Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 6 (~~4/27, 4/29~~), (5/4, 5/7)	Two-mode Squeezed states	☐ EPR pair ☐ Cat states ☐ non-Gaussian states	☐ Quantum Discord ☐ Entanglement ☐ Steering ☐ Bell's inequality
week 7 (5/11, 5/14)	Optical devices	☐ Beam splitter ☐ Mach-Zehnder interferometer	☐ linear optics ☐
week 8 (~~5/11, 5/14~~, (4/27, 4/29)	Interferometry	☐ Young's Interferometry, $g^{(1)}$ ☐ HBT-Interferometry, $g^{(2)}$	☐ Quantum Phase Estimation ☐ Quantum Fisher Information ☐

- **Assignment**

  Deadline: 4:00PM, Tuesday, May 11th

(1) Schmidt decomposition:
Theorem:
Suppose $|\psi\rangle$ is a pure state of a bipartite composite system, AB. Then there exist orthonormal states $|\phi_A\rangle$ for system A, and $|\phi_B\rangle$ for system B, s.t.

$$|\Psi\rangle = \sum_i, \lambda_i |\phi_A\rangle\, |\phi_B\rangle, \tag{1}$$

where λ_i are non-negative real numbers satisfying known as Schmidt coefficients.

1. Do the Schmidt decomposition and find the Schmidt number for a product state,

$$|\psi\rangle = \frac{|00\rangle + |01\rangle}{\sqrt{2}}. \tag{2}$$

2. Do the Schmidt decomposition and find the Schmidt number for an entangled state,

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}. \tag{3}$$

3. What is the Schmidt number for the state

$$\frac{|00\rangle + |11\rangle + |22\rangle}{\sqrt{3}}. \tag{4}$$

(2) Two-mode squeezing:

(2a) Derive the following Bogoliubov transformation:

$$\hat{S}(\xi)^\dagger\, \hat{a}\, \hat{S}(\xi) = \hat{a}\, \cosh r + \hat{b}^\dagger e^{i\phi}\, \sinh r, \tag{5}$$
$$\hat{S}(\xi)^\dagger\, \hat{b}\, \hat{S}(\xi) = \hat{b}\, \cosh r + \hat{a}^\dagger e^{i\phi}\, \sinh r, \tag{6}$$

(2b) Show that the non-zero correlation

$$\langle\xi|\hat{a}\hat{b}|\xi\rangle = \cosh r\, \sinh r e^{i\phi} \tag{7}$$
$$\tag{8}$$

- **Take-home Messages:**
  1. Two-mode Squeezed states
  2. Entanglement
  3. Schmidt decomposition
  4. Peres-Horodecki separability criterion
  5. Positive partial transpose criterion
  6. Duan's inseparability criterion

- **From Scratch !!**
- Two-mode squeezed operator:

$$\hat{S}(\xi) \equiv \exp(\xi \hat{a}^\dagger \hat{b}^\dagger - \xi^* \hat{a}\hat{b}) \tag{9}$$

- Two-mode vacuum:

$$|\xi\rangle = \hat{S}(\xi)|0,0\rangle \tag{10}$$

$$= \frac{1}{\cosh r} \sum_{n=0}^{\infty} e^{i\phi} (\tanh r)^n |n,n\rangle \tag{11}$$

- Correlations:

$$\langle\xi|\hat{a}^\dagger \hat{a}|\xi\rangle = \langle\xi|\hat{b}^\dagger \hat{b}|\xi\rangle = \sinh^2 r \tag{12}$$

$$\langle\xi|\hat{a}\hat{b}|\xi\rangle = \cosh r \, \sinh r e^{i\phi} \tag{13}$$

$$\langle\xi|\hat{a}^\dagger \hat{b}|\xi\rangle = 0 \tag{14}$$

$$\langle\xi|\hat{a}^2|\xi\rangle = \langle\xi|\hat{b}^2|\xi\rangle = \langle\xi|\hat{a}|\xi\rangle = \langle\xi|\hat{b}|\xi\rangle = 0 \tag{15}$$

- Theorem (SVD Theorem for Matrices): $\overline{\overline{A}}_{m\times n} = \overline{\overline{U}}_{m\times m} \overline{\overline{\Sigma}}_{m\times n} \overline{\overline{V}}^*_{n\times n}$

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