Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

Ray-Kuang Lee[1]

[1] *Room 911, Delta Hall, National Tsing Hua University, Hsinchu, Taiwan.*
*Tel: +886-3-5742439; E-mail: rklee@ee.nthu.edu.tw**

(Dated: Spring, 2021)

Syllabus:

Date	Topic	To Know	To Think
week 8 (5/11, 5/14)	Photon-Atom Interactions	☐ Einstein's AB coefficients ☐ Classical model ☐ Semi-Classical	☐ Rabi-frequency ☐ Wavefunction Revival ☐
weeks 9-10 (5/18, 5/21, 5/25, 5/28)	Full Quantum model	☐ Jaynes-Cummings ☐ Dicke model ☐ Cavity-QED	☐ Vacuum Rabi oscillation ☐ Collective interaction ☐ Circuit-QED ☐
week 11-12 (6/1, 6/4, 6/8, 6/11)	Open systems	☐ Weisskopf-Wigner approximation ☐ Born-Markovian approximation ☐ Master equation ☐ Lindblad equation	☐ dissipation-fluctuation theorem ☐ non-Markovian ☐

- **Assignment**

  Deadline: 4:00PM, Tuesday, May 18th

(1) Interaction Hamiltonian: for the dipolar interaction Hamiltonian,

$$\hat{H}_0 = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\nu(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \tag{1}$$

$$\hat{H}_1 = \hbar g(\hat{\sigma}_+\,\hat{a} + \hat{a}^\dagger\,\hat{\sigma}_-), \tag{2}$$

show that the corresponding interaction picture Hamiltonian is

$$\hat{H}_I(t) = \exp(i\hat{H}_0 t/\hbar)\hat{H}_1(t)\exp(-i\hat{H}_0 t/\hbar) \tag{3}$$

$$= \hbar g[\hat{\sigma}_+\hat{a}e^{i(\omega-\nu)t} + \hat{a}^\dagger\hat{\sigma}_- e^{-i(\omega-\nu)t}], \tag{4}$$

under the *rotating-wave approximation.*

(2) Vacuum Rabi oscillation: the Hamiltonian for a two-level atom interaction with a single quantized fields can be modeled by the Jaynes-Cummings Hamiltonian:

$$\hat{H} = \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\nu(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \hbar g(\hat{\sigma}_+\,\hat{a} + \hat{a}^\dagger\,\hat{\sigma}_-), \tag{5}$$

where $\Delta \equiv \omega - \nu$. Show that for the initially excited state, $c_{e,n}(0) = c_n(0)$ and $c_{g,n+1}(0) = 0$, the solutions for are

$$c_{g,n}(t) = c_n(0)[\cos(\frac{\Omega_n t}{2}) - \frac{i\Delta}{\Omega_n}\sin(\frac{\Omega_n t}{2})]e^{i\Delta t/2}, \tag{6}$$

$$c_{e,n+1}(t) = -c_n(0)\frac{2ig\sqrt{n+1}}{\Omega_n}\sin(\frac{\Omega_n t}{2})e^{i\Delta t/2}, \tag{7}$$

where the Rabi frequency is $\Omega_n^2 = \Delta^2 + 4g^2(n+1)$. Show that the population inversion of a two-level atom is,

$$W(t) = \sum_n |c_{e,n}(t)|^2 - |c_{g,n}(t)|^2 = \sum_0^\infty |c_n(0)|^2[\frac{\Delta^2}{\Omega_n^2} + \frac{4g^2(n+1)}{\Omega_n^2}\cos(\Omega_n t)]. \tag{8}$$

(3) Dressed states: consider the Jaynes-Cummings Hamiltonian,

$$\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar g(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-), \tag{9}$$

with the product states $|e\rangle|n\rangle$, $|g\rangle|n+1\rangle$, etc., referred to as the *bare states*, the interaction term in $\hat{H}$ causes only transitions of the type,

$$\begin{array}{lcl} |e\rangle|n\rangle & \leftrightarrow & |g\rangle|n+1\rangle, \\ |e\rangle|n-1\rangle & \leftrightarrow & |g\rangle|n\rangle, \end{array} \tag{10}$$

1. In therms of $|e\rangle|n\rangle$ and $|g\rangle|n+1\rangle$, we can define

$$|\psi_{1n}\rangle = |e\rangle|n\rangle \tag{11}$$
$$|\psi_{2n}\rangle = |g\rangle|n+1\rangle \tag{12}$$

write down the matrix representation of $\hat{H}^{(n)}$ for these bases, i.e., $H_{ij}^{(n)} = \langle\psi_{in}|\hat{H}^{(n)}|\psi_{jn}\rangle$.

2. Find the eigen-energy of $\hat{H}^{(n)}$.
3. Construct the *dressed state*, the associated eigen-state of $\hat{H}^{(n)}$, i.e.

$$|n,+\rangle = c_1|\psi_{1n}\rangle + c_2|\psi_{2n}\rangle, \tag{13}$$
$$|n,-\rangle = c_3|\psi_{1n}\rangle + c_4|\psi_{2n}\rangle, \tag{14}$$

find the coefficients c_i, $i = 1, 2, 3, 4$.

- **Take-home Messages:**
  1. Lorentz model
  2. Rabi oscillation
  3. Jaynes-Cummings Hamiltonian
  4. Interaction Picture
  5. Vacuum Rabi oscillation
  6. Spontaneous emission due to the vacuum fields

*Electronic address: rklee@ee.nthu.edu.tw