Optical solitons and wave-particle duality

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We investigate the propagation of optical solitons interacting with linear defects in the medium. We show that solitons exhibit a wave-particle dualism versus power, i.e., depending on the relative size of soliton and defect, responsible for a soliton trajectory dependent on its waist. © 2011 Optical Society of America

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The parallelism between optics and other areas of physics has been addressed in several cases, including fluid dynamics [1], nonrelativistic [2] and relativistic quantum mechanics [3], general relativity [4], solid state [5] and condensed matter physics [6]. In optics, an important role is played by solitons, i.e., optical wave packets undergoing transverse confinement via nonlinear effects in time or space [7]. Spatial [8] and temporal [9] solitons were first discussed in pure Kerr media, but their existence and features were later investigated both experimentally and theoretically in materials encompassing various light-matter nonlinear interactions [7,10,11]. Fundamental solitons often behave in a particlelike fashion: their transverse field distribution is strongly peaked in space/time, they are robust to external perturbations and survive as independent entities to a large set of collisions. Therefore, solitary wave packets are an ideal bench for investigating the wave-particle duality in a macroscopic scale [12–14]. Soliton interactions with inhomogeneities have been studied with reference to both longitudinally infinite [15–17] and pointlike defects [18] in refractive index, as well as to pointlike perturbations in nonlinearity [18,19], at the interface between two nonlinear media [20,21] and across a linear potential barrier [22,23]. In this Letter we try to elucidate the dual nature of self-trapped optical wave packets by investigating the interaction of solitons with localized perturbations (defects) in an otherwise uniform linear refractive index environment, showing their transition from particle to wavelike as the defect remains nearly uniform or varies appreciably across the soliton profile, respectively.

We look for solutions of the well-known one-dimensional nonlinear Schrödinger equation (NLSE) governing the nonlinear paraxial propagation of light in the form of temporal pulses in fibers or of beams in planar guides. Hereafter, we focus on the latter case, but all the results remain valid in both cases. We assume a linear index defect n_L $(x/w_p, z)$ (w_p is its width along x) superposed to an isotropic Kerr medium with background index n_0 and intensity-dependent coefficient n_2. For an electric field of amplitude A, the effective potential $V_{\mathrm{eff}}(X,Z) = -[n_L^2(X,Z) + 2n_0 n_L(X,Z)]$, and the vacuum wavenumber $k_0 = 2\pi/\lambda$ (λ is the vacuum wavelength), light propagation in the scalar approximation is governed by the modified NLSE

$$i\frac{\partial u}{\partial Z} + \frac{1}{2}\frac{\partial^2 u}{\partial X^2} + |u|^2 u - pV_{\mathrm{eff}}(X,Z)u = 0, \qquad (1)$$

with $p = k_0^2 w_p^2/2$, $u = \sqrt{pn_2}A$ the normalized field, $Z = z/L_d$ and $X = x/w_p$ the dimensionless coordinates with $L_d = k_0 w_p^2 n_0$. We consider solutions of Eq. (1) when the input is a soliton propagating along Z, i.e., $u(X, Z=0) = u_0 \mathrm{sech}(u_0 X)$. In the limit $n_L \ll n_0$, $V_{\mathrm{eff}} \approx -2n_0 n_L$. Defining the normalized intensity $\psi = |u|^2/\int |u|^2 \mathrm{d}X$, the beam position $\langle X\rangle = \int \psi X \mathrm{d}X$ is known to satisfy the Ehrenfest's theorem [12]

$$\frac{d^2\langle X\rangle}{dZ^2} = -p\int \psi \frac{\partial V_{\mathrm{eff}}}{\partial X}\mathrm{d}X. \qquad (2)$$

Expressing V_{eff} in a power series and setting $W_m(\langle X\rangle) = -\frac{1}{m!}\frac{\partial^{m+1}V_{\mathrm{eff}}}{\partial X^{m+1}}|_{X=\langle X\rangle}$, Eq. (2) becomes

$$\frac{d^2\langle X\rangle}{dZ^2} = p\sum_{m=0}^{\infty} W_m(\langle X\rangle)\langle y^m\rangle_\psi, \qquad (3)$$

with $y = X - \langle X\rangle$ and $\langle y^m\rangle_\psi = \int \psi y^m \mathrm{d}X$.

Equation (3) determines the effect of the linear defect on soliton trajectory. If the soliton is much narrower than w_p (i.e., high power), the term W_0 is the dominant contribution to the force acting on the beam; hence, the soliton path changes according to geometrical optics. Moreover, due to the small difference in defect-induced phase delay between left and right *wings* of its profile, the soliton approximately retains its shape, despite the defect extension along Z. When the soliton waist increases and becomes comparable to w_p (i.e., low power), higher order terms in Eq. (3) yield nonnegligible contributions to the force and the soliton path depends on its transverse profile. Owing to the large transverse phase modulation induced on the self-confined beam, the latter will take a multihump profile, even within the defect in the case of extended perturbations in Z, eventually generating a fan of self-localized waves [24]. Thus, at high power the soliton motion is governed by Newton's laws, i.e., it is particlelike, whereas at low excitation the soliton dynamics is ruled by wave mechanics. In summary, we expect solitons to undergo a power-dependent transition from particle- to wavelike as they interact with a

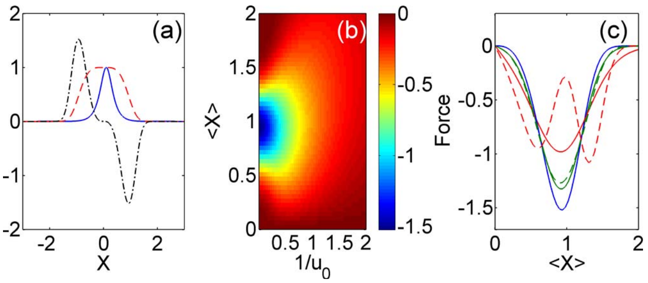

Fig. 1. (Color online) (a) Soliton profiles for $u_0 = 1$ (blue solid curve), n_L (red dashes), and $\partial n_L/\partial X$ (dashed-dotted black curve) versus X. (b) Force acting on soliton versus $\langle X\rangle$ and $1/u_0$ computed from Eq. (2). (c) Force versus soliton position $\langle X\rangle$ (solid and dashed curves correspond to all terms and terms up to $m = 2$ in Eq. (3), respectively) for soliton widths $1/u_0 = 0.01, 0.17$ and 0.37, with narrower soliton corresponding to larger forces (in modulus). Here $\Delta = 1$.

localized defect. Remarkably, such a qualitative statement holds valid for any cubic nonlinearity, including saturable [1] and nonlocal responses [21].

Let us take $2pn_0 n_L(X, Z) = \Delta \exp(-X^{2l})\mathrm{rect}_d(Z - Z_0)$, with $\Delta/(k_0^2 w_p^2 n_0)$ the jump in linear index, d the defect length along Z, and l a positive integer describing the abruptness of head and tail in $n_L(X, Z)$. Hereby we take $l = 2$ (see Fig. 1), leaving other cases to a future extended publication.

From Eq. (2), the $1/u_0$ force acting on a narrow soliton is maximum when the beam is centered on the largest gradient in $n_L(X)$ [Figs. 1(a) and 1(b)], i.e., $|\langle X\rangle| \approx 1$ in our case (the lines in Fig. 1 are graphed for $\langle X\rangle > 0$, as n_L is symmetric across $X = 0$). For broad solitons the peak force decreases and moves toward larger $\langle X\rangle$ [Fig. 1(b)], as the soliton is wide enough to overlap significantly with both maxima in $|\partial n_L/\partial X|$ [Fig. 1(a)]. Figure 1(c) plots the force versus soliton position $\langle X\rangle$ in three regimes: for solitons much narrower than $\partial n_L/\partial X$, the force given by Eq. (3) is well approximated by the W_0 term only [bottom curve in Fig. 1(c), the dashed curve perfectly overlaps with the solid one], whereas for wider solitons the terms depending on W_2 and W_m with $m > 2$ become nonnegligible [middle and top curves in Fig. 1(c), respectively].

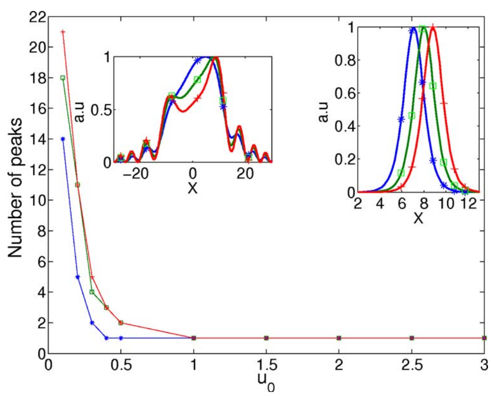

Fig. 2. (Color online) Number of interference fringes in the soliton profile versus input for $\Delta = -1$ (blue stars), -2 (green squares), -3 (red crosses) for $\langle X\rangle_0 = 2$. The insets show the corresponding profiles at $Z = 20$ for $u_0 = 0.1$ (left) and 2.5 (right), respectively.

To confirm our results, we integrated Eq. (1) with a standard beam propagation method (BPM) code, using a soliton input of the form $u_0 \mathrm{sech}[u_0(X - \langle X\rangle_0)]$, with a nonzero linear index defect $n_L(X, Z)$ in a narrow region ranging from $Z = 2$ to $Z = 2.5$, i.e., $d = 0.5$ and $Z_0 = 2.25$. We studied soliton dynamics for various amplitudes u_0 by varying the initial beam position $\langle X\rangle_0$ as well as the height Δ of the defect potential. As predicted, by increasing u_0 (i.e., decreasing the soliton waist), the beam interacting with the defect undergoes a behavior transition from wavelike to particlelike. In fact, for $1/u_0$ comparable with 1, i.e., broad solitons, the profile is strongly phase modulated and exhibits a distinct interference pattern with fringes; their number (shown for $Z = 20$ in Fig. 2) increases with the size of the potential. Noteworthy, after the interaction region each fringe undergoes self-focusing, the main peak always turning into a soliton generally noncollinear with z and wider than the input owing to the power fraction channeled in other fringes or radiated away [24]. Increasing the power, the soliton behaves as a particle, retaining its bell-shaped profile even after collision with the defect due to the small gradient in the impressed phase (inset in Fig. 2). Figure 3 shows transverse profiles in XZ for $\Delta = -1$: at low power (wave behavior), the formation of fringes due to the n_L-induced phase modulation is apparent; at higher power, in agreement with inverse scattering [7,19], small breathing oscillations arise in soliton evolution, which eventually disappear at large powers (particle behavior). When $\langle X\rangle_0 = 0$ (first row in Fig. 3) the soliton does not undergo any deflection in agreement with Eq. (2) (for large perturbations it splits into two symmetric beams [19]), whereas for $\langle X\rangle_0 \neq 0$ (second row in Fig. 3) the soliton is pushed sideways and steered in angle. To quantify the soliton angular deviation, we considered five input positions for the beam and plotted the computed results for $\gamma = \arctan[(\langle X\rangle - \langle X\rangle_0)/Z]$ (the propagation angle θ in the physical environment reads $\theta = \arctan[\tan\gamma/(k_0 n_0 w_p)]$) versus power in Fig. 4. The angle γ depends on both the input beam position $\langle X\rangle_0$ and the height Δ of the potential, in agreement with Eq. (3). γ initially increases with power, but then decreases or saturates depending on $\langle X\rangle_0$. These results perfectly agree with Fig. 1(b): for $\langle X\rangle_0$ close to the peak

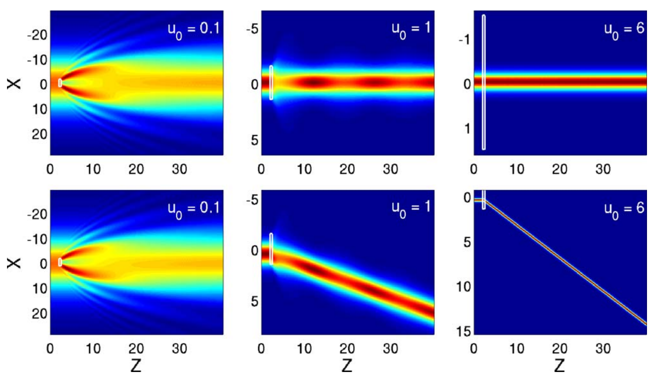

Fig. 3. (Color online) Evolution in XZ for various input powers (legends) and beam positions $\langle X\rangle_0 = 0$ (first row) and $\langle X\rangle_0 = 0.5$ (second row), with $\Delta = -1$. The white rectangles indicate the defect location.

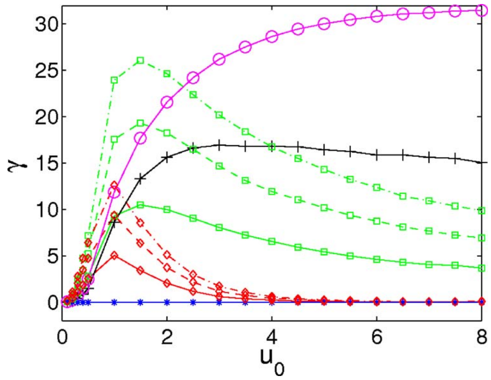

Fig. 4. (Color online) Soliton deflection versus power for $\langle X\rangle_0 = 0$ (blue stars), 0.5 (black crosses), 1 (magenta circles), 1.5 (green squares), 2 (red diamonds) and $\Delta = -1$ (solid curve), −2 (dashed curve), −3 (dash-dots).

of the index gradient (in modulus), the force monotonically grows with u_0 (power), whereas a relative maximum occurs as the soliton moves away from this point in either direction. Remarkably, for a given u_0 the effective (transverse) force is invariant with propagation distance within a short defect due to the nonspreading nature of solitons, or it can be computed at a fixed abscissa in Fig. 1(b) if variations of transverse beam position $\langle X\rangle$ within the defect are not negligible.

In conclusion, we investigated the interaction between solitons and localized linear defects, elucidating the dual nature of self-trapped waves: solitons behave like waves or particles depending on their own power.

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