Conditions to preserve quantum entanglement of quadrature fluctuation fields in electromagnetically induced transparency media

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We study the propagation of quantum fields through an electromagnetically induced transparency (EIT) medium with initially two squeezed and one coherent states. Conditions to preserve and to establish nonseparation criteria for perturbed quantized fluctuation fields are demonstrated. The results in this work provide a guideline for using EIT media as quantum light devices. © 2009 Optical Society of America

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With the quantum interference between two input fields in three level atoms, electromagnetically induced transparency (EIT) media have been widely investigated in various fields of optical and atomic physics [1–3]. Many interesting properties and applications based on EIT media are predicted and demonstrated, such as slow-light propagation [4,5] and enhanced giant Kerr nonlinearity [6,7]. In the beginning, the storage and retrieval of optical information through EIT media were implemented with classical properties of lights. Experimental demonstrations show that the amplitude as well as the phase of optical pulses are perfectly preserved in the adiabatic condition [8].

It is well known that optical pulses act as a powerful carrier not only for classical but also for quantum information sciences. In recent years, the idea of light storage and retrieval has been carried out in experiments for a single photon source [9,10]. Later on, mapping photonic entanglement into and out of an EIT medium is investigated [11], which may open a new technique to establish the crucial devices, such as quantum repeaters and quantum memories, in the quantum information processing and quantum communication [12,13].

Distinct from the single photon source, quantum noise squeezed states belong to another nonclassical photon family, resident in the continuous variable space [14]. Quantum information delays and the propagation of squeezed vacuum in EIT media was reported recently [15–17]. Along this direction, it is found that EIT media become opaque for squeezed states, and there exists an oscillatory transfer of the initial quantum properties between the probe and the pump fields [18].

In this Letter, we study the quantum properties of continuous variables in an EIT system by investigating the entanglements among three quadrature radiation fields, i.e., one coherent and two squeezed states as the inputs. The goal of our work is to provide the operation conditions to preserve and to establish quantum entanglement of quadrature fluctuation fields propagating through an EIT medium. For the possible applications in the quantum information processing, we consider two input fields labeled as $\hat{a}_1$ and $\hat{a}_2$ interacting within an EIT medium and a third one, $\hat{b}$, passing through the free space, as shown in Fig. 1. The input noise squeezed states here form a pair of two-mode squeezed states, which can be used to generate an entangled Einstein–Podolsky–Rosen (EPR) state [19–21]. Then we solve the corresponding set of Heisenberg–Langevin equations for the interacting atoms and fields with the considerations of quantum fluctuations. Within the range of suitable parameters, we show the conditions to preserve nonseparation criteria as well as to establish entanglement between the quantum fluctuations of two input fields in EIT media, which are noncorrelated in the beginning. The results we have obtained may facilitate more practical applications for quantum information processing based on EIT media.

The interacting Hamiltonian for a Λ-type EIT system, including three atomic levels interacting with two quantized electromagnetic (EM) fields ($\hat{a}_1$ and $\hat{a}_2$), along with the equation of motion for the fields is

$$\hat{H}_{\text{int}} = \hbar \sum_{j=1,2} g_j(\hat{\sigma}_{ej}\hat{a}_j + \hat{a}_j^\dagger \hat{\sigma}_{je}), \tag{1}$$

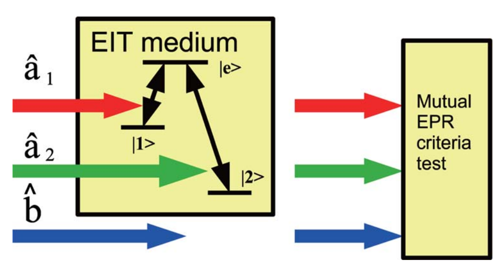

Fig. 1. (Color online) Schematic of three fields interacting in an EIT medium, where $\hat{a}_1$ (a coherent state) and $\hat{a}_2$ (a squeezed state) are quantized fields propagating through the medium, while the third field $\hat{b}$ (another squeezed state) passes through the vacuum. The mutual EPR criteria tests are performed later for the three fields.

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)\hat{a}_j=-ig_jN\hat{\sigma}_{je},\qquad(2)$$

where g_j and $\hat{\sigma}_{\mu\nu}=|\mu\rangle\langle\nu|$ correspond to the coupling constant and atomic operator in different states. N is the number of atoms, c is the velocity of light in the medium (normalized to the speed of light in the vacuum, c_0), and the three atomic levels are labeled as $|1\rangle$, $|2\rangle$, and $|e\rangle$, respectively. Based on the linearization approximation for the perturbed quantum fields, i.e., $\hat{a}_j=\alpha_j+\delta\hat{a}_j$ with α_j being the mean field, one can obtain a linear set of the equations of motion for the quantum fluctuations. With the Fourier transformation and the collective Langevin operator $\hat{f}_j$, which accounts for the noise introduced by the coupling of the atomic system to the free radiation field, the relation between the input and output quantum fluctuation fields through an EIT medium can be derived as

$$\delta\hat{a}_j(L,\omega)=G_j(L,\omega)\delta\hat{a}_j(0,\omega)+H_j(L,\omega)\delta\hat{a}_{j'}(0,\omega)+\hat{f}_j(L,\omega),\qquad(3)$$

where $\delta\hat{a}_j$ is the perturbed annihilation operator of the EM field, $j,j'=1,2(j\neq j')$, and L is the length of the EIT media. The detuning frequencies ω are introduced for the Fourier transformation of the perturbed quantum fluctuation fields, $\delta\hat{a}(t)\leftrightarrow\delta\hat{a}(\omega)$. The meaning of frequency here corresponds to the detuning between the fluctuation noises and the EIT resonance condition, not to the one- or two-photon detunings of the mean fields. Followed by [18], two characteristic functions, $G_j(L,\omega)$ and $H_j(L,\omega)$, representing the response of an EIT medium for the quantum fluctuation fields can be derived explicitly as

$$G_j(L,\omega)=\left(\frac{K_j}{K}\right)^2e^{i\beta}+\left(\frac{K_{j'}}{K}\right)^2e^{\eta+i(\beta+\chi)},\qquad(4)$$

$$H_j(L,\omega)=\frac{K_1K_2}{K^2}[e^{i\beta}-e^{\eta+i(\beta+\chi)}],\qquad(5)$$

where $K_1=g_2\Omega_1$, $K_2=g_1\Omega_2$, and $K^2=K_1^2+K_2^2$ with the corresponding Rabi frequency $\Omega_j\equiv|g_j\alpha_j|$. The formulations for β, η, and χ are $\beta(L,\omega)=L\omega/c$, $\eta(L,\omega)=-2L\gamma NK^2\omega^2/D(\omega)$, and $\chi(L,\omega)=-4LNK^2(\omega^2-\Omega^2)\omega/D(\omega)$, with $D(\omega)=c\Omega^2[\gamma^2\omega^2+4(\omega^2-\Omega^2)^2]$, $\Omega^2=\Omega_1^2+\Omega_2^2$, and $\gamma=\gamma_1+\gamma_2$, which is the total decay rate with γ_j from the exited state. From the above equations, it is obvious that the quantum properties of the output quantized fluctuation field not only depend on its own input state but also associate with the other input field through the EIT medium [18].

Since we focus on the quantum fluctuations for coherent and squeezed states, which are continuous variables, the uncertainty product of the inferred quadrature components is compared with the Heisenberg uncertainty product limit. The corresponding quadrature components for three perturbed fields in the output from Eq. (3) are defined as $\delta\hat{Y}_1$, $\delta\hat{Y}_2$, and $\delta\hat{W}$, respectively, i.e.,

$$\delta\hat{Y}_j^{\theta}(L,\omega)=\tfrac{1}{2}[\delta\hat{a}_j(L,\omega)e^{-i\theta/2}+\delta\hat{a}_j^{\dagger}(L,-\omega)e^{i\theta/2}],\qquad(6)$$

$$\delta\hat{W}^{\phi}(L,\omega)=\tfrac{1}{2}[\delta\hat{b}(L,\omega)e^{-i\phi/2}+\delta\hat{b}^{\dagger}(L,-\omega)e^{i\phi/2}],\qquad(7)$$

with the respective quadrature angles θ and ϕ. For any two fields, the appropriate quantum state basis to calculate the quantum correlations is just a direct product of them except for the two-mode squeezed state basis. The sufficient condition for the entanglement is that the inseparability criterion for bipartite continuous variables is satisfied [22],

$$\Delta I(\delta\hat{A}^{\theta},\delta\hat{B}^{\phi})\equiv16\sqrt{\Delta E(\theta,\phi,\omega)\Delta E(\theta+\pi/2,\phi-\pi/2,\omega)}<1,\qquad(8)$$

where $\Delta E(\theta,\phi,\omega)\delta(\omega+\omega')=\langle\delta[\hat{A}^{\theta}(\omega)-\delta\hat{B}^{\phi}(\omega)][\delta\hat{A}^{\theta}(\omega')-\delta\hat{B}^{\phi}(\omega')]\rangle$ is defined for any two quadrature fields, $\delta\hat{A}^{\theta}(\omega)$ and $\delta\hat{B}^{\phi}(\omega)$. In this way, we use $\Delta I(\delta\hat{A}^{\theta},\delta\hat{B}^{\phi})$ as a figure of merit for the quantum entanglement, and only when the value of $\Delta I(\delta\hat{A},\delta\hat{B})$ is smaller than 1 that one has the entangled states, which have no classical counterparts.

With the above derivations, we consider $\hat{a}_2$ and $\hat{b}$ as an EPR source generated from two-mode squeezed states as two of our three input states to study the entanglement in an EIT system, i.e.,

$$\hat{S}(\xi)=\exp[\xi^*\hat{a}_2(\omega)\hat{b}(-\omega)-\xi\hat{a}_2^{\dagger}(\omega)\hat{b}^{\dagger}(-\omega)],\qquad(9)$$

with the squeezing parameter $\xi=\gamma e^{i\delta}$, squeezing ratio γ, and squeezing angle δ. It should be noted that the nonseparation criteria for the two-mode squeezed states are only satisfied when $\xi\geq\frac{1}{2}\ln2\approx0.246$ [21].

In Fig. 2, we show the mutual quantum correlations for three output quadrature fluctuation fields, i.e., (a) $\Delta I(\delta\hat{Y}_1^{\theta},\delta\hat{Y}_2^{\phi})$, (b) $\Delta I(\delta\hat{Y}_2^{\theta},\delta\hat{W}^{\phi})$, and (c) $\Delta I(\delta\hat{Y}_1^{\theta},\delta\hat{W}^{\phi})$ for different lengths of EIT media L and different detuning frequencies ω at a fixed squeezed parameter $|\xi|=0.7$ and $\delta=\pi$. The fluctuation spectra shown here are in terms of the frequency parameter ω [23]. As mentioned before, the three input quantized fields are one coherent state ($\hat{a}_1$) and two squeezed states ($\hat{a}_2$ and $\hat{b}$), respectively. For the quantum correlation between two noncorrelated input quantized fields propagating in an EIT medium, shown as $\Delta I(\delta\hat{Y}_1,\delta\hat{Y}_2)$ in Fig. 2(a), it can be understood as another interpretation for the opaque nature of EIT media for squeezed states [18]. Only with a small frequency detuning for the quantum fluctuation fields do the two output fields propagating through the EIT medium could be both in the squeezed states. In such a case, in the marked color regions we show the conditions that satisfy nonseparation criteria.

Figure 2(b) shows the regions for the quantum correlation between two squeezed states, $\Delta I(\delta\hat{Y}_2,\delta\hat{W})$.

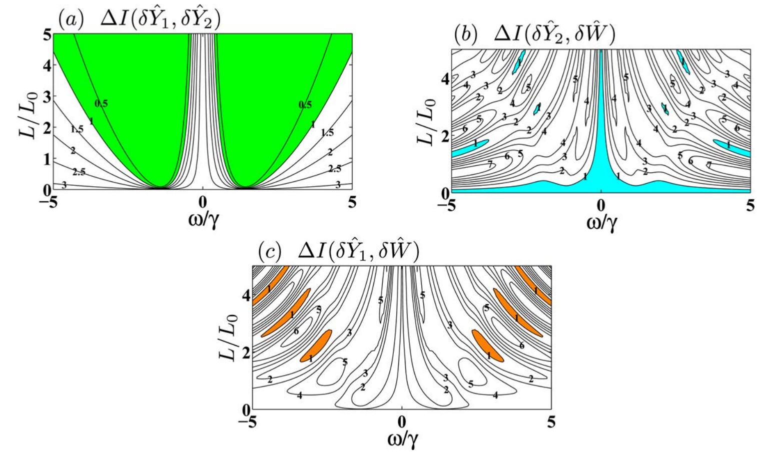

Fig. 2. (Color online) Contour plot of the quantum correlations for three quadrature fluctuation fields, i.e., (a) $\Delta I(\delta\hat{Y}_1^{\theta}, \delta\hat{Y}_2^{\phi})$, (b) $\Delta I(\delta\hat{Y}_2^{\theta}, \delta\hat{W}^{\phi})$, and (c) $\Delta I(\delta\hat{Y}_1^{\theta}, \delta\hat{W}^{\phi})$. The parameters used in simulations are $\theta = \phi = 0$, $\Omega_1 = \Omega_2 = \gamma$, $g_1 = g_2 = \gamma/60$, $N = 10{,}000$, $L_0 = \gamma c\Omega^2/NK^2 = 0.36c/\gamma$, and $c/c_0 = 1$. The number shown in the contour line is the value for the quantum correlation, ΔI. The shaded regions mark the area that satisfies the nonseparation criteria, i.e., $\Delta I < 1$.

Initially these two squeezed states form an EPR pair ($|\xi| = 0.7$ is chosen at $L = 0$) and their entanglement remains perfect only for the condition that the frequency detuning of the quantum fluctuation field is zero. But the entanglement between two input squeezed states vanishes quickly for $\omega \neq 0$. More unexpectedly, one can also find several marked shaded regions for these two perturbed fields to meet the entanglement requirement even for the case of far detuning, which we believe comes from the EIT-induced transfer of the quantum properties between the coherent and squeezed states. Again for the quantum correlation between two noncorrelated input quantized fields, $\Delta I(\delta\hat{Y}_1, \delta\hat{W})$, in Fig. 2(c) we show that there exist several regions as separated islands to satisfy the nonseparation criteria. These marked shaded regions can be used as the operation conditions to preserve and to establish quantum entanglement of quadrature fields propagating through an EIT medium.

In summary, we investigate the entanglements of the quantum fluctuation of EM fields under the EIT condition. The quantum entanglements are found to be preserved and produced by an EIT medium under certain conditions. The results in our work provide a starting step to establish quantum devices of light.

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