Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 3 (3/30, ~~4/2~~, ~~4/6~~)	Coherent states, $\|\alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐

- **What we know at this stage !**

2. Based on the definition of Wigner function,

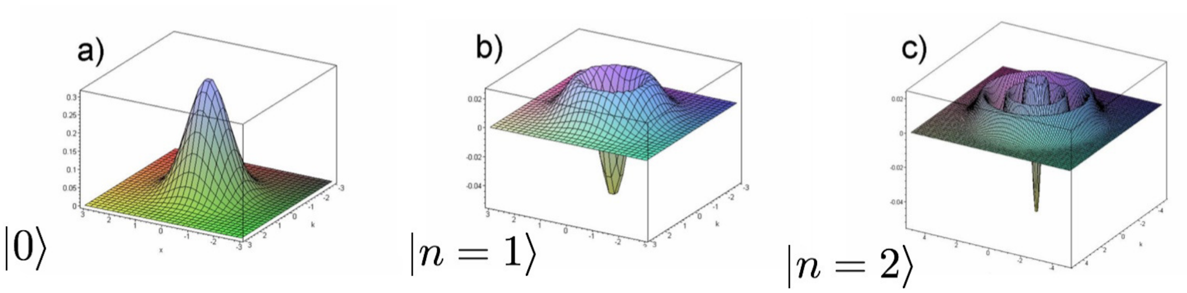
$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} d\xi \exp[-\frac{i}{\hbar}p\,\xi]\langle x + \frac{\xi}{2}|\hat{\rho}|x - \frac{\xi}{2}\rangle, \tag{1}$$

show and plot the corresponding phase space wave function:

2. (a) For the vacuum state, $\hat{\rho} = |0\rangle\langle 0|$;
2. (b) For the single-photon state, $\hat{\rho} = |1\rangle\langle 1|$;
2. (c) For the number state, $\hat{\rho} = |n\rangle\langle n|$. In general, one has the following formula for Wigner function in phase space:

$$W(x,p) = \frac{(-1)^m}{\pi\hbar}\exp[-2\eta(x,p)]\,L_m[4\eta(x,p)], \tag{2}$$

where L_m is the m-th order Laguerre polynomial.



- **Take-home Messages:**
  (a) Position distribution of Wigner function: $\int_{-\infty}^{\infty} \mathrm{d}p\, W(x,p) = W(x) = \langle x|\hat{\rho}|x\rangle$
  (b) Momentum distribution of Wigner function: $\int_{-\infty}^{\infty} \mathrm{d}x\, W(x,p) = W(p) = \langle p|\hat{\rho}|p\rangle$
  (c) Overlap of quantum strates: $\mathrm{tr}(\hat{\rho_1}\hat{\rho_2}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}x\mathrm{d}p\, W_{\hat{\rho_1}}(x,p)W_{\hat{\rho_2}}(x,p)$
  (d) von Neumann Equation:

$$\frac{\partial}{\partial t}\hat{\rho} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}], \tag{3}$$

  (d) Time evolution of Wigner function, i.e., Quantum Liouville equation:

$$[\frac{\partial}{\partial t} + \frac{p}{m}\frac{\partial}{\partial x} - \frac{d\,U(x)}{dx}\frac{\partial}{\partial p}]W(x,p;t) = \sum_{l=1}^{\infty}[\frac{(-1)^l(\hbar/2)^{2l}}{(2l+1)!}\frac{d^{2l+1}U(x)}{dx^{2l+1}}\frac{\partial^{2l+1}}{\partial p^{2l+1}}]W(x,p;t) \tag{4}$$

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