Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
Feb. 26th (Fri.)	Simple Harmonic Oscillator (SHO)	☐ classical trajectory ☐ analogue to EM waves	☐ Bohmian mechanics ☐ Inverted SHO ☐
week 1 (3/2, 3/9)	Quantum SHO	☐ Fock states, $\|n\rangle$ ☐ creation operator, $\hat{a}^\dagger$	☐ single-photon detection ☐ Wave-Particle Duality ☐ photon-number resolving ☐
(3/12, 3/16, ~~3/19~~)		☐ Vacuum state ☐ Quantum Fluctuations	☐ Shot Noise Limit ☐ Casimir Force ☐

- **Take-home Messages:**
  1. Class Materials: http://mx.nthu.edu.tw/∼rklee
  2. Discussion Channel: Quantum Optics, Lecture@NTHU, Slack, quantumoptics-zgq1695.slack.com
  3. *"Everything should be made as simple as possible, but not simpler."* – Albert Einstein
  4. SHO is RK's favorite.
  5. Applications: spring-mass, pendulum, EM-wave, two-level systems, ...
  6. Phase Space
  7. Possible **quantization**
  8. Classical analog of electromagnetically induced transparency

- **References:**

1. Chapter I, in C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *"Photons & Atoms"*, John Wiley & Sons (1989).
2. Chapter 2, in J. J. Sakurai, *"Modern Quantum Mechanics,"* Addison Wesley (1994).
3. Chapter 7, in A. Goswami, *"Quantum Mechanics,"* WCB Publishers (1992).
4. Chapters 3-4, in J. B. Marion and S. T. Thornton, *"Classical dynamics of particles and systems,"* Saunders College (1995).

- **From Scratch !!**
- Simple Harmonic Oscillator (SHO):

$$\frac{d^2}{dt^2}x(t) + \omega_0^2\, x(t) = 0, \tag{1}$$

- Hamiltonian energy

$$H = \text{K.E.} + \text{P.E.} = \frac{1}{2}\frac{p^2}{m} + \frac{1}{2}k\, x^2, \tag{2}$$

- Phase Space
- EM wave as a SHO: The *total energy* for the electric and magnetic fields, plus the particles, is

$$H = \sum_\alpha \frac{1}{2} m_\alpha v_\alpha^2(t) + \frac{\epsilon_0}{2}\int d^3\vec{r}\,[E^2(\vec{r},t) + c^2\, B^2(\vec{r},t)], \tag{3}$$

$$= \sum_\alpha \frac{1}{2} m_\alpha v_\alpha^2(t) + H_\perp + H_\parallel, \tag{4}$$

- Quantum SHO:

$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \tag{5}$$

where the $\hat{x}$ and $\hat{p}$ are non-commute operators, *i.e.*,

$$[\hat{x}, \hat{p}] = i\hbar. \tag{6}$$

- **More to know ....**
  - Canonical transformation and Quantization
  - Hamiltonian and Lagrangian mechanics, Poisson braket
  - Symplectic Approach
  - Damped SHO (over-damped, critically-damped, under-damped), Driven SHO,
  - Inverted SHO
    • V. Subramanyan et al., *"Physics of the Inverted Harmonic Oscillator: From the lowest Landau level to event horizons,"* arXiv: 2012.09875 (2020).
  - Pendulum, Double pendulum/Coupled SHOs, Nonlinear SHO
  - Periodic (Orbit) and Chaotic Trajectories
    • Tsin-Dong Lee, Chih-Yao Chen, YuanYao Lin, Ming-Chiu Chou, Te-ho Wu, and RKL, *"Surface-Structure-Assisted Chaotic Mode Lasing in Vertical Cavity Surface Emission Lasers,"* Phys. Rev. Lett. **101**, 084101 (2008).
  - Bose-Einstein Condensates in Double Wells:
    • S. Raghavan, et al., *"Coherent oscillations between two weakly coupled Bose-Einstein condensates: Josephson effects, π oscillations, and macroscopic quantum self-trapping,"* Phys. Rev. A **59**, 620 (1999).
    • Blas M. Rodriguez-Lara and RKL, *"Classical dynamics of a two-species condensate driven by a quantum field,"* Phys. Rev. E **84**, 016225 (2011)
  - Parity-Time ($\mathcal{PT}$)-symmetric SHO
    • Ludmila Praxmeyer, Popo Yang, and RKL, *"Phase-space representation of a non-Hermitian system with PT-symmetry,"* Phys. Rev. A 93, 042122 (2016).
  - EIT and Fano resonance:
    • C. L. Garrido Alzar et al., *"Classical analog of electromagnetically induced transparency,"* Am. J. Phys. **70**, 37 (2002).
  - Pseudo-Potential
    • Chun-Yan Lin, Jen-Hsu Chang, Gershon Kurizki, and RKL, *"Solitons supported by intensity-dependent dispersion,"* Opt. Lett. **45**, 1471 (2020).
  - WKB (Wentzel–Kramers–Brillouin) approximation, Bohmian, Quantum Chaos

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