Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 3 (3/30, ~~4/2~~, ~~4/6~~)	Coherent states, $\|\alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐

- **Assignment**

  Deadline: 4:00PM, Friday, April 9th

1. By referring to the Bose-Einstein statistics, we have the thermal state:

$$\rho_{th} = \sum_n P(n)\,|n\rangle\langle n|, \qquad \text{with} \quad P(n) = \frac{1}{\bar{n}+1}(\frac{\bar{n}}{\bar{n}+1})^n, \quad \bar{n} = \frac{1}{\exp[\hbar\omega/k_B T]-1}. \tag{1}$$

Instead, without assuming quantum statistical mechanics, we can also derive the quantum state of thermal light by the maximization of the entropy of thermal states in the presence of the conservation of the total probability. Show that the entropy for thermal state has its maximum value,

$$\mathcal{S} = -k_B \sum_n \rho_n \,\ln \rho_n - \mu_1(\sum_n \rho_n - 1) - \mu_2(\sum_n \rho_n E_n - E), \tag{2}$$

with E denotes the average energy of the mode, μ_1 and μ_2 are the Lagrange multipliers, resulting in

$$\rho_n = \frac{1}{Z}\,\exp(-\frac{E_n}{k_B T}), \qquad Z = \sum_n \exp(-\frac{E_n}{k_B T}). \tag{3}$$

2. Based on the definition of Wigner function,

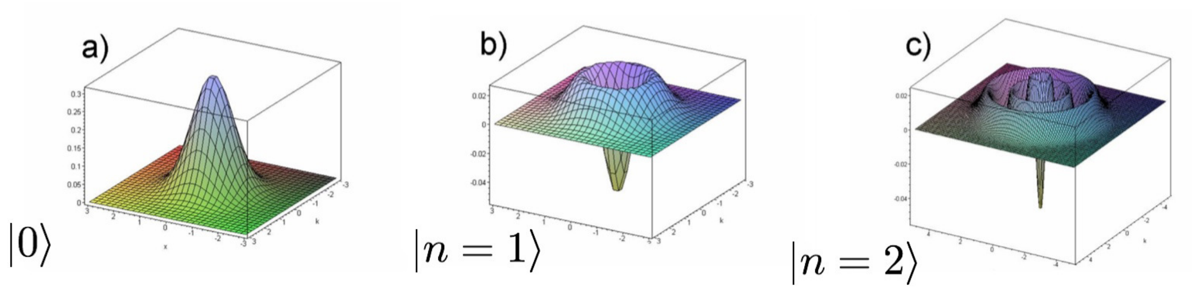
$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \mathrm{d}\xi \,\exp[-\frac{i}{\hbar}p\,\xi]\langle x+\frac{\xi}{2}|\hat{\rho}|x-\frac{\xi}{2}\rangle, \tag{4}$$

show and plot the corresponding phase space wave function:

2. (a) For the vacuum state, $\hat{\rho} = |0\rangle\langle 0|$;
2. (b) For the single-photon state, $\hat{\rho} = |1\rangle\langle 1|$;
2. (c) For the number state, $\hat{\rho} = |n\rangle\langle n|$. In general, one has the following formula for Wigner function in phase space:

$$W(x,p) = \frac{(-1)^m}{\pi\hbar}\exp[-2\eta(x,p)]\,L_m[4\eta(x,p)], \tag{5}$$

where L_m is the m-th order Laguerre polynomial.



- **Take-home Messages:**
    1. Heisenberg's Uncertainty Relation
    2. Minimum Uncertainty States
    3. Heisenberg limit
    4. Robertson–Schrödinger uncertainty relations
    5. Quantum entropic uncertainty principle

- **From Scratch !!**
- For any two non-commuting observables, $[\hat{A}, \hat{B}] = i\hat{C}$, we have the *uncertainty relation*:

$$\Delta A^2 \Delta B^2 \geq \frac{1}{4}[\langle \hat{F} \rangle^2 + \langle \hat{C} \rangle^2], \tag{6}$$

where

$$\hat{F} = \hat{A}\hat{B} + \hat{B}\hat{A} - 2\langle \hat{A} \rangle \langle \hat{B} \rangle, \tag{7}$$

where the operator $\hat{F}$ is a measure of correlations between $\hat{A}$ and$\hat{B}$.

- A *Minimum Uncertainty State* (MUS), $|\psi\rangle$, satisfies

$$[\hat{A} + i\lambda\hat{B}]|\psi\rangle = [\langle \hat{A} \rangle + i\lambda \langle \hat{B} \rangle]|\psi\rangle = z|\psi\rangle, \tag{8}$$

where z is a complex number.

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