Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
weeks 9-10 (5/18, 5/21, 5/25, 5/28)	Full Quantum model	☐ Jaynes-Cummings ☐ Dicke model ☐ Cavity-QED	☐ Vacuum Rabi oscillation ☐ Collective interaction ☐ Circuit-QED ☐
week 11-12 (6/1, 6/4, 6/8, 6/11)	Open systems	☐ Weisskopf-Wigner approximation ☐ Born-Markovian approximation ☐ Master equation ☐ Lindblad equation	☐ dissipation-fluctuation theorem ☐ non-Markovian ☐
week 13-14 (6/15, 6/18)	Selected Applications of QO	☐ Quantum Sensor ☐ Test of Quantum Mechanics ☐ Quantum Communication ☐ Quantum Computing ☐	☐ Gravitational Wave Detectors ☐ Quantum Zeno effect ☐ Quantum Key Distribution ☐ Quantum Photonic Circuit ☐

- **Take-home Messages:**
  1. Master equation
  2. Born-Markovian approximation
  3. Lindblad form:
     - (a): For any *effective Hamiltonian*, which may not be Hermitian, one can write down it as

$$\hat{H}_{eff} = \hat{H} - i\,\hat{V}, \tag{1}$$

       where $\hat{H}$ and $\hat{V}$ are Hermitian.
     - (b) However the normalization of $|\psi\rangle$ becomes

$$\frac{d}{dt}\langle\psi|\psi\rangle = -2\,\langle\psi|\hat{V}|\psi\rangle. \tag{2}$$

     - (c) To keep the normalization of states, we can introduce

$$\hat{V} = \sum_a \gamma_a\,\hat{T}_a^\dagger\hat{T}_a. \tag{3}$$

     - (d) The resulting *von Neumann equation* for the density matrix has the form,

$$\frac{d}{dt}\hat{\rho} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}] - \sum_a \gamma_a(\hat{T}_a^\dagger\hat{T}_a\hat{\rho} - 2\hat{T}_a\hat{\rho}\hat{T}_a^\dagger + \hat{\rho}\hat{T}_a^\dagger\hat{T}_a), \tag{4}$$

       which is called the *quantum master equation.*

- **Assignment**

  Deadline: 4:00PM, Tuesday, June 22nd

- (1) Consider the damping of an optical cavity mode by a two-level atomic beam reservoir.
  - (a)Derive the equation of motion for the reduced density operator $\mathrm{Tr}_R[\hat{\rho}(t)] \equiv \hat{\rho}_f(t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_f(t) = (\frac{1}{i\hbar})^2 \int_0^t \mathrm{d}t' \mathrm{Tr}_R([\hat{H}_I(t), [\hat{H}_I(t'), \hat{\rho}_f(t) \otimes \hat{\rho}_R(0)]]). \tag{5}$$

  - (b) Then, assume that r atoms are injected into the cavity per second, and they spend an average time of τ seconds inside the cavity, i.e.

$$\int_0^\tau \mathrm{d}t' r t' = \frac{1}{2} r \tau^2, \tag{6}$$

Find the diagonal elements of the reduced density matrix $\mathrm{Tr}_R[\hat{\rho}(t)]$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{n,n}(t) = -\frac{\nu}{Q_0}\{[n_{th}(n+1) - (n_{th}+1)n]\rho_{n,n} - n_{th}\rho_{n-1,n-1} - (n_{th}+1)(n+1)\rho_{n+1,n+1}\}, \tag{7}$$

$$= [-R_e(n+1) - R_g n]\rho_{nn} + R_e n \rho_{n-1,n-1} + R_g(n+1)\rho_{n+1,n+1}, \tag{8}$$

where then,

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_f(t) = -\frac{1}{2}R_e[\hat{a}\hat{a}^\dagger\hat{\rho}_f - \hat{a}^\dagger\hat{\rho}_f\hat{a}] - \frac{1}{2}R_g[\hat{a}^\dagger\hat{a}\hat{\rho}_f - \hat{a}\hat{\rho}_f(t)\hat{a}^\dagger] + \mathrm{H.C}, \tag{9}$$

where

$$R_e = r\rho_{aa}g^2\tau^2, \quad \text{and} \quad R_g = r\rho_{bb}g^2\tau^2. \tag{10}$$

and

$$\frac{\nu}{Q_0} \equiv R_g - R_e, \quad \text{and} \quad R_e(1+n_{th}) = R_g n_{th} \to n_{th} = \frac{R_e}{R_g - R_e} = \frac{1}{\exp(\hbar\omega/k_b T) - 1}. \tag{11}$$

  - (c) Show the *detailed balance* condition, with the solution

$$\rho_{n,n} = [1 - \exp(-\hbar\omega/k_B T)]\exp(-n\hbar\omega/k_B T), \tag{12}$$

with $n_{th} = \frac{1}{\exp(\hbar\omega/k_b T)-1}$.

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