Transverse instability of transverse-magnetic solitons and nonlinear surface plasmons

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We analyze stability of the TM polarized optical solitons and nonlinear guided waves localized at a metal-dielectric interface. We demonstrate, both analytically and numerically, that the spatial solitons can experience vectorial transverse modulational instability that leads to the generation of arrays of two-dimensional TM polarized self-trapped localized beams. In a sharp contrast, we reveal that the transverse instability is completely eliminated for nonlinear surface plasmons. © 2009 Optical Society of America

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Nonlinearity-induced instabilities are observed in different physical systems, and they provide the most dramatic manifestation of strongly nonlinear effects. Transverse (or symmetry breaking) instabilities of solitary waves were predicted theoretically a long time ago [1], but only recently such instabilities were observed experimentally for different types of spatial optical solitons [2–4].

When several components of an optical beam are coupled together, they form a vector soliton [3]. The most natural example of a vector soliton is the TM-polarized spatial soliton described by a system of coupled nonlinear equations for two field components [5–7]. The interest to the TM polarized nonlinear waves has been renewed recently in connection with the research on subwavelength localization in nanophotonics associated with the light localization in the form of surface plasmon polaritons [8,9].

In this Letter, we study the transverse instability of the TM polarized spatial optical solitons and demonstrate, both analytically and numerically, that the spatial solitons can experience *vectorial transverse modulational instability* observed for both the components; this instability leads to the generation of arrays of two-dimensional TM polarized self-trapped beams. In addition, we address an important question of stability of nonlinear guided waves localized at an interface between a metal and a nonlinear Kerr-like dielectric medium. In a sharp contrast to TM solitons, we reveal that the transverse instability of nonlinear plasmons is completely eliminated.

First, we consider the propagation of TM polarized light in a nonlinear dielectric. The governing Maxwell's equations for the vector field in a nonlinear material can be expressed in the well-known vectorial form [5,7]

$$[\nabla^2 + k^2 n^2]E = -\nabla[E \cdot \nabla \ln n^2]. \qquad (1)$$

To find spatially localized waves in a nonlinear medium, we present the electrical and magnetic fields in the form, $\vec{E}=[e_x(x)\hat{x}+e_z(x)\hat{z}]\exp(i\beta z)$ and $\vec{H}=h_y \exp(i\beta z)$, where β is the soliton propagation constant. Without loss of generality, we assume $(e_x, e_z) \equiv (u, iv)$ and rewrite the nonlinear equations in the form

$$\left(\frac{d^2}{dx^2} + k^2 n^2 - \beta^2\right) u = -\frac{d}{dx}\left(u \frac{d \ln n^2}{dx}\right),$$

$$\left(\frac{d^2}{dx^2} + k^2 n^2 - \beta^2\right) v = -\beta u \frac{d \ln n^2}{dx};$$

$$n^2 = n_0^2 + \alpha(|u|^2 + |v|^2), \qquad (2)$$

where u and v are the fields polarized in the x and z directions, respectively, both vanishing for $|x| \to \infty$. k is the free-space wave number, and n is the scalar refractive index of the nonlinear Kerr-like medium, as in thermal- or electrostriction-type nonlinearities [5].

The two-component localized solutions of Eq. (2) for the TM polarized vector soliton can be found numerically [5,6], and the results are summarized in Figs. 1(a) and 1(b). In Fig. 1(b), we show the power curves for two polarizations u and v, where $P_u = \int_{-\infty}^{\infty} |u|^2 dx$ and $P_v = \int_{-\infty}^{\infty} |v|^2 dx$.

To analyze the transverse instability of the TM polarized solitons, we apply a standard linear stability analysis [2]. We present the solutions in the form $\hat{E} = [u_0, 0, -iv_0]^T \exp(i\beta z) + [\Delta e_x, \Delta e_y, \Delta e_z]^T \exp(i\beta z)$, where u_0 and v_0 are the soliton components, and the

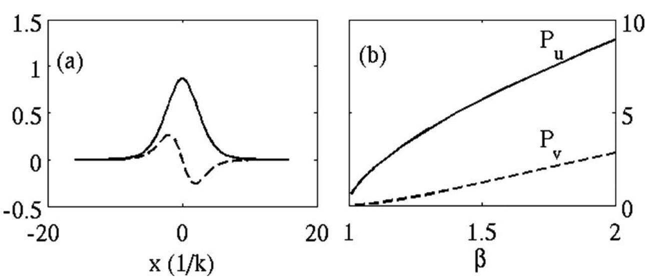

Fig. 1. (a) Spatial TM soliton. Shown are the electric field components u (solid curve) and v (dashed curve). (b) Partial powers P_u and P_v of the electric field components.

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perturbations are described by the functions $\Delta e_j=[\varepsilon(p_j+iq_j)\exp(i\lambda z+ik_y y)+\varepsilon(p_j^*+iq_j^*)\exp(-i\lambda^* z-ik_y y)]$, ε being a small parameter, and the subscripts are $j=x,y,z$, respectively. Next, we rewrite the nonlinear refractive index as $n^2=n_0^2+\alpha(|e_x|^2+|e_y|^2+|e_z|^2)$ to involve the y-polarized perturbations and substitute the ansatz into Eq. (1) for TM polarized nonlinear waves. In the first-order approximation in ε, we obtain a set of linearized equations for small perturbations and solve numerically the corresponding eigenvalue problem.

The growth rate of the linear modes responsible for the transverse instability of the TM solitons is shown in Fig. 2 for several values of the soliton propagation constant β. When the transverse modulations are void, i.e., when $k_y=0$, the amplitude growth rate of the perturbed field vanishes, which implies that all one-dimensional TM polarized vector solitons are stable for the Kerr-like nonlinearity, similar to their TE polarized scalar counterparts [3]. However, the TM spatial solitons are unstable to transverse modulations ($k_y>0$), resulting in the formation of two-dimensional array of self-trapped beams as illustrated later in Figs. 5(a) and 5(b). Moreover they can induce the E_y component, which changes their polarization [5].

Nonlinear TM polarized guided waves propagating along an interface separating a metal and nonlinear Kerr-like dielectric have been studied more than 20 years ago [10–13], and they provide a nonlinear generalization of surface plasmon polaritons actively studied these days in nanophotonics [8]. Nevertheless, the stability of such nonlinear plasmon modes has never been addressed. The nonlinear plasmon mode can be viewed as a TM polarized soliton propagating along the interface between a metal and nonlinear dielectric. In this case, the transverse field should be continuous across the interface. Using the soliton solutions found above, we construct the solutions for the nonlinear plasmons by employing a matching procedure [10]. If the interface is at a auxiliary location, $x=x_0$, the equations for the TM waves in a metal with the dielectric permittivity $\epsilon_M(\omega)=\epsilon_0[1-\omega_p^2/(\omega^2+i\omega\Gamma)]$ can be written in the form, $(\mathrm{d}^2/\mathrm{d}x^2+k^2\epsilon_M(\omega)-\beta^2)H_y=0$ for $x<x_0$; and as Eq. (2), for $x>x_0$. By matching the fields at $x=x_0$ at $\Gamma=0$, we calculate the dispersion relation for TM modes from the continuity of the transverse components [11],

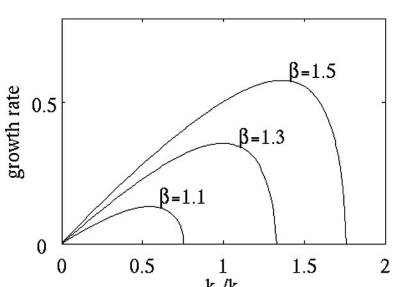

Fig. 2. Spectrum of transverse instability for TM polarized spatial soliton for three values of the propagation constant: $\beta=1.1$, 1.3, and 1.5, respectively.

$$\frac{[\beta^2-\epsilon_M(\omega)k^2]^{1/2}\epsilon_0 n^2(x_0)}{\beta\epsilon_M(\omega)}u(x_0)=v(x_0). \qquad (3)$$

For a given set of propagation constant β, and carrier frequency ω, this auxiliary location of the interface can be uniquely determined if it exists. By the translation symmetry, one can move this auxiliary location x_0 to the position where the physical interface resides. Furthermore a continuum band of nonlinear surface-guided modes can be expected. Figure 3 shows the dispersion of nonlinear surface plasmons for different x_0. Nonlinear surface modes are shown in Fig. 4, where the power is defined as electrical powers. Importantly, the soliton power is drastically reduced owing to the metallic boundary. Although higher-frequency TM solitons carry larger power in nonlinear media, the nonlinear surface plasmons, on the contrary, can only have lower power owing to matching to the plasmonic resonance [13].

In the limit where $\beta\sim kn_0$, we can derive approximate solutions for nonlinear plasmons. In this case, the power ratio P_v/P_u is small, so that we solve the first equation in Eq. (2) neglecting the field v. This equation can then further be approximated by first-order perturbation as $u=e_y\exp(i\delta\beta z)$, where $e_y=[(2\beta^2-2k^2n_0^2)(k^2\alpha_1)^{-1}]^{1/2}\mathrm{sech}([\beta^2-k^2n_0^2]^{1/2}x)\exp(i\beta z)$ is the solution for the TE polarized spatial soliton [5] and $\delta\beta$ is explicitly written in [5]. The associated electric and magnetic fields can be obtained from Maxwell's equations. Applying this approximation to the nonlinear plasmons, only for $\omega<\omega_p$ and $x_0>0$ we find solutions, which decays into the metal. Using the approximate expressions for the fields, we derive the approximate dispersion relation in the form ($\omega\ll\omega_p$)

$$\frac{\omega}{\omega_p}\approx\frac{1}{kn^2(x_0)}\sqrt{\beta^2-k^2n_0^2}\tanh(\sqrt{\beta^2-k^2n_0^2}x_0),$$

which shows an accurate fit with numerical solutions of Fig. 4 for small values of the propagation constant β. We can categorize the surface polaritons as linear-like (L), intermediate nonlinear (IN), and strong nonlinear (SN) as the shaded regions in the blown-up

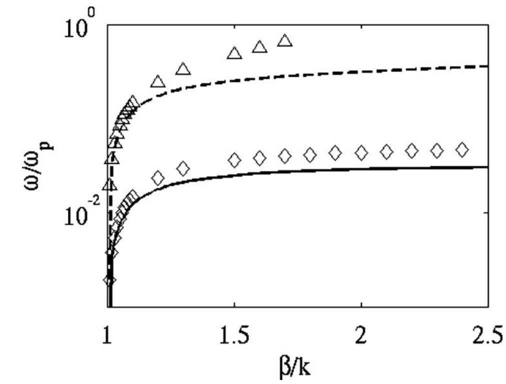

Fig. 3. Examples of the dispersion relation $\omega(\beta)$ of nonlinear surface plasmons calculated analytically for $x_0=0.1$ (◇) and $x_0=1$ (△), and from direct numerical solutions $x_0=0.1$ (solid curve) and $x_0=1$ (dashed curve), respectively. Parameters are $k=1$ and $n_0=1$.

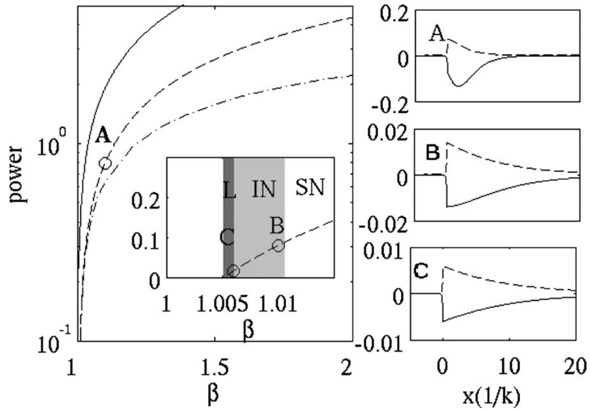

Fig. 4. Nonlinear surface plasmon polaritons. Left, power dependence of the plasmon modes versus β for different frequencies $\bar{\omega}=0.05$ (solid curve), $\bar{\omega}=0.1$ (dashed curve) and analytical solution for $\bar{\omega}=0.1$ (dotted-dashed curve), respectively. The inset is its blow up. Right, three examples of the nonlinear plasmons presented by their field components e_z (dashed curve) and h_y (solid curve), and marked as A, B, and C on the power dependencies.

panel of Fig. 4 where nonlinear polaritons are distinguished by their field profile for E_z. In the L band, all three fields exhibit a exponential decay both into the metal and nonlinear media as typical linear polaritons do [8]; in the IN band, the amplitude of the three waves decreases into both sides of the interface; moreover, in the SN band, the polariton is more nonlinear like, and its E_z field has a peak in the nonlinear dielectrics.

Now, we analyze the transverse instability of nonlinear plasmons. The dispersion relations resulting from the matching conditions at the interface can also be applied to more general equations including small perturbation fields (p_j, q_j). In the analysis, we neglect (p_y, q_y), since they are not only producing the second-order effects, but also they cannot survive at the interface owing to the dispersion relation [8]. To matching the boundary conditions, we derive a very similar dispersion relation. Our numerical analysis demonstrates that the dispersion relations for small perturbations cannot be satisfied because the frequency for the perturbed field does not exist even for finite values of Γ. Therefore, this result indicates that the perturbation field is scattered off the boundary, and no modes are localized, which may grow and cause instability to exist. As a result, we come to the conclusion that the transverse instability of the TM polarized vector solitons is completely eliminated by the presence of a metal interface, so that the nonlinear plasmons remain transversally stable.

Figures 5(a)–5(f) show the snapshots of the field components H_y and E_z for the perturbed evolution of both TM polarized spatial soliton and nonlinear plasmon polariton for $z=0$ and $z=10$, respectively. It is clear that in the presence of a metallic boundary [see Figs. 5(c)–5(f)], the transverse instability is eliminated, and the perturbed fields cease to grow. We confirm this conclusion by analyzing more realistic case of lossy metal, as shown in Figs. 5(e) and 5(f). The

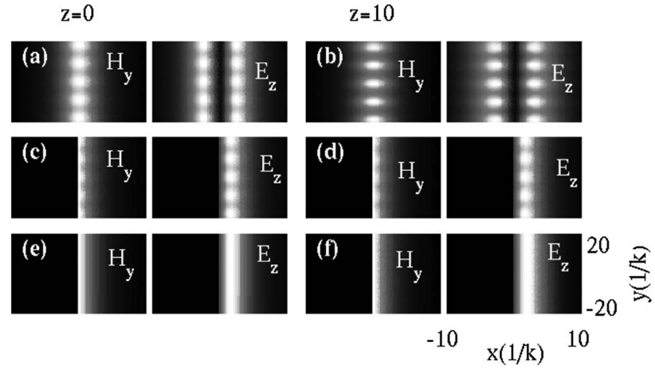

Fig. 5. Results of numerical simulations showing [(a),(b)] the development of transverse modulations for the TM polarized vector soliton; [(c),(d)] propagation of a nonlinear plasmon at a metal-dielectric interface, at $\Gamma=0$; and [(e),(f)] propagation of a nonlinear plasmon for $\Gamma=10^{-4}$. Shown are the field components H_y (left) and E_z (right) at the input ($z=0$) and after the propagation ($z=10$), in columns. The propagation constant $\beta=1.1$ (point A in Fig. 4).

surface mode becomes broader, as can be seen in Figs. 5(e) and 5(f).

In conclusion, we have demonstrated, both analytically and numerically, that the transverse instability observed for TM-polarized spatial solitons is completely eliminated for the nonlinear surface plasmons localized at an interface between a lossy metal and nonlinear dielectric.

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References

1. V. E. Zakharov and A. M. Rubenchik, Sov. Phys. JETP **38**, 494 (1974).
2. Yu. S. Kivshar and D. E. Pelinovsky, Phys. Rep. **331**, 117 (2000).
3. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, 2003).
4. C.-C. Jeng, Y. Y. Lin, R.-C. Hong, and R.-K. Lee, Phys. Rev. Lett. **102**, 153905 (2009).
5. A. W. Snyder, D. J. Mitchell, and Y. Chen, Opt. Lett. **19**, 524 (1994).
6. B. V. Gisin and B. A. Malomed, J. Opt. Soc. Am. B **18**, 1356 (2001).
7. A. Ciattoni, B. Crosignani, P. D. Porto, and A. Yariv, J. Opt. Soc. Am. B **22**, 1384 (2005).
8. A. V. Zayats, I. I. Smolyaninov, and A. A. Maradudin, Phys. Rep. **408**, 131 (2005).
9. A. R. Davoyan, I. V. Shadrivov, and Yu. S. Kivshar, Opt. Express **16**, 21209 (2008) and references therein.
10. G. I. Stegeman, C. T. Seaton, J. Ariyasu, R. F. Wallis, and A. A. Maradudin, J. Appl. Phys. **58**, 2453 (1985).
11. D. Mihalache, G. I. Stegeman, C. T. Seaton, E. M. Wright, R. Zanoni, A. D. Boardman, and T. Twardowski, Opt. Lett. **12**, 187 (1987).
12. A. D. Boardman, A. A. Maradudin, G. I. Stegeman, T. Twardowski, and E. M. Wright, Phys. Rev. A **35**, 1159 (1987).
13. M. S. Kushwaha, Jpn. J. Appl. Phys. **29**, L1826 (1990).