Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 2 (3/12, 3/16, ~~3/19~~)	Quantum Mechanics	☐ Schrödinger picture ☐ Heisenberg picture ☐ Interaction picture	☐ Uncertainty Relation ☐ Probability Interpretation ☐ Measurement problem ☐ Non-locality ☐ Macrorealism ☐
week 3 (3/30, ~~4/2~~, 4/6)	Coherent states, $\|\alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐

- **Assignment**

  Deadline: 3:30PM, Tuesday, March 30th

1.(a) Show that the mean and variance of photon number in the *Poisson distribution* are

$$\langle\hat{n}\rangle = \sum_n nP(n) = |\alpha|^2 \equiv \bar{n}, \tag{1}$$

$$\langle\Delta\hat{n}^2\rangle = \langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2 = |\alpha|^2 = \langle\hat{n}\rangle. \tag{2}$$

1.(b) Show that the mean and variance of photon number in the *Bose-Einstein distribution* are

$$\bar{n} = \sum_{n=0}^{\infty} n\,P(n) = \frac{1}{\exp[\hbar\omega/k_B T] - 1}, \tag{3}$$

$$\Delta n^2 = \bar{n} + \bar{n}^2, \tag{4}$$

which is larger than that of a Poisson distribution.

2. Show that $\hat{D}(\alpha) \equiv e^{+\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$ acts as a displacement operator upon the amplitudes $\hat{a}$ and $\hat{a}^\dagger$, *i.e.*,

$$\hat{D}^{-1}(\alpha)\,\hat{a}\,\hat{D}(\alpha) = \hat{a} + \alpha, \tag{5}$$

$$\hat{D}^{-1}(\alpha)\,\hat{a}^\dagger\,\hat{D}(\alpha) = \hat{a}^\dagger + \alpha^*. \tag{6}$$

You may apply the formula

$$e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]] + \cdots$$

3. Show that the set of all coherent states $|\alpha\rangle$ is a complete set,

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi\sum_n |n\rangle\langle n|, \quad \text{or} \quad \frac{1}{\pi}\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = 1. \tag{7}$$

You may first consider the following integral identity, with $\alpha = |\alpha|e^{i\theta}$,

$$\int (\alpha^*)^n \alpha^m e^{-|\alpha|^2}\, d^2\alpha = \int_0^\infty |\alpha|^{n+m+1} e^{-|\alpha|^2}\, d|\alpha| \int_0^{2\pi} e^{i(n-m)\theta}\, d\theta = \pi\, n!\, \delta_{mn}.$$

- **Take-home Messages:**
  1. Poisson distribution
  2. $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$
  3. Displacement operator
  4. Coherent states are Gaussian states

- **From Scratch !!**
- The coherent state $|\alpha\rangle$ has the Poisson distribution in the photon number,

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \tag{8}$$

- The coherent state is displaced from the ground state of a simple harmonic oscillator.

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = e^{+\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle. \tag{9}$$

- The set of all coherent states $|\alpha\rangle$ is a complete set,

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi \sum_n |n\rangle\langle n|, \quad \text{or} \quad \frac{1}{\pi}\int |\alpha\rangle\langle\alpha|\mathrm{d}^2\alpha = 1. \tag{10}$$

- Two coherent states corresponding to different eigenstates α and β are not orthogonal,

$$\langle\alpha|\beta\rangle = \exp(-\frac{1}{2}|\alpha|^2 + \alpha^*\beta - \frac{1}{2}|\beta|^2) = \exp(-\frac{1}{2}|\alpha - \beta|^2). \tag{11}$$

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