Quantum Optics, IPT5340

Time: T7T8F7F8 (15:30-17:20, Tuesday, and 16:00-17:20, Friday), at Room 208, Delta Hall

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Syllabus:

Date	Topic	To Know	To Think
week 3 (3/30, ~~4/2~~, ~~4/6~~)	Coherent states, $\|\alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐
week 5 (4/16, 4/20, 4/23)	Squeezed states	☐ $\|\xi\rangle$ ☐ OPO	☐ Continuous Variables ☐
week 6 (4/27, 4/29)	Two-mode Squeezed states	☐ EPR pair ☐ Cat states ☐ non-Gaussian states	☐ Quantum Discord ☐ Entanglement ☐ Steering ☐ Bell's inequality

- **Assignment**

  Deadline: 4:00PM, Friday, April 20th

1. Show that the unitary transformation of the squeeze operator,

$$\hat{S}^\dagger(\xi)\hat{a}\hat{S}(\xi) = \hat{a}\cosh r - \hat{a}^\dagger e^{i\theta}\sinh r, \tag{1}$$
$$\hat{S}^\dagger(\xi)\hat{a}^\dagger\hat{S}(\xi) = \hat{a}^\dagger\cosh r - \hat{a}e^{-i\theta}\sinh r, \tag{2}$$

with the formula $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]],\ldots$

2. Show that the variances for squeezed vacuum are

$$\Delta\hat{a}_1^2 = \frac{1}{4}[\cosh^2 r + \sinh^2 r - 2\sinh r\cosh r\cos\theta], \tag{3}$$
$$\Delta\hat{a}_2^2 = \frac{1}{4}[\cosh^2 r + \sinh^2 r + 2\sinh r\cosh r\cos\theta], \tag{4}$$

3. Show that the expectation values for squeezed coherent states are,

$$\langle\alpha,\xi|\hat{a}|\alpha,\xi\rangle = \alpha, \tag{5}$$
$$\langle\hat{a}^2\rangle = \alpha^2 - e^{i\theta}\sinh r\cosh r, \tag{6}$$
$$\langle\hat{a}^\dagger\hat{a}\rangle = |\alpha|^2 + \sinh^2 r, \tag{7}$$

with helps of

$$\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha,$$
$$\hat{D}^\dagger(\alpha)\hat{a}^\dagger\hat{D}(\alpha) = \hat{a}^\dagger + \alpha^*.$$

4. Consider squeezed vacuum state in the basis of number states,

$$|\xi\rangle = \sum_{n=0}^{\infty} C_n|n\rangle,$$

Shown that

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{m=0}^{\infty}(-1)^m\frac{\sqrt{(2m)!}}{2^m m!}e^{im\theta}\tanh^m r|2m\rangle, \tag{8}$$

where the coefficient C_0 can be determined from the normalization, *i.e.*, $C_0 = \sqrt{\cosh r}$.

- **Take-home Messages:**
  1. Squeezed Operator
  2. Squeezed Vacuum
  3. Squeezed Coherent states v.s. Coherent Squeezed states
  4. Squeezed states as the Minimum Uncertainty states
  5. Generation of Squeezed states by Optical Parametric Oscillators (OPO)
  6. Homodyne detections

- **From Scratch !!**
- One of the squeezed states can be defined as,

$$|\Psi_s\rangle = \hat{S}(\xi)|\Psi\rangle, \tag{9}$$

with the unitary Squeeze operator $\hat{S}(\xi) = \exp[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}]$.
- If $|\Psi\rangle$ is the vacuum state $|0\rangle$, then $|\Psi_s\rangle$ state is the *squeezed vacuum*,

$$|\xi\rangle = \hat{S}(\xi)|0\rangle. \tag{10}$$

- The variances for squeezed vacuum are

$$\Delta\hat{a}_1^2 = \frac{1}{4}[\cosh^2 r + \sinh^2 r - 2\sinh r\cosh r\cos\theta], \tag{11}$$

$$\Delta\hat{a}_2^2 = \frac{1}{4}[\cosh^2 r + \sinh^2 r + 2\sinh r\cosh r\cos\theta], \tag{12}$$

- For $\theta = 0$, we have

$$\Delta\hat{a}_1^2 = \frac{1}{4}e^{-2r}, \quad \text{and} \quad \Delta\hat{a}_2^2 = \frac{1}{4}e^{+2r}, \tag{13}$$

and squeezing exists in the $\hat{a}_1$ quadrature.
- Squeezed states as the Minimum Uncertainty states:

$$(\mu\hat{a} + \nu\hat{a}^\dagger)|\xi\rangle = 0, \tag{14}$$

the squeezed vacuum state is an eigenstate of the operator $\mu\hat{a} + \nu\hat{a}^\dagger$ with eigenvalue zero. Similarly,

$$\hat{D}(\alpha)\hat{S}(\xi)\hat{a}\hat{S}^\dagger(\xi)\hat{D}^\dagger(\alpha)\hat{D}(\alpha)|\xi\rangle = 0, \tag{15}$$

with the relation $\hat{D}(\alpha)\hat{a}\hat{D}^\dagger(\alpha) = \hat{a} - \alpha$, we have

$$(\mu\hat{a} + \nu\hat{a}^\dagger)|\alpha,\xi\rangle = (\alpha\cosh r + \alpha^*\sinh r)|\alpha,\xi\rangle \equiv \gamma|\alpha,\xi\rangle. \tag{16}$$

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