Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 1 (3/2, 3/9)	Quantum SHO	☐ Fock states, $\|n\rangle$ ☐ creation operator, $\hat{a}^\dagger$	☐ single-photon detection ☐ Wave-Particle Duality ☐ photon-number resolving ☐
(3/12, 3/16, ~~3/19~~)		☐ Vacuum state ☐ Quantum Fluctuations	☐ Shot Noise Limit ☐ Casimir Force ☐

- **Assignment**

  Deadline: 4:00PM, Friday, March 19th

1. Simulate *Electromagnetically Induced Transparency* (EIT) by coupled Simple Harmonic Oscillators: referring to the paper by C. L. Garrido Alzar et al., *"Classical analog of electromagnetically induced transparency,"* Am. J. Phys. **70**, 37 (2002).

   1.(a) Plot the frequency dependence of the absorption of the probe energy by particle 1, i.e., detuning v.s. probe absorption power.

   1.(b) Show the *transparency window.*

   1.(c) Instead of the symmetric profile int probe absorption power spectrum, show the condition to have *asymmetric spectrum*, i.e., Fano spectrum.

2. Quantum Simple Harmonic Oscillation: $\hat{H} = \hbar\omega(\hat{a}^\dagger\,\hat{a} + \frac{1}{2})$,

   2.(a) For the ground state eigenfunction, *i.e.*, $\hat{a}|n=0\rangle = 0$, find the corresponding wave function in the x- and p-spaces, respectively. That is to find $\Phi(x)$ and $\tilde{\Psi}(p)$, respectively,

$$\Phi(x) \equiv \langle x|0\rangle, \tag{1}$$

$$\tilde{\Psi}(p) \equiv \langle p|0\rangle. \tag{2}$$

   2.(b) Show by direct integration that

$$\langle n|\hat{p}|n\rangle = 0, \tag{3}$$

$$\langle n|\Delta\hat{p}^2|n\rangle = \hbar m\omega - m^2\omega^2\langle n|\hat{x}^2|n\rangle. \tag{4}$$

   2.(c) Use the operator method to calculate the expectation values $\langle\hat{p}^2\rangle$ and $\langle\hat{x}^2\rangle$ in the energy eigenstate $|n\rangle$. Hence prove the quantum analog of the *virital theorem*:

$$\langle V\rangle = \langle T\rangle = \langle H\rangle/2, \tag{5}$$

   the equal partition of the total energy H, divided into the kinetic and potential energies, T and V, respectively.

- **Take-home Messages:**
  1. Class Materials: http://mx.nthu.edu.tw/∼rklee
  2. Discussion Channel: Quantum Optics, Lecture@NTHU, Slack, quantumoptics-zgq1695.slack.com
  3. *"According to quantum mechanics, the vacuum is not empty, but teeming with virtual particles that constantly wink in and out of existence. "*
  4. Vacuum state $|0\rangle$
  5. eigen-energy of SHO: $E = \hbar\omega(n + \frac{1}{2}), \qquad n = 0, 1, 2, 3, \ldots.$
  6. Creation operator $\hat{a}^\dagger$
  7. Annihilation $\hat{a}$
  8. *The Force of Empty Space*, https://physics.aps.org/story/v2/st28

- **From Scratch !!**
- Quantum SHO:

$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \tag{6}$$

where the $\hat{x}$ and $\hat{p}$ are non-commute operators, *i.e.*,

$$[\hat{x}, \hat{p}] = i\hbar. \tag{7}$$

- The *Hermite-Gaussian* solutions associated with Hermite polynomials H_n

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n + 1, \qquad n = 0, 1, 2, 3 \ldots \tag{8}$$

- For the corresponding eigen-energy:

$$E = \frac{\hbar\omega}{2}\,\epsilon = \hbar\omega(n + \frac{1}{2}), \qquad n = 0, 1, 2, 3, \ldots. \tag{9}$$

- Quantum SHO:

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \tag{10}$$

- Free particle expansion: $\hat{H} = \frac{\hat{p}^2}{2m}$, with the unitary operator

$$\hat{U} = \exp(-\frac{i}{\hbar}\frac{\hat{p}^2}{2m}t). \tag{11}$$

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