Quantum Phase Transitions of Light for Two-level Atoms

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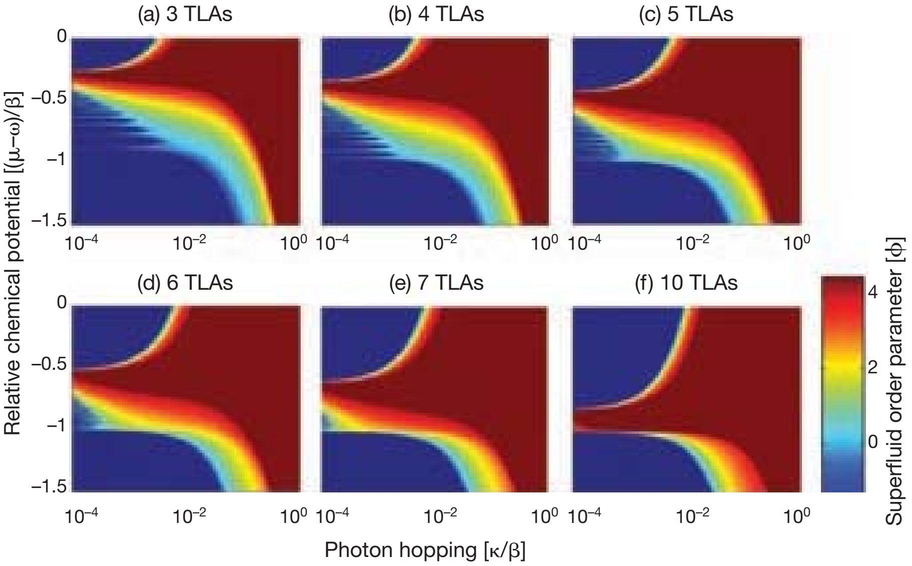
Quantum phase transition (QPT) can only be accessed at near-absolute-zero temperatures by changing an external parameter or a coupling constant driven by quantum fluctuations.[1] There have been intensive studies of QPTs in interacting many-body problems, originally for strongly correlated electronic systems in condensed matter physics, and more recently for a weakly interacting ultracold atomic system. Typically, it is difficult to control and probe such exotic quantum phenomena in strongly correlated systems of electrons.

Optical lattices—artificial crystals made by interfering laser beams—offer a versatile platform for studying trapped Bose gases.[1] In this situation, one can describe the many-body dynamics from a Mott-insulating phase to a superfluid phase in a gas of ultracold atoms with periodic potentials by using a Bose-Hubbard model that includes an on-site two-atom interaction and hopping between adjacent sites.

Photons are non-interacting bosons, and there is no possibility of having any QPTs in purely photonic systems. For a purely Bose system, the conducting phase at zero temperature is presumably always superfluid. However, engineered composites of optical cavities, few-level atoms, and laser light can form a strongly interacting many-body system to study the concepts and methods in condensed matter physics from the viewpoint of quantum optics. In this case, a photonic condensed-matter analog could be realized with state-of-the-art photonic crystals embedded with high-Q defect cavities.

Therefore, it should be easier to study critical quantum phenomena such as QPTs in conventional condensed matter systems. The simplest light-atom system is photons interacting with a single two-level atom, described by the Jaynes-Cummings model. With an array of high-Q electromagnetic cavities, each containing a single two-level atom in the photon-blockade regime, quantum phase transitions of photonic insulator (excitations localization) to superfluid (excitations delocalization) are predicted by the Bose-Hubbard model.[2, 3]



The phase diagrams for arbitrary number of two-level atoms, (a) N=3, (b) N=4, (c) N=5, (d) N=6, (e) N=7, and (f) N=10. The horizontal and vertical axes are shown by the normalized intercavity hopping energy of photons and the relative chemical potential. The phase boundary between MI-SF is the superposition states that disappear and result in a single macroscopic coherent radiation state for a large number of two-level atoms.

As the number of TLAs increases, collective effects due to the interactions of atoms among themselves give rise to intriguing many-body phenomena. With the Dicke-Bose-Hubbard Hamiltonian, we show that the Mott insulator to superfluid QPTs with photons can be realized in an extended Dicke model for an arbitrary number of two-level atoms. We illustrate the generality of the method by constructing the dressed-state basis for an arbitrary number of two-level atoms. Moreover, we show that, as the number of TLAs increases, superposition states may disappear and classically emerge.

We expect that more controllable light-wave technologies will lead to an enhanced understanding of the QPTs of light with distinctive properties, improvements in the organization of the ground-state wave function, and the introduction of new applications. With combinations of Dicke-like and Hubbard-like models to simulate strongly correlated electron systems using photons, we believe that there will be many more interesting QPTs to be demonstrated. ◭

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References

1. S. Sachdev. *Quantum Phase Transitions*, Cambridge University Press (1999).
2. A.D. Greentree et al. Nature Phys. **2**, 856 (2006).
3. M.J. Hartmann et al. Nature Phys. **2**, 849 (2006).
4. S.C. Lei and R.-K. Lee. Phys. Rev. A **77**, 033827 (2008).