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Giant Kerr nonlinearities and slow optical solitons in coupled double quantum-well nanostructure

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ABSTRACT

We investigate the enhancement of Kerr nonlinearities in an asymmetric coupled double quantum-well (CQW) by analyzing the nonlinear optical response. With the consideration of real parameters in AlGaAs-based CQWs, we indicate the possibility to obtain the giant Kerr nonlinearities with the cancellation of linear absorption simultaneously. We also reveal both analytically and numerically that under appropriate conditions a probe field with very low intensity can evolve into a stable shape-preserving waveform that propagates with a slow group velocity. The stability of such slow optical solitons is also demonstrated numerically.

1. Introduction

Optical transitions between electronic states within the conduction bands of semiconductor quantum-wells (QWs) have proved to be a promising candidate for the realization of optical devices and optical information sciences [1]. Large number of efforts have been devoted to the investigations of both linear and nonlinear optical properties in semiconductor quantum-well structures.

It is well known that the third-order Kerr nonlinearity $\chi^{(3)}$ plays an important role in nonlinear optics with applications from optical shutters [2–4] to the generation of optical solitons [5,6]. It is desirable to achieve giant Kerr nonlinearities with low light powers [7,8]. In recent years, both theoretically [9–11] and experimentally [12], the giant third-order nonlinear susceptibility with reducing linear absorption has been one of the most extensively studied phenomena. In addition, retaining the merits of the giant Kerr nonlinearities, Wu and Deng [13–15] have theoretically proposed that it is possible to form ultraslow optical bright and dark solitons for a weak light by including the self-phase modulation in cold atomic media.

However, it is more advantageous at least from the view point of practical purposes to find possible solid-state media that could permit to realize the giant Kerr nonlinearities with low pump power, low absorptions, and shape-invariant propagation of the optical field instead of the cold atom gases mentioned above. Due to strong electron–electron interactions, the two-dimensional electron gas behaves effectively as a single oscillator, with atomiclike intersubband transition (ISBT) responses [1]. Quantum interference and coherence in QWs can also produce some interesting phenomena such as strong electromagnetically induced transparency [16–18], coherent population trapping [19], lasing without inversion [20], enhancement of refractive index [21], tunneling-induced transparency [22], and so on [23–31]. More recently, the enhancement of Kerr nonlinearities based on Fano-interference with intersubband transitions [32,33] and a large cross-phase modulation (XPM) have been studied in an asymmetric QWs [34].

In this Letter, we show that asymmetric semiconductor coupled double quantum-well (CQW) can also support the propagation of optical solitons via a two-photon Raman resonance scheme. Besides, under the two-photon resonance condition and with appropriate one-photon detuning, we can obtain the cancellation of linear absorption, enhancement of Kerr nonlinearities, and slow group velocity propagation of the weak probe pulse simultaneously. Since the conduction subband energy level can be easily tuned by an external bias voltage, the proposed CQW structure

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also provide another possibility to realize electrically controllable phase modulator at low light levels [35].

2. Model and equations of motion

Let us consider an asymmetric semiconductor CQW structure consisting of 10 pairs of a 51-monolayer (145 Å) thick wide well and a 35-monolayer (100 Å) thick narrow well, separated by an $Al_{0.2}Ga_{0.8}As$ buffer layer [35,36], as shown in Fig. 1. In this quantum-well structure, the first ($n = 1$) electron levels in the wide and narrow wells can be aligned with each other by applying a static electric field, while the corresponding $n = 1$ hole levels are never aligned due to the polarization of input fields. The electrons then delocalize over both wells while the holes remain localized. For transitions from the narrow and wide wells, the Coulomb interaction between the electron and hole down shifts the value of the electric field, where the resonance condition is fulfilled to the corresponding built-in field. Level $|1\rangle$ in the narrow well and level $|2\rangle$ in the wide well are localized hole states. The energy difference 2δ between the bonding state $|3\rangle$ and antibonding state $|4\rangle$ is determined by the level splitting in the absence of tunneling and related tunneling matrix elements, which can be controlled by an electric field applied perpendicularly to CQW. For more details on this structure we refer the reader to Ref. [36].

As shown in Fig. 1(b), all the lights propagate along the z axis (parallel to the plane of the CQW) within our proposed structure, and we consider a transverse magnetically (TM) polarized probe field incident with respect to the growth direction (y axis). This configuration is preferred due to a relatively long propagation distance, of the order of millimeters. We assume the transitions $|2\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are simultaneously coupled by a control laser field with the one-half Rabi frequencies $\Omega_c = \mu_{42}E_c/2\hbar$ and $(\Omega_c\mu_{32})/\mu_{42}$, respectively. At the same time, a weak probe laser field is applied to the transitions $|1\rangle \leftrightarrow |4\rangle$ with the corresponding Rabi frequencies Ω_p. And the transitions $|1\rangle \leftrightarrow |3\rangle$ is dipole forbidden. E_c and E_p are the amplitude of the control and weak probe fields, respectively. The electric field of the system can be written as $\mathbf{E} = \vec{e}_p E_p \exp(-i\omega_p t + i\vec{k}_p \cdot \vec{r}) + \vec{e}_c E_c \exp(-i\omega_c t + i\vec{k}_c \cdot \vec{r}) + \text{c.c.}$, where $\vec{e}_j$ and $\vec{k}_j$ are related unit vector and wave vector for the slowly varying envelope E_j, respectively. In the present analysis we assume that the semiconductor quantum-well are designed with low dopings such that electron–electron effects have very small influences in our results. Many-body effects (for example, the depolarization effect, which renormalizes the free-carrier and carrier-field contributions) are not included in our study [37]. Working in the interaction picture, utilizing the rotating-wave approximation and the electric-dipole approximation, we derive the Hamiltonian for our quantum-well structure as

$$H = -\Delta_p|4\rangle\langle 4| - (\Delta_p + \delta)|3\rangle\langle 3| - (\Delta_p - \Delta_c)|2\rangle\langle 2| \\ - \big(\Omega_c|4\rangle\langle 2| + q\Omega_c|3\rangle\langle 2| + \Omega_p|4\rangle\langle 1| + \text{H.c.}\big), \tag{1}$$

where $2\delta = E_4 - E_3$ is the energy splitting between the upper levels. $\Delta_c = \omega_c - \omega_{42}$ and $\Delta_p = \omega_p - \omega_{41}$ are detunings of the coupling and probe fields with transitions $|2\rangle \leftrightarrow |4\rangle$ and $|1\rangle \leftrightarrow |4\rangle$, respectively. $\Omega_p = \mu_{41}E_p/(2\hbar)$ denotes one half Rabi frequency for the transition $|1\rangle \leftrightarrow |4\rangle$, the coefficient $q = \mu_{42}/\mu_{32}$ describes the ratio of a pair of dipole moments with μ_{ij} being the dipole moment for the corresponding transitions $|i\rangle \leftrightarrow |j\rangle$. The system dynamics can be described by the equations of motion for the probability amplitudes of the electronic wave functions, i.e.,

$$\frac{\partial A_1}{\partial t} = i\Omega_p^* A_4, \tag{2}$$

$$\frac{\partial A_2}{\partial t} = -\big[\gamma_2 - i(\Delta_p - \Delta_c)\big]A_2 + i\Omega_c^* A_4 + iq\Omega_c^* A_3, \tag{3}$$

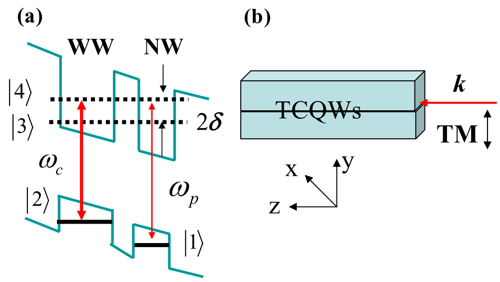

Fig. 1. (Color online.) (a) Conduction subband energy level diagram for a single period of the coupled double quantum-well structure consisting of a wide well (WW) and a narrow well (NW). (b) Possible launching geometry. Lights are injected with the wave vectors k ($k_{p,c}$) parallel to the plane of the CQW (z axis) and y direction denotes the growth axis. The polarization state of the field (TM) is also indicated. This configuration is preferred due to a relatively long propagation distance, one the order of millimeters, in order to observe the soliton formation.

$$\frac{\partial A_3}{\partial t} = -\big[\gamma_3 - i(\Delta_p - \delta)\big]A_3 + iq\Omega_c A_2 + \kappa A_4, \tag{4}$$

$$\frac{\partial A_4}{\partial t} = -(\gamma_4 - i\Delta_p)A_4 + i\Omega_p A_1 + i\Omega_c A_2 + \kappa A_3, \tag{5}$$

where A_j ($j = 1, 2, 3$) being the amplitudes of subbands $|j\rangle$. The total decay rates γ_i are given by $\gamma_i = \gamma_{il} + \gamma_i^{dph}$, where γ_i^{dph} is the dephasing decay rates and can be determined by the intrasubband phonon scattering, electron–electron scattering, and inhomogeneous broadening due to the scattering on the interface roughness. The population decay rates γ_{il}, determined by longitudinal optical (LO) phonon emission events at low temperature, can also be calculated by solving the effective mass Schrödinger equation [38]. For the temperatures up to 10 K and a carrier density smaller than 10^{12} cm^{-2}, the dephasing decay rates γ_i^{dph} can be estimated according to Ref. [14]. $\kappa = \sqrt{\gamma_{3l}\gamma_{4l}}$ represents the cross-coupling of states $|3\rangle$ and $|4\rangle$ via the LO phonon decay. Note that a comprehensive treatment of the decay rates would involve incorporation of the decay mechanisms into the Hamiltonian of our system. However, in this work we have adopted the phenomenological approach to treat the decay mechanisms, in order to illustrate the main physical picture here.

Under weak probe approximation ($(|\Omega_p| \ll |\Omega_c|)$), almost all of the population are assumed to remain in the ground state $|1\rangle$. Also, we assume that the excited state $|4\rangle$ can be adiabatically eliminated when the variation of the probe field's envelope is slow compared to the decay of the excited states. There is no population transfer of the ground state $|1\rangle$. With these assumptions, it can be shown that $|A_1| \approx 1$, $A_{2,3,4}^{(0)} = 0$. With two-photon resonance condition ($\Delta_p = \Delta_c = \Delta$), we obtain the solutions of A_j to the first-order of Ω_p from Eqs. (2)–(5)

$$A_2^{(1)} = -\frac{(b - q\kappa)\Omega_c^*\Omega_p}{abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \tag{6}$$

$$A_3^{(1)} = -\frac{i(a\kappa + q|\Omega_c|^2)\Omega_p}{-abc + a\kappa^2 - (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \tag{7}$$

$$A_4^{(1)} = -\frac{i(ab + q^2|\Omega_c|^2)\Omega_p}{abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2}, \tag{8}$$

with $a = -\gamma_2$, $b = -[\gamma_3 - i(\Delta + \delta)]$, and $c = -(\gamma_4 - i\Delta)$. The induced polarization at the probe frequency is $P(\omega_p) = \chi(\omega_p)E_p$. The susceptibility is written as $\chi^{(1)}(\omega_p) + 3\chi^{(3)}|E_p|^2$, where we just consider the susceptibility to the third-order and neglect higher-order terms. The first-order $\chi^{(1)}(\omega_p)$ and third-order $\chi^{(3)}$ susceptibility of the probe pulse are given by [13–15]

$$\chi^{(1)} = -\frac{N|\mu_{14}|^2}{\hbar\varepsilon_0}\frac{A_4^{(1)}A_1^{(1)*}}{\Omega_p}, \tag{9}$$

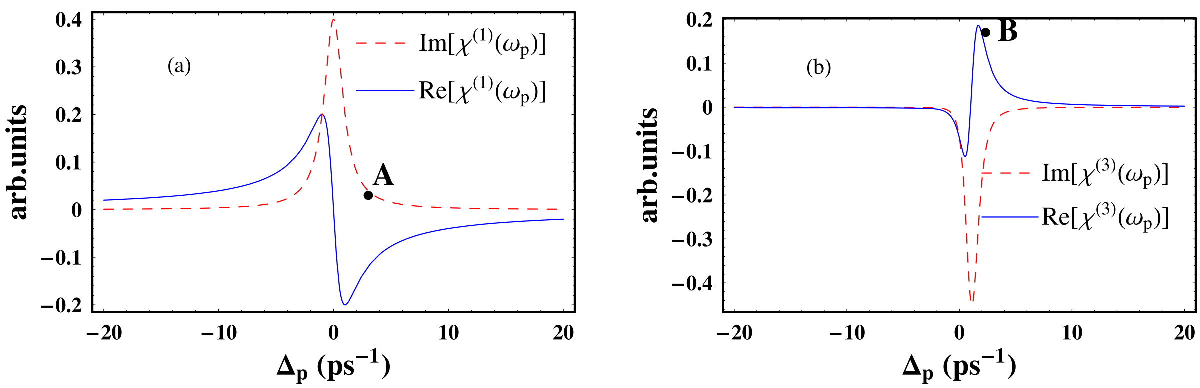

Fig. 2. Dependencies of Im[$\chi^{(1)}$] and Re[$\chi^{(1)}$] in (a) and Im[$\chi^{(3)}$] and Re[$\chi^{(3)}$] in (b) versus the probe detuning Δ_p. We have set $N|\mu_{14}|^2/\hbar\varepsilon_0$ and $N|\mu_{14}|^4/3\hbar^3\varepsilon_0$ as units, respectively. Other parameters used are $F=1$, $\gamma_3=4.2$ ps^{-1}, $\gamma_4=3.7$ ps^{-1}, $\gamma_3^2\simeq 0$, $\delta=4$ ps^{-1}, $\Omega_c=2$ ps^{-1}, and $q=1.25$.

$$\chi^{(3)}=-\frac{N|\mu_{14}|^4}{3\hbar^3\varepsilon_0}\frac{A_4^{(1)}(|A_4^{(1)}|^2+|A_3^{(1)}|^2+|A_2^{(1)}|^2)}{|\Omega_p|^2\Omega_p}, \quad (10)$$

where N is the electron volume density. For the CQW structure considered here, we take $\gamma_2=0$ by consulting Ref. [39]. Thus Eq. (9) can be simplified as

$$\chi^{(1)}=-\frac{N|\mu_{14}|^2}{\hbar\varepsilon_0}\frac{iq^2|\Omega_c|^2}{-(b+cq^2-2q\kappa)|\Omega_c|^2}. \quad (11)$$

Based on Eq. (11), one can find that the first-order susceptibility is mainly induced by the cross coupling from the driving field Ω_c and via the LO phonon decay through states $|3\rangle$ and $|4\rangle$. In order to examine the linear absorption and dispersion, we derive the imaginary and the real parts of the first-order susceptibility explicitly,

$$\mathrm{Im}\left[\chi^{(1)}\right]=-\frac{N|\mu_{14}|^2}{\hbar\varepsilon_0}\frac{2q^3\kappa+q^2\gamma_3+q^4\gamma_4}{(\delta+\Delta+q^2\Delta)^2+(2q\kappa+\gamma_3+q^2\gamma_4)^2}, \quad (12)$$

$$\mathrm{Re}\left[\chi^{(1)}\right]=\frac{N|\mu_{14}|^2}{\hbar\varepsilon_0}\frac{q^2\delta+q^2\Delta+q^4\Delta}{(\delta+\Delta+q^2\Delta)^2+(2q\kappa+\gamma_3+q^2\gamma_4)^2}. \quad (13)$$

Eqs. (12) and (13) show that the absorption Im[$\chi^{(1)}$] and dispersion Re[$\chi^{(1)}$] of the probe field do not depend on the intensity of the control field Ω_c. When $\Delta=-\delta/(1+q^2)$, there appears an absorption peak, and the corresponding dispersion is zero. In Fig. 2(a) we show the imaginary and real parts of the linear susceptibility versus the single-photon detuning Δ. It is clearly seen that far from the point $\Delta=-\delta/(1+q^2)$, the linear absorption will be closed to zero.

3. Giant Kerr nonlinearities in quantum-well nanostructure

In Section 2, we have derived the third-order nonlinear susceptibility for our four-level system. Note that for the simplified three-level Λ configuration ($|1\rangle$, $|2\rangle$, and $|4\rangle$), the third-order nonlinear susceptibility is zero under the two-photon resonance condition. But in this four-level two-photon resonance Raman system ($\Delta_p=\Delta_c$), the coupling of the control field with transition $|2\rangle\leftrightarrow|3\rangle$ destroys the coherence between states $|1\rangle$ and $|2\rangle$, which causes the linear absorption of the probe field and also leads to the nonlinear effect. As a result, $q\neq 0$ indicates constructive interference in the XPM nonlinearities.

We perform a numerical calculation of the third-order nonlinear susceptibility in Eq. (10). As shown in Fig. 2, for a certain detuning, for example, at the marker **B** in Fig. 2(b) (corresponds to the marker **A** in Fig. 2(a) with the same frequency detuning), linear absorption is vanished while the strength of XPM is large, which suggests that a large XPM can be achieved as linear absorption vanishes simultaneously. This interesting result is produced by the cross coupling in the nonlinear susceptibility associated with XPM, which is the behavior of a quantum interference between the two different excitation channels: cross-coupling and back-coupling channels. If the probe field is so strong that it can couple the channels $|1\rangle\leftrightarrow|3\rangle$ and $|1\rangle\leftrightarrow|4\rangle$, in this case the back-coupling channel $|1\rangle\leftrightarrow|4\rangle$ will simultaneously give rise to a destructive interference path way, resulting in the suppression of XPM. In other words, this is a consequence of quantum coherence by cross-coupling excitation of control field Ω_c under weak probe approximation. In addition, one can readily checked that the decay-induced interference, κ, only has a small effect on the XPM because here the XPM is mainly induced by the control field Ω_c under the condition $\kappa\ll\Omega_c$. The same concept we demonstrate here can also be applied to a four-level atomic system. But unlike the cold atomic systems with specific four-level atoms, the conduction subband energy varies with the bias voltage in QWs. When we adjust the energy level of the bonding state $|3\rangle$ and the antibonding state $|4\rangle$ at different bias voltages, different nonlinear phase shifts can be obtained by such a giant Kerr nonlinearity. Thus our proposed CQW structures could be provided as a flexible device to realize voltage controllable, solid-based phase modulators at low light powers.

4. Optical solitons in quantum-well nanostructure

If the losses of the probe pulse are small enough to be neglected, the balance between the nonlinear self-phase modulation and group velocity dispersion (GVD) may keep a pulse with shape-invariant propagation. From above numerical discussions, as long as probe detuning Δ_p is far from the point corresponding to the absorption peak, the linear absorption of the probe pulse is negligible and the nonlinear self-phase modulation is enhanced. The evolution for the electric-field is governed by the Maxwell equation in one dimension,

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\frac{1}{\varepsilon_0 c^2}\frac{\partial^2\mathbf{P}}{\partial t^2}, \quad (14)$$

with

$$\begin{aligned}\mathbf{P}=N\big(&\vec{\mu}_{14}A_4A_1^*\exp\left[i(\vec{k}_p\cdot\vec{r}-\omega_p t)\right]\\&+\vec{\mu}_{23}A_3A_2^*\exp\left[i(\vec{k}_c\cdot\vec{r}-\omega_c t)\right]\\&+\vec{\mu}_{24}A_4A_2^*\exp\left[i(\vec{k}_c\cdot\vec{r}-\omega_c t)\right]+\text{c.c.}\big),\end{aligned} \quad (15)$$

where N, c, and ε_0 being the concentration, velocity of light in vacuum, and vacuum dielectric constant, respectively. Under the slowly varying envelope approximation, Eq. (14) can be reduced to describe the evolution for the probe field, i.e.,

$$\frac{\partial \Omega_p(z,t)}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p(z,t)}{\partial t} = iFA_4A_1^*, \tag{16}$$

where $F = N\Omega_p|\vec{e}_p \cdot \vec{\mu}_{14}|^2/(2\varepsilon_0\hbar c)$. For simplicity, we have assumed $\vec{k}_p = \vec{e}_z\vec{k}_p$. In order to provide a clear picture of the interplay between the dispersion and nonlinear effects for the CQW system interacting with two optical fields, we first investigate the dispersion properties of the system. This requires a perturbation treatment of the system response to the first-order of weak probe field Ω_p, while keeping all orders due to the control field Ω_c. Below, we demonstrate that due to the balance between higher-order terms and related dispersion effect one can have the formation of slow optical solitons.

We assume that $A_j = \Sigma_k A_j^{(k)}$, where $A_j^{(k)}$ is the k-th-order part of A_j in terms of Ω_p. Within an adiabatic framework it can be shown that $A_j^{(0)} = \delta_{j0}$ and $A_1^{(1)} = 0$. Taking the time Fourier transform of Eqs. (3)–(5) and keeping up to the first-order of Ω_p, we have

$$(\omega + \Delta_p - \Delta_c + i\gamma_2)\beta_2^{(1)} + \Omega_c^*\beta_4^{(1)} + q\Omega_c^*\beta_3^{(1)} = 0, \tag{17}$$

$$(\omega + \Delta_p - \delta + i\gamma_3)\beta_3^{(1)} + q\Omega_c\beta_2^{(1)} + \kappa\beta_4^{(1)} = 0, \tag{18}$$

$$(\Delta_p + i\gamma_4)\beta_4^{(1)} + \Lambda_p + \Omega_c\beta_2^{(1)} + \kappa\beta_3^{(1)} = 0, \tag{19}$$

where $\beta_j^{(1)}$ and Λ_p are the Fourier transforms of $A_j^{(1)}$ and Ω_p, respectively. ω is the Fourier-transform variable. The above equations (17)–(19) can be solved analytically, yielding

$$\Lambda_p(z,\omega) = \Lambda_p(0,\omega)\exp(iKz), \tag{20}$$

where

$$K = \frac{\omega}{c} + F\frac{(\omega - \Delta - \delta - i\gamma_3)(\omega + i\gamma_2) - q^2|\Omega_c|^2}{abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2} = K_0 + \frac{\omega}{V_g} + K_2\omega^2 + \cdots. \tag{21}$$

The physical interpretation of Eq. (21) is rather clear. $K_0 = \phi + i\alpha/2$ describes the phase shift ϕ per unit length and absorption coefficient α of the probe field, $K_1 = 1/V_g$ gives the propagation velocity, and K_2 represents the group-velocity dispersion that contributes to the probe field.

With the dispersion coefficients obtained in Eq. (21), we now investigate the nonlinear evolution of the probe field. Following the method in Refs. [13,14], we take a trial function $\Omega_p(z,t) = \Omega_p(z,t)\exp(iK_0z)$ and substitute it into the wave equation to obtain the following nonlinear wave equation for the slowly varying envelope $\Omega_p(z,t)$, which is accurate up to the third-order, i.e.,

$$i\frac{\partial}{\partial\xi}\Omega_p - K_2\frac{\partial^2}{\partial\eta^2}\Omega_p = We^{-\alpha\xi}|\Omega_p|^2\Omega_p, \tag{22}$$

where $\xi = z$, $\eta = t - z/V_g$, the absorption coefficient $\alpha = 2\,\mathrm{Im}(K_0)$, the group velocity V_g and the dispersion coefficient K_2 are determined by Eq. (21). The nonlinear coefficient W is explicitly given by

$$W = -\frac{Fi(ab + q^2|\Omega_c|^2)[(b - q\kappa)^2|\Omega_c|^2 - (a\kappa + q|\Omega_c|^2)^2 - (ab + q^2|\Omega_c|^2)^2]}{D|D|^2}, \tag{23}$$

with $D = abc - a\kappa^2 + (b + cq^2 - 2q\kappa)|\Omega_c|^2$. If a reasonable and realistic set of parameters can be found so that $\exp(-\alpha L) \simeq 1$ (L is the length of the CQW system), $K_2 = K_{2r} + iK_{2i} \simeq K_{2r}$, and

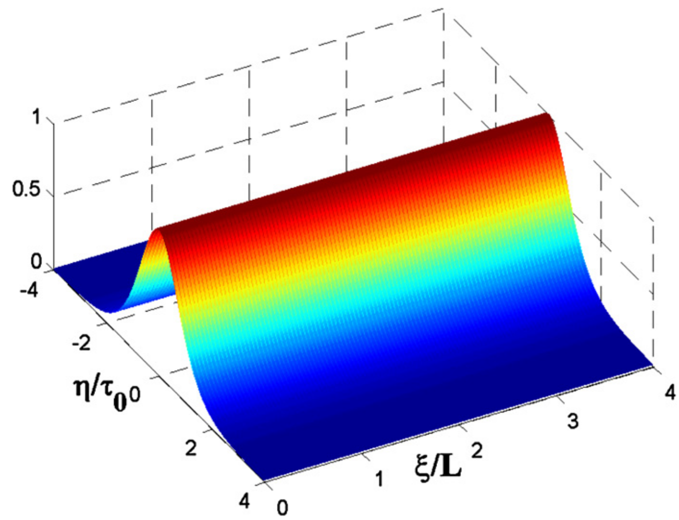

Fig. 3. (Color online.) Surface plot of the probe intensity $|\Omega_p/\Omega_{p0}|^2\exp(-\alpha\xi)$ versus dimensionless time η/τ_0 and distance z/L obtained by solving Eq. (22) numerically without neglecting the imaginary part of coefficients with the initial condition given in Eq. (26). Here $L = 1$ mm, $\tau_0 = 1.6 \times 10^{-8}$ s, and other parameters are explained in the main text.

$W = W_r + iW_i \simeq W_r$, then Eq. (22) can be reduced to the standard nonlinear Schrödinger equation, i.e.,

$$i\frac{\partial}{\partial\xi}\Omega_p - K_{2r}\frac{\partial^2}{\partial\eta^2}\Omega_p = W_r|\Omega_p|^2\Omega_p, \tag{24}$$

which admits solutions describing dark ($K_{2r}W_r < 0$) and bright ($K_{2r}W_r > 0$) solitons including N-soliton ($N = 1, 2, 3, \ldots$) for dark and bright solitons [40]. Fundamental dark soliton with $K_{2r}W_r < 0$ takes the form

$$\Omega_p = \Omega_{p0}\tanh(\eta/\tau_0)\exp\big(-iW_r\xi|\Omega_{p0}|^2\big), \tag{25}$$

where the amplitude Ω_{p0} and the width τ_0 are arbitrary constants, subjected only to the constraint $|\Omega_{p0}\tau|^2 = -2K_{2r}/W_r$. For $K_{2r}W_r > 0$, one has fundamental bright soliton and bright 2-soliton (bright soliton of second-order) given by

$$\Omega_p = \Omega_{p0}\,\mathrm{sech}(\eta/\tau_0)\exp\left(-\frac{1}{2}iW_r\xi|\Omega_{p0}|^2\right), \tag{26}$$

$$\Omega_p = \Omega_{p0}\frac{4[\cosh(3\eta/\tau_0) + 3\exp(-8iK_{2r}\xi/\tau_0^2)\cosh(\eta/\tau_0)]\exp(-iK_{2r}\xi/\tau_0^2)}{\cosh(4\eta/\tau_0) + 4\cosh(2\eta/\tau_0) + 3\cos(8K_{2r}\xi/\tau_0^2)}, \tag{27}$$

with the amplitude Ω_{p0} and width τ_0 being arbitrary constants, subjected only to the constraint $|\Omega_{p0}\tau_0|^2 = -2K_{2r}/W_r$. We note that the bright 2-soliton solution in Eq. (27) satisfies $\Omega_p(\xi = 0, \eta) = 2\Omega_{p0}\,\mathrm{sech}(\eta/\tau_0)$.

To check the validity of our assumptions that leads to Eqs. (24)–(27), below we give a practical example for a realistic CQW system, with $F = 1.6\ \mathrm{mm}^{-1}\,\mathrm{ps}^{-1}$, $q = 1.2$, $\Omega_c = 2\ \mathrm{ps}^{-1}$, decay rates $\gamma_3 = 4.2\ \mathrm{ps}^{-1}$, $\gamma_4 = 3.7\ \mathrm{ps}^{-1}$, $\gamma_2 \simeq 0$, detuning $\Delta = 3\ \mathrm{ps}^{-1}$, the energy splitting $\delta = 4\ \mathrm{ps}^{-1}$, $K_2 \simeq (-1.76 - 0.045i) \times 10^{-22}\ \mathrm{mm}^{-1}\,\mathrm{s}^2$, and $W \simeq (-1.23 + 0.032) \times 10^{-20}\ \mathrm{mm}^{-1}\,\mathrm{s}^2$. Clearly, for all complex coefficients the imaginary parts are indeed much smaller than their corresponding real parts. Based on this parameters, we obtain $\alpha \simeq 0.0031\ \mathrm{mm}^{-1}$ and $v_g/c \simeq 3.5 \times 10^{-2}$. Then the standard nonlinear Schrödinger equation in Eq. (24) with $K_{2r}W_r > 0$ is well characterized, and hence the existence of bright solitons that travels with slow group velocity in the CQW structures is supported. In Fig. 3, we show the result of numerical simulation on the soliton wave shape versus dimensionless time η/τ_0 and distance ξ/L with

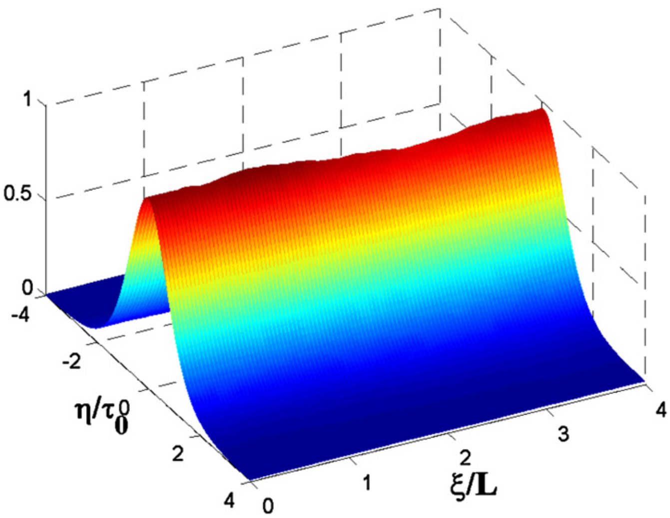

Fig. 4. (Color online.) Surface plot of the solitary intensity $|\Omega_p/\Omega_{p0}|^2\exp(-\alpha\xi)$ versus dimensionless time η/τ_0 and distance z/L obtained by directly simulating Eqs. (2)–(5), and Eq. (16) without using any approximation. The initial condition and the parameters are the same as those in Fig. 3.

the full complex coefficients by taking Eq. (26) as an initial condition. One can find that in this case the soliton is fairly stable during propagation, which is mainly produced from the balance between dispersion and nonlinearity. Thus the result of numerical simulation shows excellent agreement with the exact soliton solution in Eq. (22). To make further confirmation on the optical soliton solutions obtained above and check their stability, we also perform additional numerical simulations starting directly from Eqs. (2)–(5) and (16) without using any approximation. Fig. 4 is the propagation for the probe field, with Eq. (22) as the input condition. One can find that, except for small ripples appearing on its peak due to higher-order dispersions and higher-order nonlinear effects that have not been included, the optical solitons produced here is rather stable as expected.

It is worth noting that the cross coupling of control field may be viewed as the perturbation to the two-photon resonance condition, which comes from the closely separated two upper levels instead of coming from the perturbation by introducing another laser field or taking two-photon detuning [13–15], thus our scheme is a very stable system to form slow optical solitons. There are four adjustable parameters in our proposed system, i.e., the intensity of the driving field, probe detuning Δ_p, the energy splitting 2δ between the two upper levels, and the relative coupling ratio q. The control field only need to be strong enough to couple two transitions $|2\rangle \leftrightarrow |4\rangle$ and $|2\rangle \leftrightarrow |3\rangle$. On the other hand, relatively lower intensity of control field can lead to better effects in formation of slow optical solitons. In addition, we have used assumption of $|\Omega_{p0}|^2 \ll |\Omega_c|^2$ in our calculations, so that the pulse width of the probe field τ_0 should be chosen to satisfy $|\Omega_{p0}\tau_0|^2 = -K_{2r}/W_r \ll |\Omega_c\tau_0|^2$. Fig. 2 shows that there is a very large range validity for the parameter Δ_p. For the parameters δ and q, smaller separation δ will be better, which is adjusted and there is no strict requirement for q.

5. Conclusion

Before conclusion, we note that we have used the one-dimensional model in analysis, and correspondingly, the momentum-dependency of subband energies was ignored [41]. In fact, there is no large discrepancy between the reduced one-dimensional calculation and the full two-dimensional calculation, and the related theoretical discussions can be found in Refs. [17,23].

In short, based on the two-photon Raman resonance scheme in an asymmetric coupled double quantum-wells, we have shown that the quantum interference caused by cross coupling of a control field not only suppresses linear absorption loss, but also enhances Kerr nonlinearities of the weak probe pulse. With the unique feature of controllable balance between linear dispersion and nonlinear effects in these solid-state devices, we also demonstrate the possibility to form ultraslow optical solitons. Slow optical solitons discussed in the present work may lead to important applications such as high fidelity optical delay lines and optical buffers. Compared to conventional EIT scheme, the two-photon Raman scheme in this work represents an alternative method, and may lead to new phenomena that manifest themselves under well controlled balance of dispersion and nonlinear effects.

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References

[1] H.C. Liu, F. Capasso, Intersubband Transitions in Quantum Wells: Physics and Device Applications, Academic Press, New York, 2000.
[2] H. Schmidt, A. Imamoğlu, Opt. Lett. 21 (1996) 1936.
[3] H. Kang, Y. Zhu, Phys. Rev. Lett. 91 (2003) 093601, 4 pp.
[4] Y.F. Chen, C.Y. Wang, S.H. Wang, Ite A. Yu, Phys. Rev. Lett. 96 (2006) 043603.
[5] V. Tikhonenko, J. Christou, B. Luther-Davies, Phys. Rev. Lett. 76 (1996) 2698.
[6] Y.Y. Lin, R.K. Lee, Opt. Exp. 15 (2007) 8781.
[7] S.E. Harris, L.V. Hau, Phys. Rev. Lett. 82 (1999) 4611.
[8] A. Imamoğlu, H. Schmidt, G. Woods, M. Deutsch, Phys. Rev. Lett. 79 (1997) 1467.
[9] A.B. Matsko, I. Novikova, G.R. Welch, M.S. Zubairy, Opt. Lett. 28 (2003) 96.
[10] Yueping Niu, Shangqing Gong, Ruxin Li, Zhizhan Xu, Xiaoyan Liang, Opt. Lett. 30 (2005) 3371.
[11] X.J. Liu, H. Jing, M.L. Ge, Phys. Rev. A 70 (2004) 055802.
[12] H. Wang, D. Goorskey, M. Xiao, Phys. Rev. Lett. 87 (2001) 073601.
[13] Y. Wu, L. Deng, Phys. Rev. Lett. 93 (2004) 143904.
[14] Y. Wu, L. Deng, Opt. Lett. 29 (2004) 2064.
[15] Y. Wu, X. Yang, Appl. Phys. Lett. 91 (2007) 094104.
[16] M.C. Phillips, H. Wang, Phys. Rev. Lett. 91 (2003) 183602.
[17] L. Silvestri, F. Bassani, G. Czajkowski, B. Davoudi, Eur. Phys. J. B 27 (2002) 89.
[18] E. Paspalakis, C. Simserides, S. Baskoutas, A.F. Terzis, Physica E 40 (2008) 1301.
[19] J.F. Dynes, M.D. Frogley, J. Rodger, C.C. Philips, Phys. Rev. B 72 (2005) 085323.
[20] M.D. Frogley, J.F. Dynes, M. Beck, J. Faist, C.C. Philips, Nat. Mater. 5 (2006) 175.
[21] S.M. Sadeghi, H.M. van Driel, J.M. Fraster, Phys. Rev. B 62 (2000) 15386.
[22] H. Schmidt, K.L. Campman, A.C. Gossard, A. Imamoğlu, Appl. Phys. Lett. 70 (1997) 3455.
[23] Jin-Hui Wu, Jin-Yue Gao, Ji-Hua Xu, L. Silvestri, M. Artoni, G.C. La Rocca, F. Bassani, Phys. Rev. Lett. 95 (2005) 057401.
[24] P.C. Ku, Opt. Lett. 29 (2004) 2291.
[25] Phedon Palinginis, Forrest Sedgwick, Shanna Crankshaw, Michael Moewe, Connie J. Chang-Hasnain, Opt. Exp. 13 (2005) 9909.
[26] W.X. Yang, J.M. Hou, R.K. Lee, Phys. Rev. A 77 (2008) 033838.
[27] E. Paspalakis, Phys. Rev. B 67 (2003) 233306.
[28] E. Paspalakis, A. Kalini, A.F. Terzis, Phys. Rev. B 73 (2006) 073305.
[29] S.M. Sadeghi, S.R. Leffer, J. Meyer, E. Mueller, J. Phys.: Condens. Matter 10 (1998) 2489.
[30] J.-H. Li, Phys. Rev. B 75 (2007) 155329.
[31] W.X. Yang, J.M. Hou, Y.Y. Lin, R.K. Lee, Phys. Rev. A 79 (2009) 033825, 8 pp.
[32] J. Faist, C. Sirtori, F. Capasso, Sung-Nee G. Chu, Loren N. Pfeiffer, Ken W. West, Opt. Lett. 21 (1996) 985.
[33] J. Faist, F. Capasso, C. Sirtori, Ken W. West, L.N. Pfeiffer, Nature 390 (1997) 589.
[34] H. Sun, et al., Opt. Lett. 32 (2007) 2475.
[35] W.X. Yang, R.K. Lee, Opt. Exp. 16 (2008) 17161.
[36] H.G. Roskos, M.C. Nuss, J. Shah, Karl Leo, D.A.B. Miller, A.M. Fox, Stefan Schmitt-Rink, K. Köhler, Phys. Rev. Lett. 68 (1992) 2216.
[37] D.E. Nikonov, A. Imamoglu, L.V. Butov, H. Schmidt, Phys. Rev. Lett. 79 (1997) 4633.
[38] D. Ahn, S.L. Chuang, Phys. Rev. B 34 (1986) R9034.
[39] A. Neogi, Opt. Commun. 133 (1997) 479.
[40] G.P. Agrawal, Nonlinear Fiber Optics, 2nd ed., Academic Press, 1995.
[41] I. Waldmüller, et al., Phys. Rev. B 69 (2004) 205307, 9 pp.