the increase of the disorder strength. These observations are fundamentally different from the quantum decay process of identical particles into the continuum of ordered lattices [15], where the survival probability is completely suppressed for the antisymmetric biphoton.

2. Stability of the bound states in the continuum against the disorder

Before elaborating on the results, it will be instructive to make some general remarks on the differences between the BICs and the disorder-induced Anderson localized states. A BIC is represented by a wave function with its extent bounded in a certain subset, assuming we divide the structure into two subsets [12]. Most theoretically studied and all experimentally available BICs are observed in uniform structures. Among them many are periodic in one direction which produces the continuous spectrum, and meanwhile the BIC is bounded in the other directions [12]. Therefore, a BIC state can be considered as a nonradiative resonance with its corresponding energy embedded in the continuum. Whereas the disorder-induced localized states are resulted from the spatial disorder and phase coherence together. They necessarily involve the interference of waves that follow various multiple-scattering paths in the disordered medium. They manifest themselves as an exponential decay in real space and are characterized by a localization length. Meanwhile, the disorder leads to the point energy spectrum. Therefore, the diffusion of waves will be completely suppressed by the disorder, as demonstrated by the paradigmatic phenomenon of Anderson localization of light [19]. Hereafter, the disorder-induced localized states will be referred to as the localized states so as to distinguish them from the BICs.

As an example, in this work we specify the symmetry-protected BICs in photonic lattices [7]. The photonic lattice system under investigation is formed by single-mode waveguides, as depicted schematically in Fig. 1(a). The waveguides are evanescently coupled and thus the input light will experience discrete diffraction during the evolution [30]. Such a photonic structure has been used to implement the 1D and 2D quantum walk of photons [1, 3, 31, 32]. As illustrated in Fig. 1(a), a pair of side-coupled waveguides symmetrically is attached above and below the leftmost extremity of the main chain. Apparently, this photonic structure is symmetric about x axis. When a system occupies a reflection symmetry, modes of different symmetry classes will be decoupled. It is found that a bound state belonging to a symmetry class resides in the continuum of another symmetry class [12]. In this photonic structure the states of the main chain

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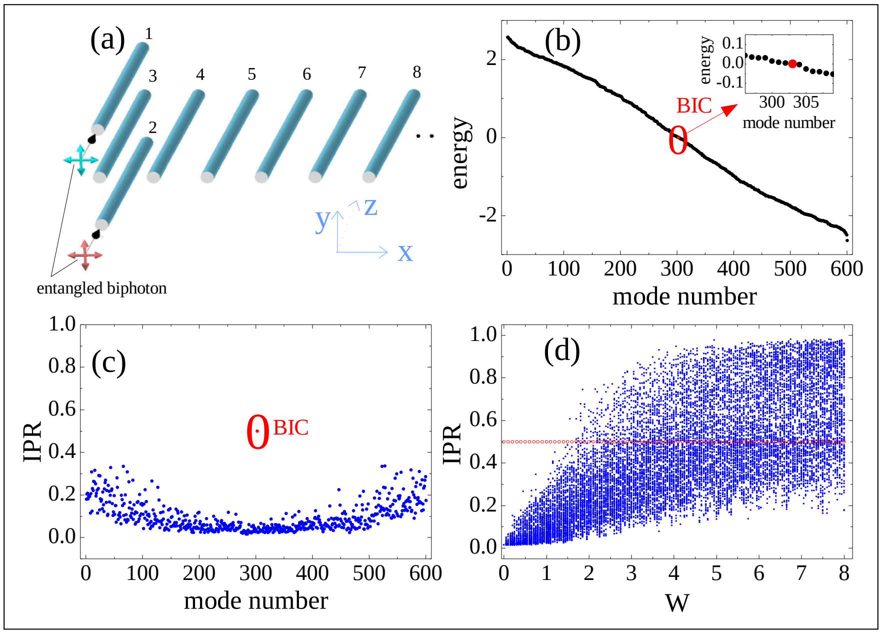

Fig. 1. (a) A sketch of a photonic implementation of the Fano-Anderson model [Eq. (1) in the text]. The light can hop along the transverse directions x, y and propagate along the longitudinal direction z. The nonclassical excitations correspond to the polarization-entangled biphoton (arrows) launched into the two side-coupled waveguides $n = 1, 2$. (b) Energy spectrum and (c) the corresponding inverse participation ratio (IPR) of eigenmodes for the disordered lattices with disorder strength $W = 2$. The inset in (b) gives a close-up of the energy spectrum around zero energy. (d) Evolution of IPRs of the eigenstates in a disordered BIC photonic lattice, calculated as function of the disorder strength W. Red empty circles indicates the IPR values of the BIC states formed, exactly equal to $1/2$.

constitute the continuous band. Meanwhile a bound state emerges in the vertical configuration, with its energy exactly equal to zero. This state is exclusively bounded in the side-chain and out-of-phase along the side-chain, and thus decoupled from the continuous band, as shown in Fig. 2(a). This is the very BIC mode [7]. The Fano-Anderson model that applies to such an array of evanescently-coupled optical waveguides reads

$$
\begin{aligned}
i\frac{da_{1,2}^{\dagger}}{dz} &= \beta_{1,2}a_{1,2}^{\dagger} + \kappa_{\perp}a_{3}^{\dagger},\\
i\frac{da_{3}^{\dagger}}{dz} &= \beta_{3}a_{3}^{\dagger} + \kappa_{\perp}a_{1}^{\dagger} + \kappa_{\perp}a_{2}^{\dagger} + \kappa_{\parallel}a_{4}^{\dagger},\\
i\frac{da_{n}^{\dagger}}{dz} &= \beta_{n}a_{n}^{\dagger} + \kappa_{\parallel}a_{n-1}^{\dagger} + \kappa_{\parallel}a_{n+1}^{\dagger}, \qquad n \geq 4.
\end{aligned} \tag{1}
$$

Here, $a_n^{\dagger}$ denotes the photonic creation operator of the guided mode at the nth-lattice site, κ the coupling constant between neighboring waveguides n and $n-1$, and β_n the propagation constant at site n. Inside the main chain the coupling coefficient between neighboring waveguides is $\kappa_{\parallel}$, while the two side waveguides are coupled to the main chain with strength $\kappa_{\perp}$. Here, the axial propagation coordinate z plays the role of time. Therefore, this equation is equivalent to the discrete Schrödinger equation, provided that the role of the time variable t is now played by the propagation distance z.

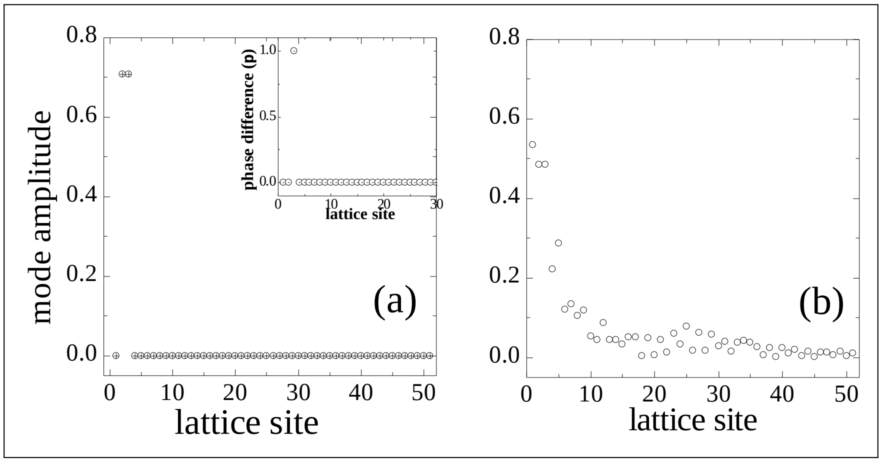

Fig. 2. Mode profiles of the BIC (a) and a typical localized state (b) in the disorder scenarios. In the inset, a phase difference at π is revealed for the BIC.

To make it more interesting, we add some disorder physics. In the integrated photonic lattices fabricated by the femtosecond direct-writing technique, the randomness can be achieved through varying the inscription velocity of the light pulses [33,34]. We first address the stability of the BIC against the local perturbations. In general, we find numerically that local fluctuations that preserve the symmetry are not harmful to BIC. We have computed the modes of the Fano-Anderson model in Eq. (1) with disordered propagation constants for sites $n \geq 3$ solely. The random propagation constants are distributed uniformly in the interval $\beta_n \in [-W, W]$. To quantify the localization properties of the wave functions, we calculate the inverse participation ratio (IPR) defined as follows [18]:

$$\text{IPR}^{(i)} = \frac{\sum_n |\psi_n^{(i)}|^4}{(\sum_n |\psi_n^{(i)}|^2)^2}, \tag{2}$$

where the superscript i denotes the ith mode $\psi^{(i)}$, which is sorted in descending order according to their energies. For spatially extended states IPR approaches zero, whereas it is finite for localized states. As shown in Fig. 1(b) for the disorder strength $W = 2$, the continuous spectrum of the ordered lattice [7] is transformed into the point spectrum by the randomness in the backbone lattice. However, one single state sustains its eigenenergy $E = 0.0$ regardless of the disorder, as marked with the Red point in the inset of Fig. 1(b). Meanwhile, Fig. 1(c) illustrates that its corresponding IPR value falls just outside the spectra of the localized states. Put more formally, because the structure's reflection symmetry about the horizontal axis is preserved, this state is robust against the random perturbations introduced in the main chain. This state is exactly the BIC. This is substantiated by Fig. 1(d), showing that the IPR of this state remains invariant, being always one-half, throughout scanning the disorder strength. We also display the mode profiles of this BIC state and a typical localized state in Fig. 2(a) and 2(b), respectively. One can see that the amplitude distribution of this BIC state is equally weighted for the $1st$ and $2nd$ sites whereas with the phase difference π. Therefore, this state is identical to the bound state in the continuum of the ordered lattice, which has been previously observed in [7]. This is why we still call it BIC state. It should be noted that this bound state now resides in the point spectrum characteristic of the disordered photonic lattices, as shown in Fig. 1(b), rather than the continuum of the ordered case. The point spectrum is related to the exponentially localized states. Therefore the diffusion of excitation will be hindered in the main chain. On the other hand, note that there exists spatial overlapping between the BIC and the localized states. The overlapping of wave functions causes the interference between the BIC and the localized states. Thus, we expect that new quantum behaviors of nonclassical light will emerge due to such an interplay.

We first examine the decay dynamics of classical light waves in the disordered photonic networks. A light beam coupled to the $1st$ waveguide will excite a superposition of different modes, having among them the largest overlapping with the BIC mode. We monitor the dynamical evolution of light intensity, depicted in Fig. 3(a), inclusive of the fractional intensity at 1st site $I(z) = |a_{i=1}(z)|^2$, as shown Fig. 3(b). Figure 3(a) shows that the excitation is transferred to the backbone lattice during the temporal evolution. Note that a nonvanishing value of $I(z)$ in Fig. 3(b) is being approached after some transient time, signaling a fractional decay is attained. This corresponds clearly to the trapping of the light at the $1st$ site and is related to the coexistence of the BIC and localized states. It is worthy noting that the single-particle quantum walk can be classically implemented [35]. Thus, the classical fractional intensity measured here is essentially equivalent to the quantum decay dynamics of the single photon.

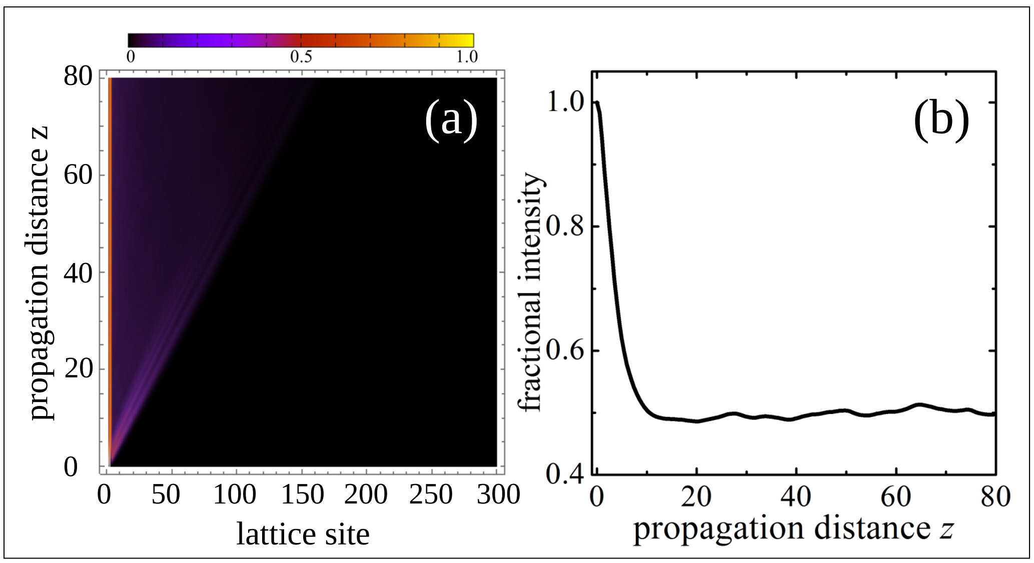

Fig. 3. (a) Evolution of light intensity and (b) the survival probability for the classical excitation at the waveguide $n = 1$. The color bar indicates the normalized light intensity.

3. Statistical characteristics of the nonclassical excitations

Having described the classical dynamic properties of the disordered BIC photonic lattices, we set about solving the nonclassical dynamics of entangled photons with different exchange symmetries. The involved photons themselves are bosons. However, polarization-entangled biphoton, $\Psi = \frac{1}{\sqrt{2}}(|HV\rangle + e^{i\phi}|VH\rangle)$, can be exploited to simulate various statistical behaviors of identical particles. These states are an entanglement of horizontally polarized photon $|H\rangle$ and vertically polarized photons $|V\rangle$, wherein with a phase parameter ϕ introduced. By interchanging the horizontal and vertical polarizations a phase ϕ is acquired, *viz.*, $\Psi = \frac{1}{\sqrt{2}}(|VH\rangle + e^{i\phi}|HV\rangle)$. Thus, $\phi = 0$ (π) corresponds to the symmetric and antisymmetric wave functions, respectively. Therefore, bosonic and fermionic statistics can be simulated by setting the entanglement phase ϕ. Two groups have performed the experiments by using the polarization-entangled biphoton walking in integrated photonic circuits [36,37]. With the adjustment of the entanglement phase ϕ, they successfully simulated the symmetric and antisymmetric exchanges of identical particles. In this way they have been able to investigate how the exchange symmetry influences the dynamics

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of identical multiparticles traveling in quantum networks, either ordered [38] or disordered [28].

We now show the fate of nonclassical excitations of the biphoton propagating through the random Fano-Anderson photonic lattices. The introduction of disorder will exert an important influence on the dynamics of multiple identical particles. After coupling the entangled biphoton into the waveguide modes $|1\rangle$, $|2\rangle$ simultaneously, we inspect the evolution of survival probability $S(z) = \langle |\langle \Psi_{(0)} | \Psi_{(z)} \rangle|^2 \rangle$ of the identical photons, which represents the detecting probability of the biphoton still being in the initial states $|\Psi_0\rangle$ during the propagation. Here the outer $\langle ... \rangle$ signifies averaging over an ensemble of disorder configurations. To be precise, we inject a polarization-entangled biphoton $|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger b_2^\dagger + e^{i\phi} a_2^\dagger b_1^\dagger)|0\rangle$ into the waveguide modes $|1\rangle$, $|2\rangle$, where $a^\dagger$and $b^\dagger$ are the creation operators of horizontally and vertically polarized photons, respectively. The biphoton will evolve into $|\Psi(z)\rangle$ when the dynamics is described by Eq. (1). By integrating Eq. (1) one obtains the creation operator at the propagation distance z: $a_n^\dagger(z) = \sum_m U_{n,m}(z) a_m^\dagger(z = 0)$. Here the time-evolution operator $U_{n,m}(z) = (\exp[-i\widehat{H}z])_{n,m}$ is the unitary evolution operator given by the Hamiltonian, and describes the amplitude for transition of a single photon initially located at site m to site n at the propagation distance z [39]. The probability to find the time-evolving state $|\Psi(z)\rangle$ in its initial state $|\Psi(0)\rangle$ is referred as the survival probability $S(z) = |\langle \Psi(0)|\Psi(z)\rangle|^2$. Since the variable t can be mapped onto the propagation distance z along the photonic lattices, one can measure the survival probability $S(z)$ at the output exit. Figure 4(a) depicts the temporal evolution of $S(z)$ of entangled biphoton respecting symmetric and antisymmetric exchange symmetry. Clearly, because of the light trapping originating from both the BIC and localized states, the corresponding evolution of $S(z)$ reaches an asymptotic steady value after some initial transient time. Moreover, a striking observation is the nonvanishing survival probability for the antisymmetric polarization-entangled input. This forms a sharp contrast with the ordered lattice, where the survival is completely suppressed. This distinct difference is associated with the interplay of the BIC and localized states. In the following, we will elaborate the corresponding mechanism.

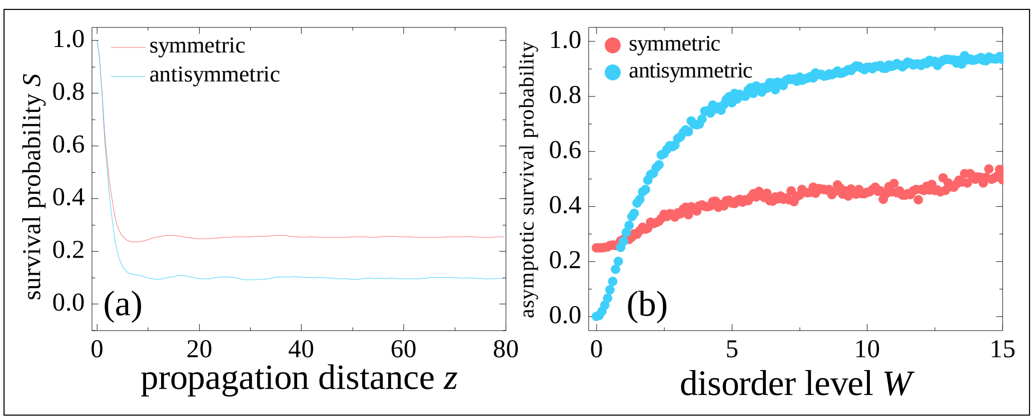

Fig. 4. (a) Ensemble-averaged decay dynamics of the survival probability of the entangled biphoton in the disordered BIC photonic lattices. (b) Asymptotic survival probability $S(z \rightarrow \infty)$ as a function of the disorder level W in the disordered BIC lattices.

To identify the role of the interplay between the BIC and the localized states, we numerically investigate the asymptotic survival probability of biphoton $S(z \rightarrow \infty)$. In determining the asymptotic behavior of survivals, we evolve the biphoton in disordered photonic lattices and calculate $S(z)$ after a long propagation distance z, whereupon the ensemble-averaged light intensity becomes stationary. By increasing the amount of disorder in the backbone lattice, we obtain $S(z \rightarrow \infty)$ as a function of disorder strength W. We plot in Fig. 4(b) this asymptotic value against the disorder strength W. A feature visible in the figure is that the asymptotic probability for both of exchange symmetries undergoes a gradual increase at weak disorder strength W, followed by a steady raise up to a saturation value with growing disorder level, i.e., red and green doted lines in Fig. 4(b). Surprisingly, in the course of increasing the disorder we observe that after the intersection point the survival probability of antisymmetric biphoton far exceeds that of symmetric biphoton. This signifies that the survivability of antisymmetric biphoton is stronger than that of symmetric biphoton. Meanwhile, a much stronger disorder strength leads to an estimate of survival probabilities for the symmetric and antisymmetric entanglements, 0.5 and 1.0, respectively. This reveals that a strongly disordered BIC lattice supports a nearly complete recovery of initial excitations with antisymmetric exchange, being of near-unity probability. The ordered photonic lattice, by contrast, has the complete suppress of antisymmetric biphoton [15]. Since the ordered photonic lattice has no localized states, we attribute these intriguing features to the interplay of the BIC and localized states. In detail, disorder leads to localized states sitting around the left extremity of the main lattice, which overlap with the BIC state in the vertical configuration, as illustrated in Figs. 2(a) and 2(b). That is to say, jointly with the localized states the BIC traps the nonclassical excitation at the two side-coupled waveguides during the time evolution. Moreover, it is worthy noticing that the mode overlapping also implies that there exists several states that can accommodate the two photons. As a consequence, these will enhance the survival probability of the initial states.

Here, we further elaborate upon the working mechanism in terms of the exchange symmetry. Let us first consider the decay dynamics involving the antisymmetric biphoton $|\Psi(\phi = \pi)\rangle$. Given the configuration in Fig. 1(a), the ordered photonic lattice hosts one single BIC state [7]. Thus, it is the Fermi-Dirac statistics, abided by the antisymmetric biphoton, that determines the survival of the biphoton: It prohibits the two photons from occupying simultaneously the single BIC state. In other words, the BIC expels the biphoton from the subspace of side-chain [15]. However, for the disordered photonic lattices, the randomness leads to the localization of the nonclassical excitations around the input positions. The antisymmetric exchange tends to anti-bunch the two photons and thus splits them across the opposite extremities of the side-chain [27]. Obviously, the BIC expels the antisymmetric biphoton while the localized states trap them. Therefore, the blankness of the measured survival in the periodic lattice, which is entirely associated with the single BIC, is compensated by the localized states in the disordered scenario. The survival probability measured at the input modes $|1\rangle$ and $|2\rangle$ takes a non-zero value. Consequently, the asymptotic survival probability $S(\infty)$ is expected to grow monotonically with the amount of disorder. In particular, as the disorder is very large, the localization effect will become more pronounced: The localized states arrest the biphoton completely and cause the localization process by the disorder to overwhelm the BIC's expelling; accordingly, the biphoton cannot leave from the input modes $|1\rangle$ and $|2\rangle$, and then the survival probability is nearly 1. That is, the antisymmetric biphoton stays on at $|1\rangle$ and $|2\rangle$ modes in the strongly disordered lattices. Therefore, by tuning the disorder, it is accessible to almost completely revive an antisymmetric biphoton state in the BIC photonic lattices.

We then shift our focus to the biphoton with symmetric entanglement phase $|\Psi(\phi = 0)\rangle$ in the disordered photonic lattices. Totally different from the antisymmetric biphoton, the symmetric biphoton obeys the Bose-Einstein statistics. When the two-particle is injected into the device, the BIC can trap the photons on the modes $|1\rangle$ and $|2\rangle$ simultaneously with a certain probability [15]. At the same time, the localized states can also support the retention of the symmetric biphoton in the side-chain. This fact indicates that both kinds of states can jointly retain the biphoton on the side-chain, and thus leads to the growing of the asymptotic survival probability as W is increased, as demonstrated in Fig. 4. However, in this case the largest survival probability observed is restricted around 0.5 even for very strong disorder. This is distinct from the antisymmetric biphoton. The reason lies in the fact that the two photons are more likely to be bunched at the same site in the disordered lattices [27]. This bunching behavior suggests that the photon pairs prefer co-occurring in the middle of the side-chain, namely, the mode $|3\rangle$. Yet, BIC favors the biphoton's appearance in the modes $|1\rangle$ and $|2\rangle$ simultaneously, as pointed in [15]. This mutual competition suggests that the probability that both particles remain in the launch waveguides $|1\rangle$ and $|2\rangle$ will be halved. This is consistent with what we observed, indicating the fundamental difference between the symmetric and antisymmetric biphoton in the disordered systems.

The sections above highlight the interesting nonclassical phenomena occurring when a BIC energy is immersed in the point spectra of disordered lattices. In order to clarify that the light trapping in the side-coupled waveguides is due to the interplay between the BIC and localized states, rather than the disorder-induced localized states solely, we introduce an asymmetric detuning between the side-coupled waveguides, i.e., $\beta_1 \neq \beta_2$. This asymmetry destroys the BIC mode. In this situation the asymptotic survival probability of antisymmetric biphoton always exceeds that of symmetric biphoton for any amount of disorder. The curve $S(\infty)$—W of antisymmetric biphoton always lies above that of symmetric case for any amount of disorder. While for the ordered photonic lattice, this detuning leads to the vanishing survival for biphoton with any exchange symmetries.

4. Conclusion

Summarizing, we have presented the explorations of the nonclassical decay dynamics of entangled biphoton walking in the disordered photonic lattices. Such photonic lattices are associated with the coexistence of the BIC and the disorder-induced Anderson localized states, and thus the interplay between them. Using the definition of survival probability, we have shown its influence on the decay dynamics of the symmetric and antisymmetric polarization-entangled biphoton, obeying Bose-Einstein and Fermi-Dirac distribution, respectively. The survival probability of either of entangled biphoton can be enhanced by such an interplay. We have demonstrated that the nonclassical excitations acquire characteristic features specific to their exchange symmetries. We highlight the occurrence of nearly complete survival of the antisymmetric biphoton. These behaviors are in contrast with the quantum decay process of identical particles into the continuum of ordered systems [15]. Our work is a hint of the richness of the statistical properties of disordered systems, which one can get with the photonic lattices by choosing the nonclassical inputs and lattice parameters. One interesting line is along the topological BICs in photonic networks with entangled biphoton walking inside [40, 41]. Our study is expected to find broad applications across a variety of fields. Prominent examples include ultracold atoms in optical potential. In this context significant progress has been made on the single atom addressing and imaging through quantum gas microscopes, and the Anderson localization of ultracold atoms in disorder potentials [42]. All these advances will facilitate the further studies of the many-body statistical properties.

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