Nonlocal dark solitons under competing cubic–quintic nonlinearities

L. Chen,[1] Q. Wang,[1] M. Shen,[1,*] H. Zhao,[1] Y.-Y. Lin,[2] C.-C. Jeng,[3] R.-K. Lee,[2] and W. Krolikowski[4]
[1]*Department of Physics, Shanghai University, Shanghai 200444, China*
[2]*Institute of Photonics Technologies, National Tsing-Hua University, Hsinchu 300, Taiwan*
[3]*Department of Physics, National Chung-Hsing University, Taichung 402, Taiwan*
[4]*Laser Physics Center, Research School of Physics and Engineering, Australian National University, Canberra ACT 0200, Australia*
**Corresponding author: shenmingluck@shu.edu.cn*

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We investigate properties of dark solitons under competing nonlocal cubic–local quintic nonlinearities. Analytical results, based on a variational approach and confirmed by direct numerical simulations, reveal the existence of a unique dark soliton solutions with their width being independent of the degree of nonlocality, due to the competing cubic–quintic nonlinearities.

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In optical nonlocal nonlinear media, the nonlinear response to the intensity of an optical beam in a particular spatial location is determined by the integrated light intensity (or power) within a certain neighborhood of this location. It appears that nonlocality is an inherent property in a variety of physical systems including plasmas, optics, and condensed matter [1–4]. Nonlocality has a profound impact on the propagation of spatial optical solitons, in particular, formation of dark solitons [5–8] and their interactions [9,10]. Recently, nonlocal media with competing nonlinearities have drawn much attention [11,12]. Competing nonlinearities occur in systems where few different physical processes contribute to the overall nonlinear response. This is, e.g., the case of Bose Einstein, condensate with simultaneous local and long range bosonic interaction [13] or nematic liquid crystals with comparable thermal and orientational nonlinearities [11]. It has been shown that the competing nonlinearities can stabilize many complex soliton structures, which are otherwise unstable in a medium with one type of nonlocal nonlinearity [14–18]. The competing nonlocal nonlinearities can also destabilize dark soliton states [19] and enable coexistence of dark and bright spatial solitons [12]. In a recent study, Tsoy [20] analyzed the effect of local quintic contribution to nonlocal Kerr-like nonlinearity on bright and dark solitons in the regime of weak nonlocality. Here, we investigate the effect of competing nonlocal cubic and local quintic nonlinearities on the properties of dark solitons for an arbitrary degree of nonlocality.

We consider propagation of a dark soliton with the slowly varying amplitude $u(x,z)$ governed by the following normalized nonlinear Schrödinger equation (NLS) [8,15,21]:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}-u\int R(x-\xi)|u(\xi,z)|^2\mathrm{d}\xi+\alpha_2|u(x,z)|^4u=0, \tag{1}$$

where $R(x)$ is the normalized nonlocal response function, and α_2 is the relative strength of the local component of the nonlinear response. The particular form of the response function is determined by the physics of underlying processes responsible for the nonlinearity [22]. Beside optics, Eq. (1) may represent other nonlocal systems such as Tonks–Girardeau gas with dipolar interactions [23] or Bose–Einstein condensate with contact and long range interaction [24].

To analyze this nonlocal NLS equation, we first employ the variational approach to Eq. (1), with the following Lagrangian density corresponding to [8,21,25]:

$$\begin{aligned}\mathcal{L}=&\frac{i}{2}\left(u^*\frac{\partial u}{\partial z}-u\frac{\partial u^*}{\partial z}\right)\left(1-\frac{1}{|u|^2}\right)-\frac{1}{2}\left|\frac{\partial u}{\partial x}\right|^2\\&-\frac{1}{2}(|u|^2-1)\int R(x-\xi)(|u(\xi,z)|^2-1)\mathrm{d}\xi\\&+\frac{\alpha_2}{3}(|u|^6-3|u|^2+2).\end{aligned} \tag{2}$$

In general, for an arbitrary form of the response function $R(x)$, Eq. (1) can be solved only numerically. To make the problem analytically tractable, without loss of generality, we consider here a phenomenological model of the rectangular profile for the nonlocal response function $R(x)$:

$$R(x)=\begin{cases}\frac{1}{2\sigma}; & -\sigma\le x\le\sigma,\\ 0; & \text{otherwise}\end{cases} \tag{3}$$

with σ defining the degree of nonlocality. To proceed further we must postulate the form of the slowly varying amplitude $u(x,z)$. We should emphasize that some previous works have obtained the exact dark solitons solutions under local cubic and quintic competing nonlinearities [26,27]. However, in order to investigate the dark solitons analytically for arbitrary degree of nonlocality, we consider here the general ansatz of dark solitons in the following form:

$$u(x,z)=B\tanh[D(x-x_0)]+iA \tag{4}$$

with $A^2+B^2=1$ and all the parameters A, B, D, and x_0 assumed to be functions of the propagation variable z.

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Substituting Eqs. (3) and (4) into Eq. (2) and integrating over x, we can obtain the effective Lagrangian:

$$L=\int_{-\infty}^{\infty}\mathcal{L}(u)\mathrm{d}x=2\frac{dx_0}{dz}\left[\tan^{-1}\left(\frac{B}{A}\right)-AB\right]-\frac{2}{3}B^2D$$
$$+\frac{B^4}{D}\left[\mathrm{csch}^2(D\sigma)-\frac{\coth(D\sigma)}{D\sigma}\right]+\alpha_2\frac{4B^4}{3D}\left(1-\frac{4}{15}B^2\right). \quad (5)$$

From the corresponding Euler–Lagrangian equations, we can get that $B=$ const, and

$$\frac{\coth(D\sigma)}{D\sigma}\left[\frac{1}{D^2}-\sigma^2\,\mathrm{csch}^2(D\sigma)\right]-\frac{2\alpha_2}{3D^2}\left(1-\frac{4B^2}{15}\right)=\frac{1}{3B^2}, \quad (6)$$

$$\frac{dx_0}{dz}=A\left[\frac{D}{3B}-\frac{B}{D}\left(\mathrm{csch}^2(D\sigma)-\frac{\coth(D\sigma)}{D\sigma}\right)\right]$$
$$-\frac{4\alpha_2AB}{3D}\left(1-\frac{2B^2}{5}\right). \quad (7)$$

Equations (6) and (7) represent analytical relations for the parameters of dark solitons in nonlocal media with competing nonlinearities.When $\alpha_2=0$, we recover the previous result describing dark nonlocal solitons in a Kerr-like medium [8]. In the following, we consider black soliton solutions, i.e., $B\approx 1$ and $A\approx 0$. We find that the dark soliton solutions only exist when the nonlocal cubic nonlinearity is self-defocusing. For the focusing nonlocal cubic nonlinearity, we have numerically tested (not shown) that the dark solitons will diffract quickly for $\alpha_2>0$ or break into two kinks moving in opposite directions for $\alpha_2<0$ [20]. In Fig. 1(a), we show the soliton width ($\propto D^{-1}$) as a function of the degree of nonlocality σ and the strength of local quintic nonlinearity α_2.

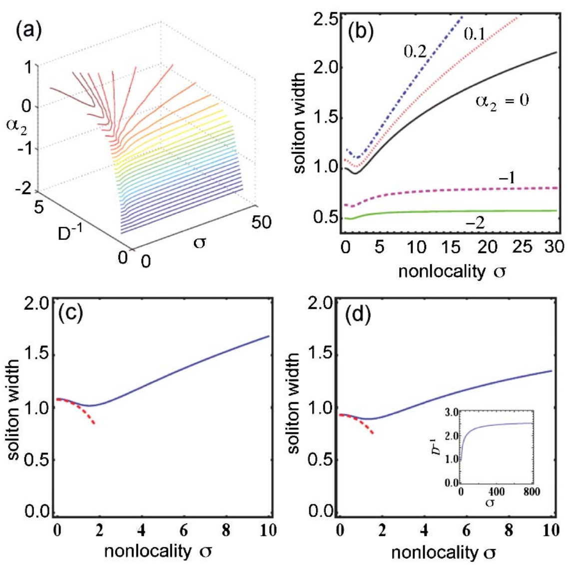

Fig. 1. (Color online) Illustration of nonlocality and competing nonlinearities on the width of dark soliton (D^{-1}): (a) D^{-1} versus σ and α_2 and (b) D^{-1} versus σ for different values of α_2. The comparison between general nonlocal (solid-curve) and weakly nonlocal (dashed-curve) solutions for (c) $\alpha_2=0.1$ and (d) $\alpha_2=-0.1$, respectively.

Figure 1(b) shows examples of the dependence of soliton width on degree of nonlocality for different values of α_2. It can clearly be seen that the self-defocusing and self-focusing quintic nonlinearities lead to the decrease or increase of the soliton width, respectively. This is because the self-defocusing (self-focusing) nonlinearity strengthens (weakens) the overall nonlinear response and subsequently the self-trapping of dark solitons, resulting in the increase (decrease) of the width. Moreover, the beam width increases with σ for both $\alpha_2>0$ and $\alpha_2<0$.

For weak nonlocality ($\sigma\ll D^{-1}$), Eq. (6) yields:

$$\frac{1}{D^2}+\frac{2}{15}\sigma^2-\frac{2\alpha_2}{D}\left(1-\frac{4}{15}B^2\right)=\frac{1}{B^2}. \quad (8)$$

This relation is depicted in dashed lines of Figs. 1(c) and 1(d). The competition between cubic and quintic nonlinearities in this regime has been recently studied in [20].

We see that for both $\alpha_2>0$ and $\alpha_2<0$ the soliton width decreases with σ for a lower degree of nonlocality. Only for a higher degree of nonlocality the soliton width increases with σ. This effect is independent of the sign of α_2 and can be explained by the fact that a weak nonlocality causes the nonlinear index change to advance toward the regions with a lower light intensity. As a result, the waveguide induced by the soliton becomes slightly narrower, and so does the soliton [8]. The increase of the width in the highly nonlocal limit comes from the fact that the effective waveguide induced by solitons gets wider for a large σ [28]. However, we notice here that in the highly nonlocal regime, the soliton width strongly depends on the sign of local quintic nonlinearity. Namely, when the quintic nonlinearity is self-focusing ($\alpha_2>0$), the width increases monotonically with the degree of nonlocality σ; while the situation becomes very different for the self-defocusing quintic nonlinearity ($\alpha_2<0$). In the latter case, the soliton width increases first but then tends to saturate for a large value of σ, as shown in the inset of Fig. 1(d).

The reason for such drastically different behavior lies in the competition between nonlocal Kerr and local quintic nonlinearities. Both nonlinearities strongly affect the soliton-induced waveguide, which determines the localization of the wave. In all cases, the nonlocality tends to weaken the waveguide, resulting in a weaker localization. When the quintic nonlinearity is self-focusing (nonlocal Kerr and quintic terms are of opposite signs), its effect is to decrease the contrast of the soliton-induced waveguide even further. Consequently, the corresponding soliton width keeps increasing. On the other hand, when the quintic term is self-defocusing, it not only enhances the self-induced waveguide, but also counteract the detrimental effect from the nonlocality. As a result, the soliton width tends to saturate with σ.

In Fig. 2, we illustrate the effect of nonlocality and the competition of nonlinearities on the propagation of a single dark soliton in nonlocal cubic media with local self-focusing and self-defocusing quintic nonlinearities,

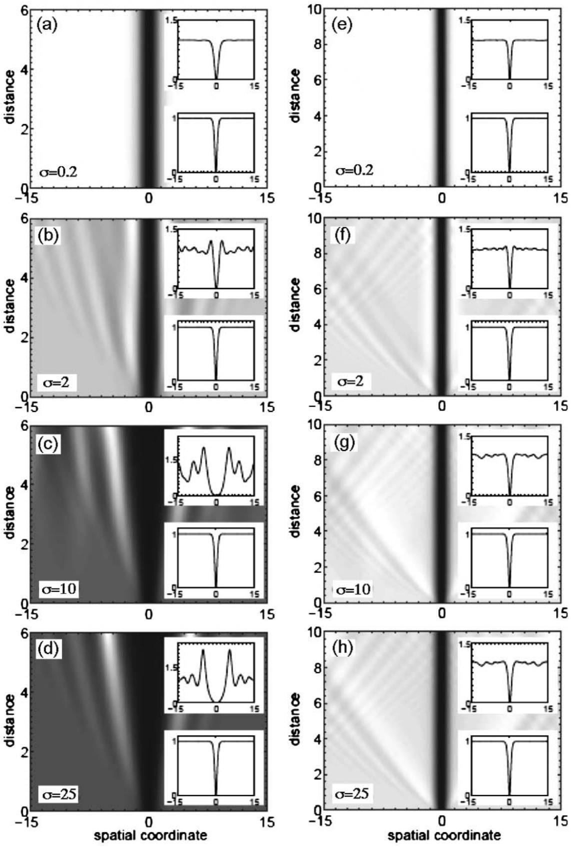

Fig. 2. Dynamics of dark solitons with different degrees of nonlocalities σ, for (a)–(d) focusing ($\alpha_2 = 0.1$, $D^{-1} = 1.09$); and (e)–(h) defocusing ($\alpha_2 = -1$, $D^{-1} = 0.65$) quintic nonlinearities. The insets show the corresponding soliton profiles at the initial (bottom) and final (top) propagation distances.

respectively. We numerically integrate Eq. (1) with the split step Fourier method. Variational soliton solutions are used as the initial conditions. The left column in Fig. 2 represents results for the focusing quintic nonlinearity ($\alpha_2 > 0$). In the limit of a weak nonlocality, for example $\sigma = 0.2$ shown in Fig. 2(a), the soliton width does not change; the nonlocality actually enhances soliton localization in this regime. For a higher degree of nonlocality, Figs. 2(b)–2(d), the soliton width increases more than that predicted by the variational solution. This is because the ansatz we used to derive the analytical solution is no longer adequate in the highly nonlocal regime [8]. Different behavior is seen for a defocusing quintic nonlinearity, as shown in Figs. 2(e)–2(h). Almost stationary propagations for well-localized solitons are observed thanks to the beneficial influence from the local self-defocusing quintic term, which counteracts the deleterious role of nonlocality.

In summary, we have investigated analytically dark solitons under competing nonlocal cubic–local quintic nonlinearities for an arbitrary degree of nonlocality. We show that a self-focusing quintic nonlinearity weakens soliton localization, whereas a defocusing quintic nonlinearity enhances soliton-induced index change and counteracts the detrimental effect from a strong nonlocality.

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