Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 2 (3/12, 3/16, ~~3/19~~)	Quantum Mechanics	☐ Schrödinger picture ☐ Heisenberg picture ☐ Interaction picture	☐ Uncertainty Relation ☐ Probability Interpretation ☐ Measurement problem ☐ Non-locality ☐ Macrorealism ☐

- **Take-home Messages:**
  1. "Nothing that is done in one system can instantaneously affect the density matrix of another isolated system, although it can affect the state vector." from "Quantum mechanics without state vectors", Steven Weinberg, Phys. Rev. A 90, 042102 (2014).
  2. Pure and Mixed states
  3. Density Operator
  4. Poisson distribution
  5. Bose-Einstein distribution

- **From Scratch !!**
- Examples:
  1. $|\Psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$, where $\langle\phi_i|\phi_j\rangle = \delta_{ij}$, orthonormal.
  2. $\langle\Psi|\Psi\rangle = 1$, normalization condition: $|c_1|^2 + |c_2|^2 = 1$.
  3. Pure states: let $|\Psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$, then we have

$$\hat{\rho}_1 = |\phi_1\rangle\langle\phi_1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1}; \tag{1}$$

$$\hat{\rho}_2 = |\phi_2\rangle\langle\phi_2| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1}; \tag{2}$$

$$\hat{\rho}_\Psi = |\phi_1\rangle\langle\phi_1| = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^{-1}; \tag{3}$$

  4. Mixed states: let $\hat{\rho}_{mix} = \frac{1}{2}\hat{\rho}_1 + \frac{1}{2}\hat{\rho}_2$, then we have

$$\hat{\rho}_{mix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{d}\bar{\bar{I}}_d; \quad (d = 2) \tag{4}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1}; \tag{5}$$

  5. Purity:

$$\mathrm{tr}(\hat{\rho}_1^2) = 1; \tag{6}$$

$$\mathrm{tr}(\hat{\rho}_2^2) = 1; \tag{7}$$

$$\mathrm{tr}(\hat{\rho}_\Psi^2) = 1; \tag{8}$$

$$\mathrm{tr}(\hat{\rho}_{mix}^2) = 1/2; \tag{9}$$

$$\tag{10}$$

- Poisson distribution Coherent state $|\alpha\rangle$ has the Poisson distribution in the photon number,

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{11}$$

- i.i.d. limit:

$$P(n) = \frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}, \tag{12}$$

  which is a *binomial distribution*.
- Boltzmann's law:

$$P(n) \propto \exp[-E_n/k_B T],$$

  where T denotes the temperature, and k_B is the Boltzmann's constant.
- Thermal states:

$$\rho_{th} = \sum_n = \frac{1}{\bar{n}+1}(\frac{\bar{n}}{\bar{n}+1})^n\,|n\rangle\langle n|. \tag{13}$$

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