Quantum squeezing and correlation of self-induced transparency solitons

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A general quantum theory of self-induced transparency (SIT) solitons is developed with nonlinear quantum effects of atoms taken into account. The quantization starts from the coarse-grain-averaged light-atom interaction Hamiltonian by which the quantum effects of ensemble atoms are modeled. The calculation of quantum properties is performed by the backpropagation method, which takes into account the contribution of the field continuum parts and the atomic fluctuations. The detectable squeezing ratio with a general homodyne local oscillator can be simulated for practical experimental realizations. Schemes for generating quantum correlation through SIT soliton interaction are also proposed and analyzed.

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I. INTRODUCTION

In nonlinear optical physics, the self-induced transparency (SIT) soliton phenomenon in two-level atomic systems is one of the most well-known coherent pulse propagation phenomena that have been intensively investigated [1,2]. For possible applications of quantum communication and information processing, different ways of manipulating the nonclassical states of photons with multilevel atomic ensembles have attracted extensive research interests both in the fields of quantum optics and quantum information science [3–7]. Optical solitons in glass fibers described by the nonlinear Schrödinger equation (NLS) have also played an important role for generating macroscopic nonclassical states exhibiting quadrature squeezing [8–10], amplitude squeezing [11,12], or intrapulse and/or interpulse quantum correlation [13].

As a member of the optical soliton family, the SIT solitons have also been suggested to be potentially capable of playing an important role in pulsed squeezed state generation [14,15], quantum nondemolition (QND) measurements [14], and quantum information storage and retrieval [16]. In contrast to the electromagnetic-induced-transparency phenomena that have been widely utilized for applications such as slow lights and quantum memories [3,4], the SIT phenomena are intrinsic nonlinear coherent pulse propagation effects that may have more advantages for short optical pulse applications. With the recent advances of microstructured fiber technologies [17], several experiments of wavelength conversion and resonant light-matter interaction by using photonic crystal fibers filled with active atoms have been demonstrated [18,19]. It has also been shown that the photonic crystal fibers can help to reduce the dominant acoustic wave Brillouin scattering noises by tenfold [20]. The generation of squeezing via SIT solitons inside hollow-core photonic crystal fibers filled with active atom vapors is also likely to happen soon [21]. In view of these development trends, an usable quantum theory for SIT solitons is urgently needed in order to provide guidelines for experiments and to help predict new phenomena.

In the literature, a quantum theory of SIT solitons has been developed previously by one of the authors based on the linearization approach within the framework of the inverse-scattering method [15]. Only the quantum noises for the perturbed soliton parameters (i.e., photon number, phase, frequency, and position) were calculated accurately, whereas the contributions from the field continuum parts are totally ignored by assuming that special homodyne local oscillators are used to project out only the soliton parts. In the present work, a more general quantum theory of SIT solitons is successfully developed. The quantization starts from the coarse-grain-averaged light-atom interaction Hamiltonian by which the quantum effects of ensemble atoms are modeled rigorously. The calculation of the quantum noise properties are performed by the backpropagation method [22], which can take into account the field continuum contributions and the atomic fluctuations generally. The detectable squeezing ratio with a general local oscillator pulse shape in the homodyne detection can be calculated accurately. In particular, for the most experimentally practical case where the output mean-field pulse is directly used as the homodyne local oscillator, the present theoretical framework can be used to predict the obtainable squeezing ratio under different system parameters and thus can be helpful for future SIT soliton squeezing experiments. The theory has also been used to investigate the possibility of generating quantum correlations between two SIT solitons through the mediation of the atomic medium. It is found that strong quantum correlations of two SIT solitons can be established through the nonlinear light-atom interaction. Unlike NLS solitons, an almost nonoverlapping SIT soliton pair can still have reasonable large long-range quantum correlations with a very short propagation distance due to the longer-lived atomic response.

This work is organized as follows. We first derive the coarse-grain-averaged light-atom interaction Hamiltonian to determine the quantum operator evolution equations for the whole system in Sec. II. Then the detailed formula for calculating the required quantum noise properties based on the backpropagation method is presented in Sec. III. The ob-

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tained results for quantum squeezing and correlation are subsequently shown and analyzed in Sec. IV. Finally, a brief conclusion is given.

II. QUANTIZATION OF SIT SOLITONS

In this section, we begin our studies by deriving the valid quantum operator equations for the SIT soliton system. In the Heisenberg picture, the (fine-grain) interaction Hamiltonian of ensemble two-level atoms with a traveling-wave light field can be written as

$$H_{int} = iK\sum_j \{\hat{U}^\dagger(z_j,t)\hat{p}_j(t) - \hat{p}_j^\dagger(t)\hat{U}(z_j,t)\}. \quad (1)$$

Here $\hat{U}(z_j,t)$ is the (envelope) light field operator, $\hat{p}_j(t)$ is the dipole moment operator of the jth atom at position z_j, and K is the coupling constant between the light filed and the atoms. For simplicity, a one-dimensional identical atomic distribution and a single polarization light resonant with the atoms have been assumed. From basic quantum optics, the atomic operators obey the following commutation relations:

$$[\hat{p}_j,\hat{p}_j^\dagger] = -\hat{n}_j, \quad (2)$$

$$[\hat{p}_j^\dagger,\hat{n}_j] = -2\hat{p}_j^\dagger, \quad (3)$$

$$[\hat{p}_j,\hat{n}_j] = 2\hat{p}_j. \quad (4)$$

Here $\hat{n}_j$ is the population inversion operator of the jth atom. By defining the coarse-grain-averaged dipole moment and population inversion density operators according to the following two expressions:

$$\int_z^{z+\Delta z}\hat{P}(z,t)dz = \sum_{z\le z_j\le z+\Delta z}\hat{p}_j(t), \quad (5)$$

$$\int_z^{z+\Delta z}\hat{N}(z,t)dz = \sum_{z\le z_j\le z+\Delta z}\hat{n}_j(t), \quad (6)$$

then one can rewrite the interaction Hamiltonian as follows:

$$H_{int} = i\hbar K\int_{-\infty}^{\infty}\{\hat{U}^\dagger(z,t)\hat{P}(z,t) - \hat{P}^\dagger(z)\hat{U}(z,t)\}dz. \quad (7)$$

This is the coarse-grain-averaged interaction Hamiltonian suitable for the SIT soliton studies. It is easy to prove that the dipole moment and population inversion density operators have to obey following commutation relations:

$$[\hat{P}(z_1,t),\hat{P}^\dagger(z_2,t)] = -\hat{N}(z_1,t)\delta(z_1-z_2), \quad (8)$$

$$[\hat{P}^\dagger(z_1,t),\hat{N}(z_2,t)] = -2\hat{P}^\dagger(z_1,t)\delta(z_1-z_2), \quad (9)$$

$$[\hat{P}(z_1,t),\hat{N}(z_2,t)] = 2\hat{P}(z_1,t)\delta(z_1-z_2). \quad (10)$$

With above results, the complete Hamiltonian for a traveling light pulse inside a two-level medium is then given by

$$H = i\hbar\int_{-\infty}^{\infty}\frac{c}{2}\left\{\hat{U}(z,t)\frac{\partial\hat{U}^\dagger(z,t)}{\partial z} - \hat{U}^\dagger(z,t)\frac{\partial\hat{U}(z,t)}{\partial z}\right\}dz + i\hbar K\int_{-\infty}^{\infty}\{\hat{U}^\dagger(z,t)\hat{P}(z,t) - \hat{P}^\dagger(z,t)\hat{U}(z,t)\}dz. \quad (11)$$

Here c is the light speed in the vacuum. We will require the light field operator $\hat{U}(z_1,t)$ to satisfy the following equal-time commutation relation:

$$[\hat{U}(z_1,t),\hat{U}^\dagger(z_2,t)] = \delta(z_1-z_2). \quad (12)$$

This commutation relation also implies that $\int\hat{U}^\dagger(z,t)\hat{U}(z,t)dz$ has the physical meaning of the photon number inside the whole light pulse.

From above Hamiltonian and related commutation relations, one finally arrives at the following quantum operator evolution equations in the Heisenberg picture:

$$\frac{\partial\hat{U}(z,t)}{\partial t} = -c\frac{\partial\hat{U}(z,t)}{\partial z} + K\hat{P}(z,t), \quad (13)$$

$$\frac{\partial\hat{P}(z,t)}{\partial t} = K\hat{N}(z,t)\hat{U}(z,t), \quad (14)$$

$$\frac{\partial\hat{N}(z,t)}{\partial t} = -2K\{\hat{P}^\dagger(z,t)\hat{U}(z,t) + \hat{U}^\dagger(z,t)\hat{P}(z,t)\}. \quad (15)$$

These equations model the quantum effects of decayless ensemble atoms rigorously and will be the starting point of our quantum noise calculation. To further simplify the notation, the evolution equations can be cast into the following normalized form by introducing suitable normalization units:

$$\frac{\partial\hat{U}(z,t)}{\partial t} = -\frac{\partial\hat{U}(z,t)}{\partial z} + \frac{r}{2}\hat{P}(z,t), \quad (16)$$

$$\frac{\partial\hat{P}(z,t)}{\partial t} = \frac{1}{2}\hat{N}(z,t)\hat{U}(z,t), \quad (17)$$

$$\frac{\partial\hat{N}(z,t)}{\partial t} = -\{\hat{P}^\dagger(z,t)\hat{U}(z,t) + \hat{U}^\dagger(z,t)\hat{P}(z,t)\}. \quad (18)$$

Here the normalization units for $\{\hat{P}(z_1,t),\ \hat{P}^\dagger(z_2,t),\ \hat{N}(z_2,t)\}$ have been chosen to be the density of atoms N_a, the normalization units t_0 for time t and u_0 for light field $\{\hat{U}(z,t),\ \hat{U}^\dagger(z,t)\}$ have been chosen to satisfy the following requirement: $Ku_0t_0=1/2$. The normalization units z_0 for position z have been chosen to be ct_0. Finally, the normalized coupling coefficient $r=2KN_at_0/u_0=N_a/u_0^2=N_az_0/(u_0^2z_0)=n_a/n_p$ now has a simple physical meaning: the ratio between the atom density N_a and the photon number density units u_0^2 (since $n_p=u_0^2z_0$ is the normalization units for the photon number). Under such a normalization scheme, the commutation relations for all the operators now become

$$[\hat{P}(z_1,t),\hat{P}^\dagger(z_2,t)] = -\frac{\hat{N}(z_1,t)}{n_a}\delta(z_1-z_2), \tag{19}$$

$$[\hat{P}^\dagger(z_1,t),\hat{N}(z_2,t)] = -2\frac{\hat{P}^\dagger(z_1,t)}{n_a}\delta(z_1-z_2), \tag{20}$$

$$[\hat{P}(z_1,t),\hat{N}(z_2,t)] = 2\frac{\hat{P}(z_1,t)}{n_a}\delta(z_1-z_2), \tag{21}$$

$$[\hat{U}(z_1,t),\hat{U}^\dagger(z_2,t)] = \frac{1}{n_p}\delta(z_1-z_2). \tag{22}$$

Here $n_a = N_a z_0$ is the number of atoms within one unit length. One important remark should be made here. When n_a and n_p are large, the collective quantum fluctuations of the atomic and light field operators become much smaller compared to their mean values. This is the case when the linearization approximation can be justified for calculating the quantum noise properties of the nonlinear problem, which is also the scope considered in the present work.

Equations (16)–(18) have the following fundamental soliton solution when the light filed is at exact resonance condition with the atoms [23]:

$$U_0(z,t) = \mathrm{sech}\left\{\frac{1}{2}[t-(1+r)z]\right\}, \tag{23}$$

$$P_0(z,t) = \mathrm{sech}\left\{\frac{1}{2}[t-(1+r)z]\right\}\tanh\left\{\frac{1}{2}[t-(1+r)z]\right\}, \tag{24}$$

$$N_0(z,t) = \mathrm{sech}^2\left\{\frac{1}{2}[t-(1+r)z]\right\} - \tanh^2\left\{\frac{1}{2}[t-(1+r)z]\right\}. \tag{25}$$

By adopting this solution form, we have implicitly assumed that the normalization units u_0 for the light field has been chosen to be the peak amplitude of the fundamental soliton and thus n_p is simply one-fourth of the soliton photon number in free space (since $\int_{-\infty}^{\infty}|U_0|^2 dz = 4$). The n_a can then be physically interpreted to be the number of atoms seen by the soliton. Such interpretation can help us to understand the quantum noise calculation formula derived in the next section. It should also be noted that the SIT solitons are also slow lights with a reduced group velocity $c/(1+r)$.

We should also note that although the above model is only derived for the simplest case of SIT solitons, in principle, it can be easily generalized to more complicated cases. For examples, the frequency detuning case can be simply derived by introducing a frequency shift term for the light field. The inhomogeneous broadening effects can be included by adding more atomic equations to represent atoms with distributed resonance frequencies. The effects of atomic decay can be included by adding the atomic decay terms together with the corresponding Langevin noise operators. All of these can be implemented by the standard techniques and will not be included here so as not to obscure the main points. Generalization of the present theory to nonlinear quantum pulse propagation effects in multilevel atomic systems including slow light solitons and quantum memories should also be possible.

III. QUANTUM NOISE CALCULATION

In this section, we derive the required formula for calculating the quantum noise properties of SIT solitons based on the general backpropagation method under the linearization approximation framework [22]. By assuming that the photon number and atom number seen by the soliton are both large enough, the linearization approximation can then be justified as stated in the previous section. By writing the perturbed field operators of the light and the perturbed dipole moment and population inversion operators of the atoms as follows $\hat{U} = U_0 + \hat{u}$, $\hat{P} = P_0 + \hat{p}$, and $\hat{N} = N_0 + \hat{n}$, the equations of motion in the Heisenberg picture for the perturbed quantum operators $\hat{u}$, $\hat{p}$, and $\hat{n}$ can be derived from Eqs. (16)–(18) and are given by

$$\frac{\partial}{\partial t}\hat{u} = -\frac{\partial}{\partial z}\hat{u} + \frac{r}{2}\hat{p}, \tag{26}$$

$$\frac{\partial}{\partial t}\hat{p} = \frac{1}{2}(U_0\hat{n} + N_0\hat{u}), \tag{27}$$

$$\frac{\partial}{\partial t}\hat{n} = -(P_0^*\hat{u} + U_0\hat{P}^\dagger + U_0^*\hat{p} + P_0\hat{u}^\dagger). \tag{28}$$

Under the linearization approximation framework, the perturbed operators $\hat{u}$, $\hat{p}$, and $\hat{n}$ now obey the following commutation relations:

$$[\hat{p}(z_1,t),\hat{p}^\dagger(z_2,t)] = -\frac{N_0(z_1,t)}{n_a}\delta(z_1-z_2), \tag{29}$$

$$[\hat{p}^\dagger(z_1,t),\hat{n}(z_2,t)] = -2\frac{P_0^*(z_1,t)}{n_a}\delta(z_1-z_2), \tag{30}$$

$$[\hat{p}(z_1,t),\hat{n}(z_2,t)] = 2\frac{P_0(z_1,t)}{n_a}\delta(z_1-z_2), \tag{31}$$

$$[\hat{u}(z_1,t),\hat{u}^\dagger(z_2,t)] = \frac{1}{n_p}\delta(z_1-z_2). \tag{32}$$

In particular, if all the atoms are originally in the ground state at $t = t_b$, then the results are the simplest, i.e.,

$$[\hat{p}(z_1,t_b),\hat{p}^\dagger(z_2,t_b)] = \frac{1}{n_a}\delta(z_1-z_2), \tag{33}$$

$$\hat{n}(z,t_b) = 0. \tag{34}$$

From these results and the assumption that the atomic system is originally in the ground state, one can infer that before interaction (at $t = t_b$) the atomic fluctuations only comes from the dipole moment fluctuations with its four correlation functions given by $\langle\hat{p}(z_1,t_b)\hat{p}(z_2,t_b)\rangle = \langle\hat{p}^\dagger(z_1,t_b)\hat{p}^\dagger(z_2,t_b)\rangle = \langle\hat{p}^\dagger(z_1,t_b)\hat{p}(z_2,t_b)\rangle = 0$ and $\langle\hat{p}(z_1,t_b)\hat{p}^\dagger(z_2,t_b)\rangle = \frac{1}{n_a}\delta(z_1-z_2)$.

Now by requiring the time conservation of the inner product defined below

$$\langle u^A,p^A,n^A|\hat{u},\hat{p},\hat{n}\rangle \equiv \int_{-\infty}^{\infty}[u^{A*}\hat{u}+u^A\hat{u}^\dagger+p^{A*}\hat{p}+p^A\hat{p}^\dagger+n^A\hat{n}]dz, \tag{35}$$

one can derive the following evolution equations for the adjoint system:

$$\frac{\partial}{\partial t}u^A=-\frac{\partial}{\partial z}u^A-\left(\frac{1}{2}N_0p^A-P_0n^A\right), \tag{36}$$

$$\frac{\partial}{\partial t}p^A=-\frac{r}{2}u^A+U_0u^A, \tag{37}$$

$$\frac{\partial}{\partial t}n^A=-\frac{1}{2}(U_0p^{A*}+U_0^*p^A). \tag{38}$$

In order to calculate the variance of the following general measurement operator at time $t=t_e$ by the homodyne detection:

$$\hat{M}(t_e)=\int_{-\infty}^{\infty}[f_L^*(z)\hat{u}(z,t_e)+f_L(z)\hat{u}^\dagger(z,t_e)]dz, \tag{39}$$

one can simply backpropagate the adjoint system from $t=t_e$ to $t=t_b$ with the initial conditions at $t=t_e$ given by $u^A(z,t_e)=f_L(z)$, $p^A(z,t_e)=0$, and $n^A(z,t_e)=0$. Due to the time conservation of the inner product between the two systems (linearized and adjoint), the measured operator can be rewritten as

$$\hat{M}(t_e)=\int_{-\infty}^{\infty}dz[u^{A*}(z,t_b)\hat{u}(z,t_b)+u^A(z,t_b)\hat{u}^\dagger(z,t_b)+p^{A*}(z,t_b)\hat{p}(z,t_b)+p^A(z,t_b)\hat{p}^\dagger(z,t_b)+n^A(z,t_b)\hat{n}(z,t_b)]. \tag{40}$$

Since all the quantum properties of the operators at the original time t_b are known, the variance of $\hat{M}(t_e)$ can be calculated according to

$$\mathrm{Var}[\hat{M}(t_e)]=\frac{1}{4}\int\left[\frac{1}{n_p}|u^A(z,t_b)|^2+\frac{1}{n_a}|p^A(z,t_b)|^2\right]dz. \tag{41}$$

Here we have used the fact that the atoms are in the ground state at $t=t_b$ such that the atomic fluctuations only comes from the dipole moment fluctuations. The squeezing ratio of the measurement then is

$$S=\frac{\mathrm{Var}[\hat{M}(t_e)]}{\mathrm{Var}[\hat{M}(t_b)]}=\frac{\int_{-\infty}^{\infty}\left[|u^A(z,t_b)|^2+\frac{n_p}{n_a}|p^A(z,t_b)|^2\right]dz}{\int_{-\infty}^{\infty}|f_L(z)|^2dz}. \tag{42}$$

Equation (42) has an interesting physical implication. When the atomic characteristic number n_a (i.e., atom number seen by the soliton) is much large than the photonic characteristic number n_p (i.e., one-fourth of the soliton photon number), the contribution from the initial atomic fluctuations will be reduced by a factor n_p/n_a. Physically, this is of course a signature of the noise averaging effects caused by the ensemble atoms. We shall also emphasize that the dependence of the quantum squeezing on the atomic density is not expected from the classical SIT theory and can only be derived by a quantum theory from the fundamental light-atom interaction Hamiltonian. In classical SIT soliton theories, the coefficient $r=n_a/n_p$ in Eq. (42) can be totally eliminated by transforming to the moving coordinate and rescale the units of z. This is not rigorously possible in the quantum theory developed here.

If the following two operators are measured by two homodyne detectors:

$$\hat{M}_1(t_e)=\int_{-\infty}^{\infty}[f_{L1}^*(z)\hat{u}(z,t_e)+f_{L1}(z)\hat{u}^\dagger(z,t_e)]dz, \tag{43}$$

$$\hat{M}_2(t_e)=\int_{-\infty}^{\infty}[f_{L2}^*(z)\hat{u}(z,t_e)+f_{L2}(z)\hat{u}^\dagger(z,t_e)]dz, \tag{44}$$

then the corresponding quantum correlation between the two operators can be calculated according to the following formula:

$$C_{12}=\frac{\langle u_1^A(z,t_b)|u_2^A(z,t_b)\rangle+\frac{n_p}{n_a}\langle p_1^A(z,t_b)|p_2^A(z,t_b)\rangle}{\sqrt{\left(\langle u_1^A(z,t_b)|u_1^A(z,t_b)\rangle+\frac{n_p}{n_a}\langle p_1^A(z,t_b)|p_1^A(z,t_b)\rangle\right)\left(\langle u_2^A(z,t_b)|u_2^A(z,t_b)\rangle+\frac{n_p}{n_a}\langle p_2^A(z,t_b)|p_2^A(z,t_b)\rangle\right)}}. \tag{45}$$

Here $u_{1,2}^A(z,t_b)$ and $p_{1,2}^A(z,t_b)$ are the backpropagated adjoint solutions with $f_{L1,2}(z)$ as the initial conditions, respectively, and the inner product of two functions is defined according to

$$\langle f_1(z,t)|f_2(z,t)\rangle \equiv \int_{-\infty}^{\infty}\mathrm{Re}[f_1^*(z,t)f_2(z,t)]dz, \tag{46}$$

Equations (42) and (45) are the main results of the present work, which will be used to study the quantum properties of SIT solitons in the following sections.

IV. QUANTUM SQUEEZING AND INTERSOLITON CORRELATION

In this section, we use the derived formula to calculate the quantum squeezing and correlation of SIT solitons after passing through a two-level medium of length L. The fundamental soliton solution in Eq. (23) will be used as the classical soliton solution for performing linearization. Equation (23) is the field solution inside the medium. Before and after the medium, the solution is still given by Eq. (23) but with $r = n_a / n_p = 0$. If the homodyne detection is used for the detection of the quantum fluctuations in SIT solitons, then the measured operator is given by Eq. (39) with $f_L(z)$ being the local oscillator of the homodyne detection. Note that in the free space after the medium, $f_L(z)$ is not only the local oscillator field distribution in the normalized z space but also the local oscillator field distribution in the normalized time domain with the two normalization units related by $z_0 = ct_0$. In the following, for convenience, we stay in the z coordinate to express the projection operation of the homodyne detection, even though in practice it is integrated over the t coordinate outside the medium. According to the soliton perturbation theory developed previously [9,15], one can use the following projection functions to project out the four perturbed soliton parameters: photon number, phase, momentum (frequency), and position,

$$f_n(z) = \mathrm{sech}\left(\frac{z}{2}\right), \tag{47}$$

$$f_\theta(z) = i\left[1 - \frac{z}{2} * \tanh\left(\frac{z}{2}\right)\right]\mathrm{sech}\left(\frac{z}{2}\right), \tag{48}$$

$$f_p(z) = i \tanh\left(\frac{z}{2}\right)\mathrm{sech}\left(\frac{z}{2}\right), \tag{49}$$

$$f_z(z) = z\ \mathrm{sech}\left(\frac{z}{2}\right). \tag{50}$$

A local oscillator that detects only the soliton parts can be expressed in general by

$$f_L(z) = c_1 * f_n(z) + c_2 * f_\theta(z) + c_3 * f_p(z) + c_4 * f_z(z), \tag{51}$$

with the linear superposition coefficients $c_{1,2,3,4}$. By minimizing the squeezing ratio with respect to the four coefficients c_1 to c_4, monotonically increasing squeezing as a function of the propagation distance can be detected as shown in Fig. 1, when the optical soliton is at exact resonance with the two-level medium. The results are calculated by the analytical formula presented in our previous work [15] and they can serve as a good checking point for the correctness of the numerical backpropagation calculation developed in the present work.

By examining the optimum local oscillator pulse shapes determined this way, we find that when at exact resonance, the maximum squeezing is solely caused by the coupling of the perturbed photon number and position operators, not including the perturbed momentum and phase operators. In contrast to the case of NLS solitons, this fact implies that at exact resonance there will be no squeezing at all if the output sech soliton pulse with only a possible phase shift is directly used as the local oscillator. Since in practical soliton squeezing experiments, the output sech pulses are the most convenient local oscillator source for the homodyne detection, the above-stated fact may seriously limit the squeezing detection for SIT solitons. To overcome this limitation, one possibility is to detune the soliton center frequency to be a little off resonance. When not at exact resonance, the perturbed photon number operator begins to influence the perturbed phase operator and the optimum local oscillator pulse shape should be more close to the sech soliton pulse shape with a possible phase shift. However, it has also been well known that when the sech soliton pulse shape is used as the local oscillator, the contribution of the field continuum parts will enter the detection. For such cases, the squeezing ratio can only be analyzed by numerical calculation like the backpropagation method [22].

Figure 2 shows the calculated squeezing ratio for different atomic medium lengths and light frequency detunings with $r=1$. It can be seen that although the detectable squeezing ratio is somewhat reduced, monotonically increasing squeezing still can be expected when the frequency detuning is suitably adjusted to be around 0.5 normalization units. Another possibility of further squeezing detection enhancement

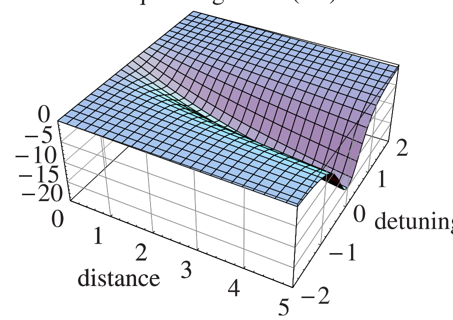

FIG. 1. (Color online) Optimum squeezing ratio versus propagation distance and frequency detuning by detecting only the soliton parts and with $r=1$. Calculation from the analytic perturbation theory.

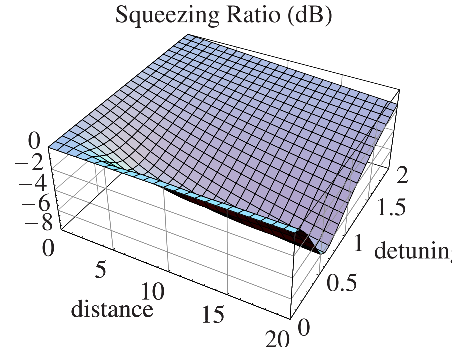

FIG. 2. (Color online) Optimum squeezing ratio versus propagation distance and frequency detuning by using a sech local oscillator pulse shape with a phase shift only. $r=1$ is assumed.

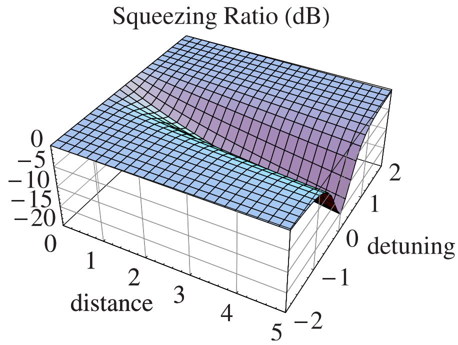

FIG. 3. (Color online) (a) Optimum squeezing ratio versus propagation distance and frequency detuning by using a sech local oscillator pulse shape, but with both position and phase shifts. $r=1$ is assumed.

is to stay at exact resonance, but the output sech soliton pulse shape is now also position shifted instead of phase shifted only. This can be expected by the observation that the linear combination of $f_n(z)$ and $f_z(z)$ should be very close to a position-shifted sech function. Figure 3 shows the detectable squeezing ratio for different medium lengths and frequency detunings when the sech local oscillator pulse is both optimally phase shifted and position shifted. The obtained results in Fig. 3 are almost the same as those in Fig. 1. For comparison, in Fig. 4 we show the difference of achievable squeezing ratio in dB scale between the results in Figs. 1 and 3 (i.e., Figs. 1–3). The squeezing degradation mainly shows up for the cases under off-resonance condition and long propagation distance. At resonance, the achievable squeezing ratio is very close to the soliton part result. In fact, it can be even a little smaller than the soliton part case due to the fact that the linear combination of $f_n(z)$ and $f_z(z)$ is not exactly a position-shifted sech function and that the introduced field continuums may help to improve the squeezing in some cases. In this way, the required local oscillator can be simply the sech pulse instead of a much more complicated pulse shape given by Eq. (51). This should be the easiest way for practical experiments to perform the homodyne detection and the obtained results here should be very useful for helping design future SIT squeezing experiments.

One interesting question here is how much contribution comes from the initial atomic noises? We find that for all the

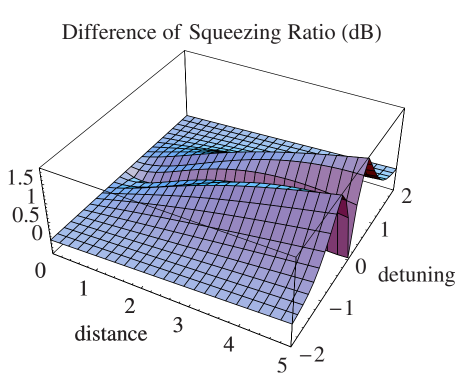

FIG. 4. (Color online) Difference of achievable squeezing ratio in dB between the cases in Figs. 1 and 3.

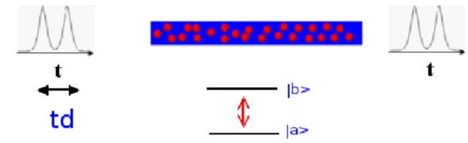

FIG. 5. (Color online) Schematic illustration of quantum correlation generation for a SIT soliton pair.

soliton cases considered above, the impacts of the initial atomic noises on the achievable squeezing ratio can be totally ignored, as long as the simulation time window $[t_b, t_e]$ is large enough to cover almost all the response tails of the backpropagation. Intuitively, this is because the backpropagated adjoin system solution travels mostly backward outside the medium and, thus, the left excitation of the dipole moment part at t_b will be very small when t_b is early enough in relative to the collision time. The generation of squeezing in these cases is thus mainly through the nonlinear mediation of the atoms on the optical field. This should not be true if the input pulses are not exact solitons.

In our studies, we have also found that two-time-multiplexed SIT solitons can become quantum correlated through the mediation of the atomic medium. The photon number correlation can be very large if the two in-phase solitons are close to each other and the propagation distance is long enough, just as the NLS solitons in optical fibers [24]. Figure 5 shows an illustration on how to establish the quantum correlations between two SIT solitons by varying the separation t_d. After passing through the atomic media, we calculate the quantum correlations of the photon number operators between the soliton pair. In Fig. 6, we show that a certain degree of quantum correlation in photon numbers can be established when the two solitons are in phase. The quantum correlation increases as the separation decreases due to stronger overlapping. It also increases as the propagation distance increases due to accumulated nonlinear effects. A more interesting result found is the situation when the two solitons are almost nonoverlapping (i.e., t_d=20). One will expect that some quantum correlations may still get established through the long-live atomic perturbations. The following cases of number-number, number-phase, phase-number, and phase-phase correlations are more interesting and unexpected. Figure 7 shows the dependence of all the four quantum correlations as functions of the propagation distance when the two

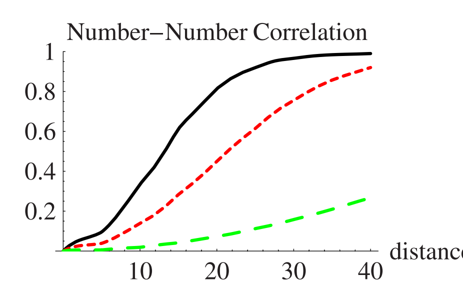

FIG. 6. (Color online) Photon number correlation established between two in-phase solitons as a function of the propagation distance for several soliton separation t_d, i.e., solid line (black): t_d $=14$; short dashed line (red): t_d=16; and long dashed line (green): t_d=20. $r=1$ is assumed.

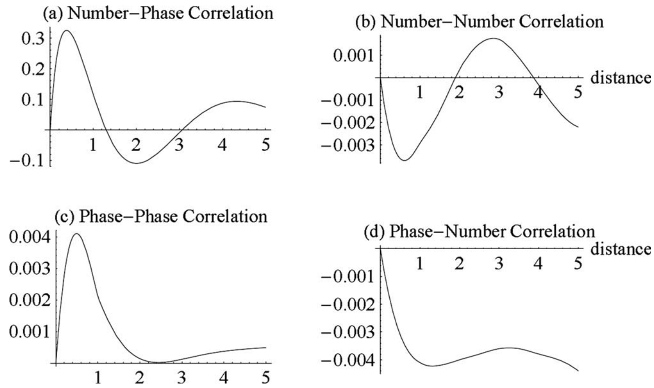

FIG. 7. Quantum correlations established between two $\pi/2$ out-of-phase solitons as functions of the propagation distance for the components in (a) Number (first soliton) phase (next soliton); (b) number number; (c) phase phase; and (d) phase number. Soliton separation $t_d=20$ and $r=1$ are assumed. Local oscillator functions in the form of sech and $i*$sech are used for the photon number and phase detections, respectively.

SIT solitons are $\pi/2$ out of phase. Due to the oscillating field continuum and atomic noises in space, the curves of quantum correlation also oscillate as functions of the propagation distance. The separation of the two solitons here is larger and the interaction through tail overlapping is expected to be very small as can be seen from Fig. 6. It is thus surprising to see that the number (first soliton)-phase (next solition) correlation can reach above 0.32 after a very short propagation distance (only 0.5 unit length). The values of the quantum number-phase correlation are about tenfold larger than those in other three correlation functions when the same interaction length is considered. In particular, the number(first soliton)-phase (next soliton) correlation and the phase (first soliton)-number (next soliton) correlation are also not the same. These results are mainly due to the fact that the atomic responses of the linearized system (original or adjoint) are relatively longer lived. Therefore, the quantum noises of the first soliton will be able to more strongly influence the second soliton to establish the unsymmetrical quantum correlations between the two solitons. The achieved soliton number-phase correlation in this case is of particular interest for the potential application of QND measurement. The photon number of the first soliton is a conserved quantity during propagation and its quantum noises can be somewhat inferred by detecting the phase of the second soliton, even though the achieved correlation coefficient is only around 0.32 from the present numerical simulation. In principle, it may still be possible to detect even stronger quantum correlation between different operator pairs if more optimized local oscillator pairs are used, just as in the case of NLS solitons [25]. Cascaded QND schemes may also be used to obtain more information about the measured soliton photon number. These are just some of the possible directions for further studies along this line.

V. CONCLUSION

In conclusion, a quantum theory that can accurately model and calculate the quantum noises and quantum correlations of self-induced transparency solitons has been developed. The nonlinear quantum effects of atoms are taken into account rigorously. SIT solitons are not only slowing down but also get squeezed due to the nonlinear light-atom interaction. We have shown that for the detection of SIT soliton squeezing, a sech local oscillator pulse shape with a suitably adjusted position shift as well as phase shift can be used to achieve a close-to-optimum squeezing ratio at the case of exact resonance. Similar to the NLS solitons, strong quantum number-number correlation between two in-phase SIT solitons can also be established through the tail overlapping interaction. Most important of all, unlike NLS solitons, an almost nonoverlapping $\pi/2$ out-of-phase SIT soliton pair can still establish a reasonably large long-range quantum number-phase correlation through the longer-lived atomic response. Further generalization of the theory to slow/fast solitons in multilevel atomic systems [26,27] can also be carried out. All of these findings should open the whole possibilities of utilizing the solitons in atomic systems for quantum squeezing/correlation generation and applications.

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