Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 6 (~~4/27, 4/29~~), (5/4, 5/7)	Two-mode Squeezed states	☐ EPR pair ☐ Cat states ☐ non-Gaussian states	☐ Quantum Discord ☐ Entanglement ☐ Steering ☐ Bell's inequality
week 7 (5/11, 5/14)	Optical devices	☐ Beam splitter ☐ Mach-Zehnder interferometer	☐ linear optics ☐
week 8 (~~5/11, 5/14~~, (4/27, 4/29)	Interferometry	☐ Young's Interferometry, $g^{(1)}$ ☐ HBT-Interferometry, $g^{(2)}$	☐ Quantum Phase Estimation ☐ Quantum Fisher Information ☐

- **Assignment**

  Deadline: 4:00PM, Tuesday, May 4th

  1. Try to unveil the difference between *squeezed coherent states* $|\xi,\alpha\rangle$ and *coherent squeezed states* $|\alpha,\xi\rangle$ by expanding these states in the number state basis. Here, show that

$$\textit{squeezed coherent states:}\quad |\xi,\alpha\rangle = \hat{S}(\xi)\,\hat{D}(\alpha)|0\rangle = \hat{S}\xi\sum_{n=0}^{\infty} C_n|n\rangle \tag{1}$$

$$\textit{coherent squeezed states:}\quad |\alpha,\xi\rangle = \hat{D}(\alpha)\,\hat{S}(\xi)|0\rangle = \hat{D}(\alpha)\sum_{m=0}^{\infty} C_{2m}|2m\rangle = \sum_{m=0}^{\infty} C_{2m}|2m\rangle, \tag{2}$$

here

$$C_n = exp[\frac{-1}{2}|\gamma|^2|\frac{1}{2}\gamma^2 e^{-i\theta}\tanh(r)]\,[n!\cosh(r)]^{-1/2}[\frac{1}{2}e^{i\theta}\tanh(r)]^{n/2}H_n(\gamma[e^{i\theta}\sinh(2r)]^{-1/2}), \tag{3}$$

$$C_m = exp[\frac{-1}{2}|\alpha|^2 - \frac{1}{2}\alpha^{*2}e^{i\theta}\tanh(r)]\,[n!\cosh(r)]^{-1/2}[\frac{1}{2}e^{i\theta}\tanh(r)]^{n/2}H_n(\gamma[e^{i\theta}\sinh(2r)]^{-1/2}), \tag{4}$$

with the squeezing parameter $\xi = r\,e^{i\theta}$ and $\gamma = \alpha\cosh(r) + \alpha^* e^{i\theta}\sinh(r)$.

Ref. J. J. Gong, and P. K. Aravind, "Expansion coefficients of a squeezed coherent state in the number state basis," American Journal of Physics 58, 1003 (1990).

- **Take-home Messages:**
  1. Correlation (Coherence) functions
  2. First-order Correlation: Young's Interferometry
  3. Second-order Correlation: Hanbury Brown and Twiss (HBT) Interferometry
  4. Photon Bunching and Anti-bunching

- **From Scratch !!**
- With the analogy to the *classical* correlation function, we have the first-order quantum correlation function by normalizing it to one:

$$g^{(1)}(x_1, x_2) = \frac{G^{(1)}(x_1, x_2)}{[G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)]^{1/2}}, \tag{5}$$

where $G^{(1)}(x_1, x_2) = \text{Tr}\{\hat{\rho}\hat{E}^{(-)}(x_1) \cdot \hat{E}^{(+)}(x_2)\}$. Again, as the classical one, we have the degree of coherence, if
- The classical second-order coherence function is defined as

$$\gamma^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2} = \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle^2}. \tag{6}$$

- We define the normalized second-order quantum correlation function,

$$g^{(2)}(x_1, x_2) = \frac{G^{(2)}(x_1, x_2)}{[G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)]}, \tag{7}$$

where $g^{(2)}(x_1, x_2)$, is the joint probability of detecting one photon at (r_1, t_1) and (r_2, t_2). At a fixed position, $g^{(2)}$ depends only on the time difference,

$$g^{(2)}(\tau) = \frac{\langle \hat{E}^{(-)}(t)\hat{E}^{(-)}(t+\tau)\hat{E}^{(+)}(t+\tau)\hat{E}^{(+)}(t)\rangle}{\langle \hat{E}^{(-)}(t)\hat{E}^{(-)}(t)\rangle\langle \hat{E}^{(-)}(t+\tau)\hat{E}^{(-)}(t+\tau)\rangle}. \tag{8}$$

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