Date	Topic	To Know	To Think
Feb. 23rd (Tue.)	Introduction	Scope	☐ Your and My Expectations. ☐ What is the nature of light? ☐ Anything else ?
Feb. 26th (Fri.)	Simple Harmonic Oscillator (SHO)	☐ classical trajectory ☐ analogue to EM waves	☐ Bohmian mechanics ☐ Inverted SHO ☐
week 1 (3/2, 3/9)	Quantum SHO	☐ Fock states, $\|n\rangle$ ☐ creation operator, $\hat{a}^\dagger$	☐ single-photon detection ☐ Wave-Particle Duality ☐ photon-number resolving ☐
(3/12, 3/16, ~~3/19~~)		☐ Vacuum state ☐ Quantum Fluctuations	☐ Shot Noise Limit ☐ Casimir Force ☐
week 2 (3/23, 3/26)	Quantum Mechanics	☐ Schrödinger picture ☐ Heisenberg picture ☐ Interaction picture	☐ Uncertainty Relation ☐ Probability Interpretation ☐ Measurement problem ☐ Non-locality ☐ Macrorealism ☐
week 3 (3/30, ~~4/2~~, 4/6)	Coherent states, $\|\alpha\rangle$	☐ photon statistics ☐ bunching ☐ Correlation function	☐ Minimum Uncertainty States ☐ Classical-Quantum boundary ☐
week 4 (4/9, 4/13)	Quantum Phase Space	☐ Wigner function	☐ Quasi-probability ☐ Quantum State Tomography ☐
week 5 (4/20, 4/23)	Squeezed states	☐ $\|\xi\rangle$ ☐ OPO	☐ Continuous Variables ☐
week 6 (4/27, 4/29)	Two-mode Squeezed states	☐ EPR pair ☐ Cat states ☐ non-Gaussian states	☐ Quantum Discord ☐ Entanglement ☐ Steering ☐ Bell's inequality ☐
week 7 (5/4, 5/7)	Optical devices	☐ Beam splitter ☐ Mach-Zehnder interferometer	☐ linear optics ☐
week 8 (5/11, 5/14)	Interferometry	☐ Young's Interferometry, $g^{(1)}$ ☐ HBT-Interferometry, $g^{(2)}$	☐ Quantum Phase Estimation ☐ Quantum Fisher Information ☐

From Scratch !!

- **How much do you known about Quantum SHO ?**

Note: Quantum SHO

- □ Quantum Simple Harmonic Oscillator, qSHO
- □ Hamiltonian
- □ Number operator
- □ Energy Quantization (equally spacing in energy)
- □ Vacuum state with zero-point energy
- □ Schrodinger picture
- □ Heisenberg picture
- More on qSHO
- □ Quantum SHO in phase space
- □ Parity-Time symmetric SHO

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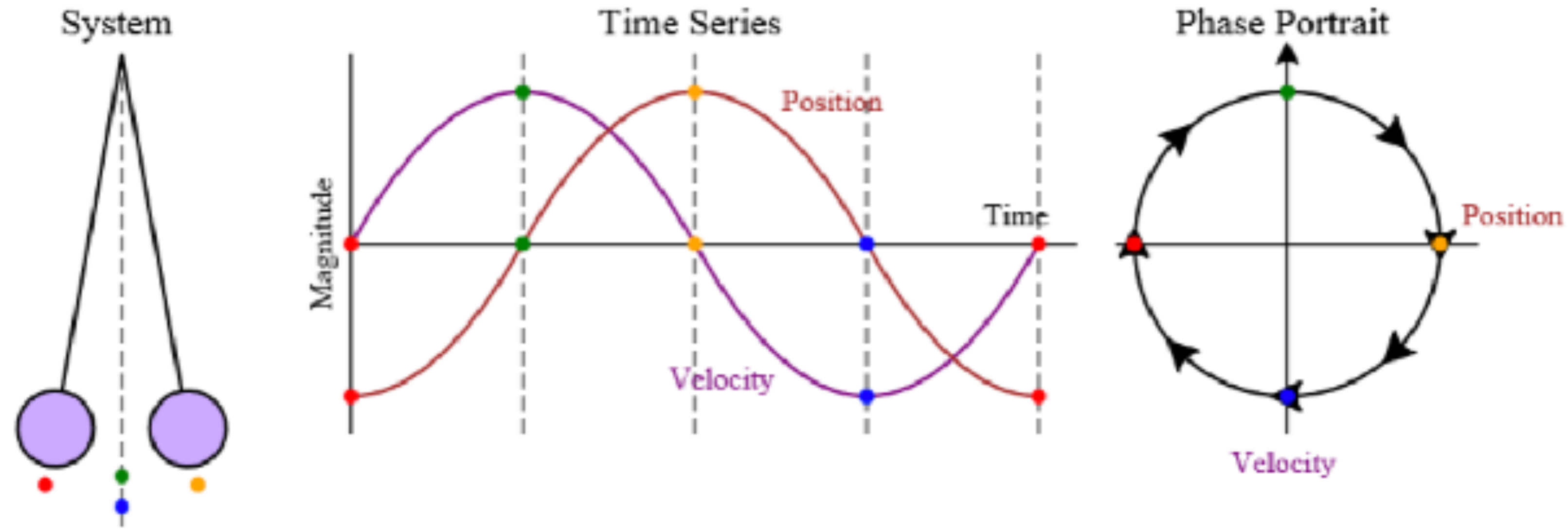
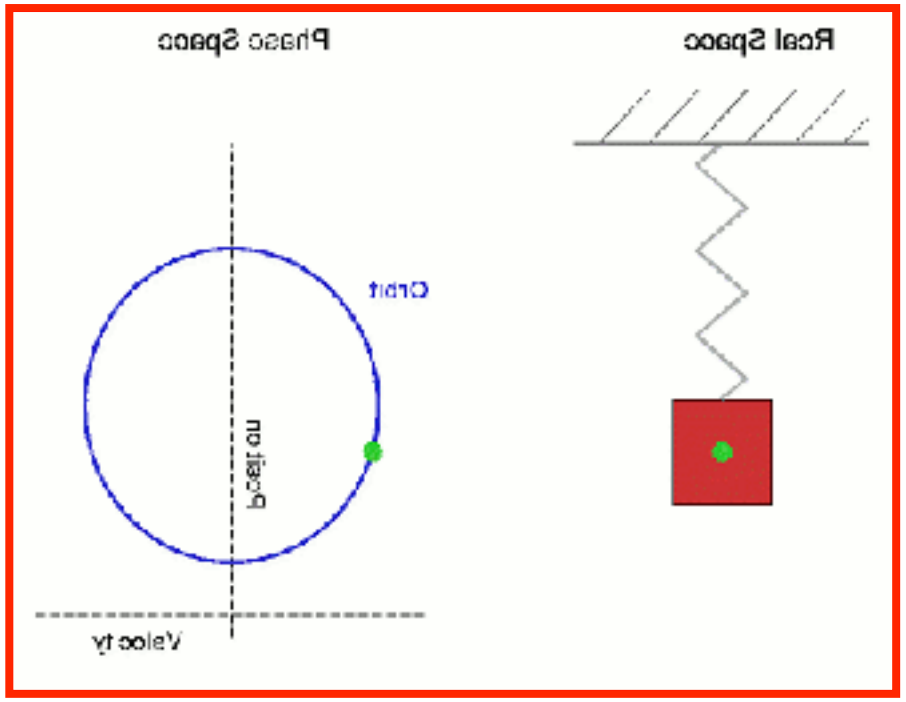
*http://mx.nthu.edu.tw/~rklee

Simple Harmonic Oscillator, SHO

- Classical Mechanics





$$\vec{F} = m\vec{a} = m\frac{d^2\,\vec{x}}{dt^2} = -k\,\vec{x}.$$

$$\vec{x}(t) = A\,\cos(\omega_0 t + \phi_0), \qquad \omega_0^2 = \frac{k}{m},$$

$$H = \text{K.E.} + \text{P.E.} = \frac{1}{2}\frac{p^2}{m} + \frac{1}{2}k\,x^2.$$

- Newton's law
- Hooke's law
- Linear force (parabolic potential)
- Hamiltonian (energy) is constant (conserved).
- Equally distributions in KE and PE
- Sinusoidal motion
- Periodic orbit (trajectory) in phase space
- Response bandwidth

Simple Harmonic Oscillator, SHO

SHO	EM waves	Quantum
particle	(transverse) mode	wave-particle
Newton's law	Maxwell's eqs	Schrodinger/Heisenberg eq. Quantum Liouville eq. Dirac eq.
sinusoidal sol.	plane wave sol.	harmonic waves
KE+PE	Poynting energy	Hamiltonian energy
Kinetic Energy	Diffraction/Dispersion	free-particle expansion
Potential Energy	refractive index change (GRIN lens)	Potential Energy
Trajectory	Phasor	Probability distribution (Husimi function)
x, p canonical coordinates	quadrature	$[\hat{X}, \hat{P}]$

What is Quantum !?

Quantum Mechanics

Non-Classicality

Bell's inequality
Non-locality

Energy Quanta

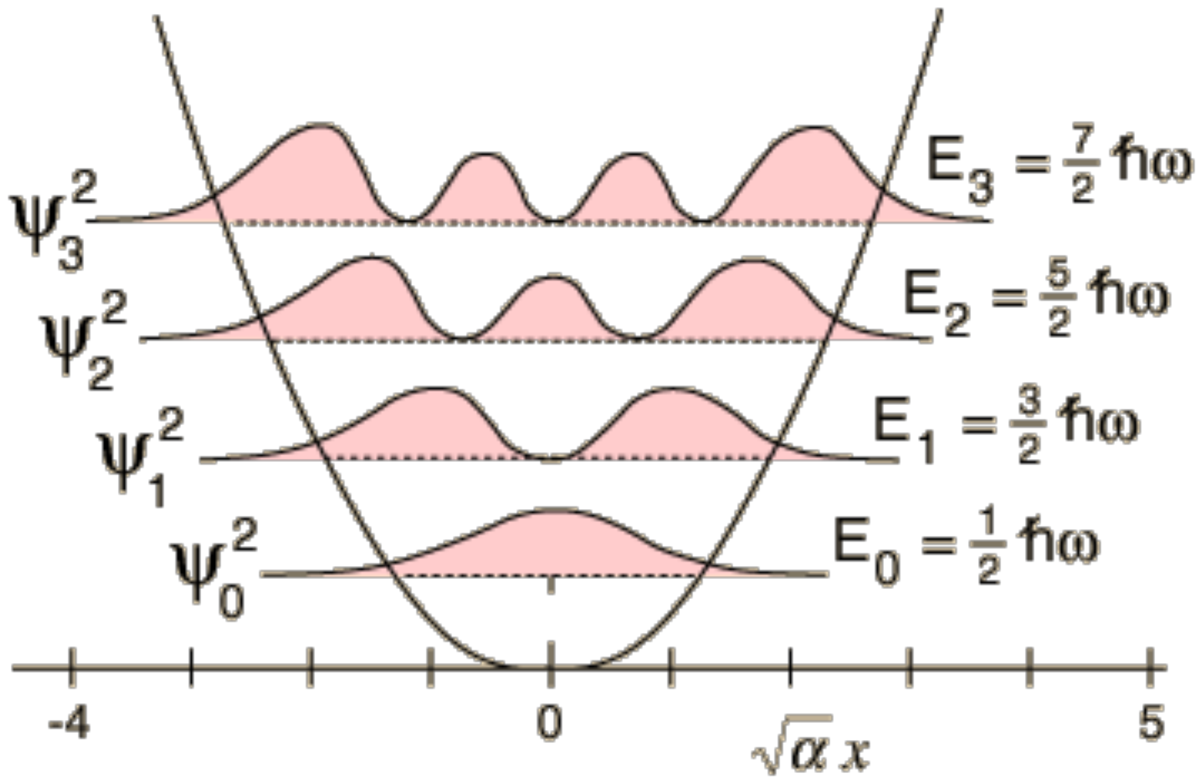
Schrödinger's Cat
Realism

Steering

Wavefunction Collapse
(decoherence)

Entanglement
(EPR-pair)

Quantum Simple Harmonic Oscillator (SHO)



$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \quad [\hat{x},\hat{p}] = i\hbar.$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \quad [\hat{a},\hat{a}^\dagger] = 1,$$

$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle, \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle, \\ E_n &= \hbar\omega(n+\frac{1}{2}). \end{aligned}$$

- Energy quantization
- Equally spacing in energy difference
- Zero-point energy $\neq 0$

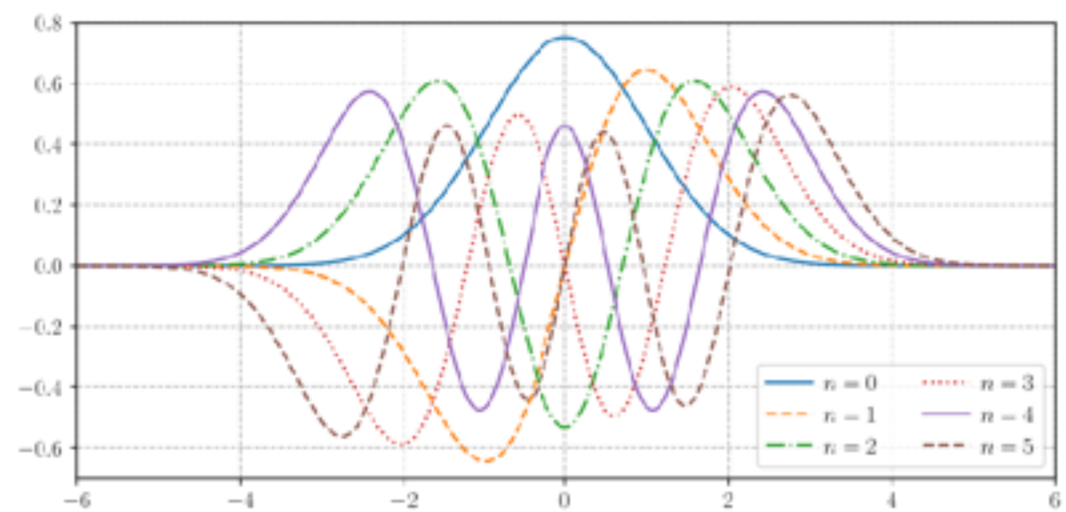
$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n+1, \qquad n = 0,1,2,3\ldots$$

$$E = \frac{\hbar\omega}{2}\epsilon = \hbar\omega(n+\frac{1}{2}), \qquad n = 0,1,2,3,\ldots.$$

Quantum Mechanics: Schrodinger's picture

The equation of motion of a quantum state is described by the Schrödinger's equation:

$$i\hbar\frac{\partial}{\partial t}|\phi\rangle = \hat{H}|\phi\rangle.$$



For the eigen-state of SHO Hamiltonian,

$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2,$$

we have

$$[\frac{d^2}{dx^2} + \frac{2m}{\hbar^2}(E - \frac{1}{2}kx^2)]\psi(x) = 0,$$

$$[\frac{d^2}{d\xi^2} + (\epsilon - \xi^2)]\psi(\xi) = 0, \qquad \epsilon = \frac{2E}{\hbar\omega},$$

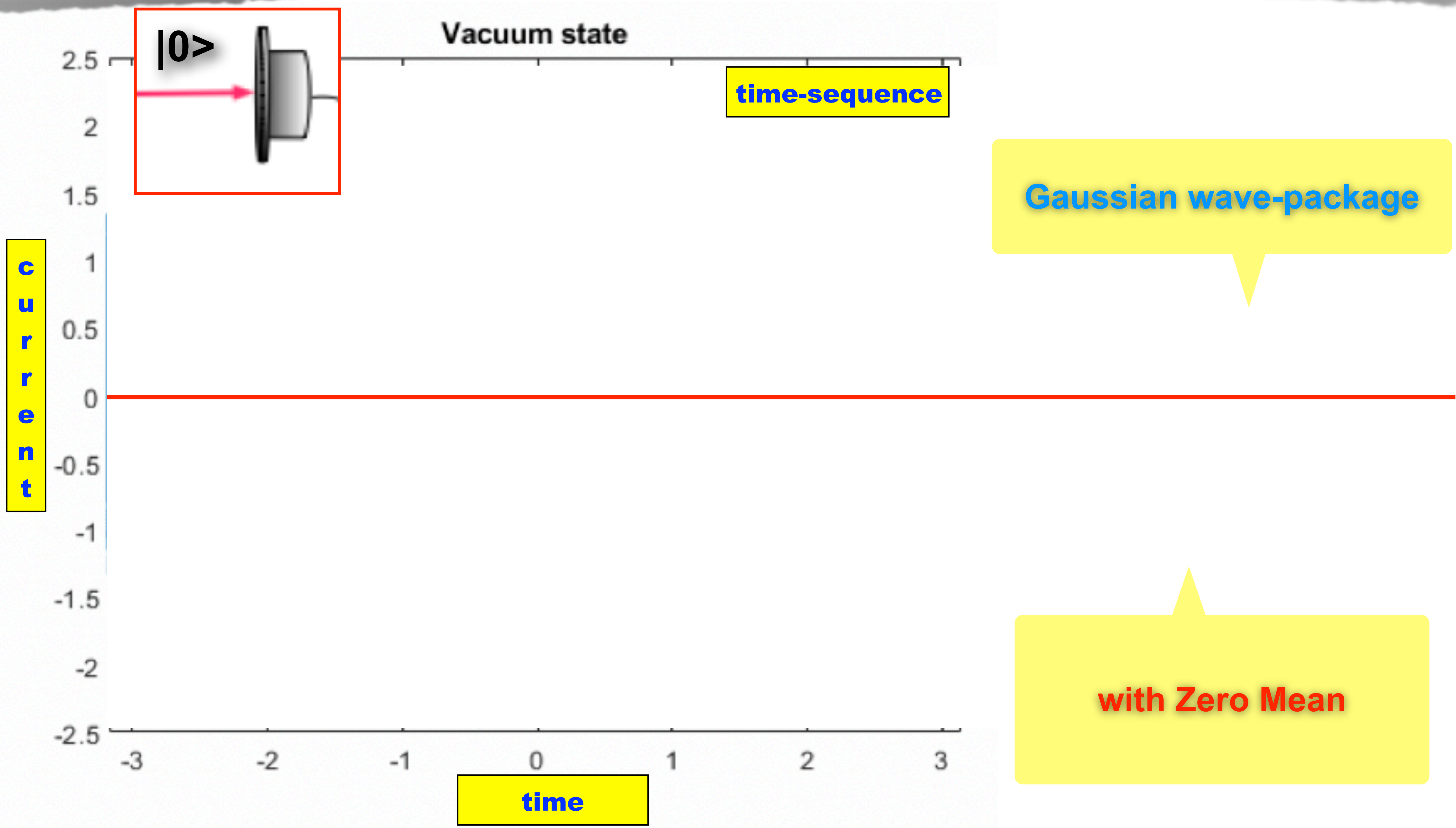
which gives the *Hermite-Gaussian* solutions associated with Hermite polynomials H_n

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n+1, \qquad n = 0, 1, 2, 3\ldots$$

For the corresponding eigen-energy:

$$E = \frac{\hbar\omega}{2}\,\epsilon = \hbar\omega(n + \frac{1}{2}), \qquad n = 0, 1, 2, 3, \ldots.$$

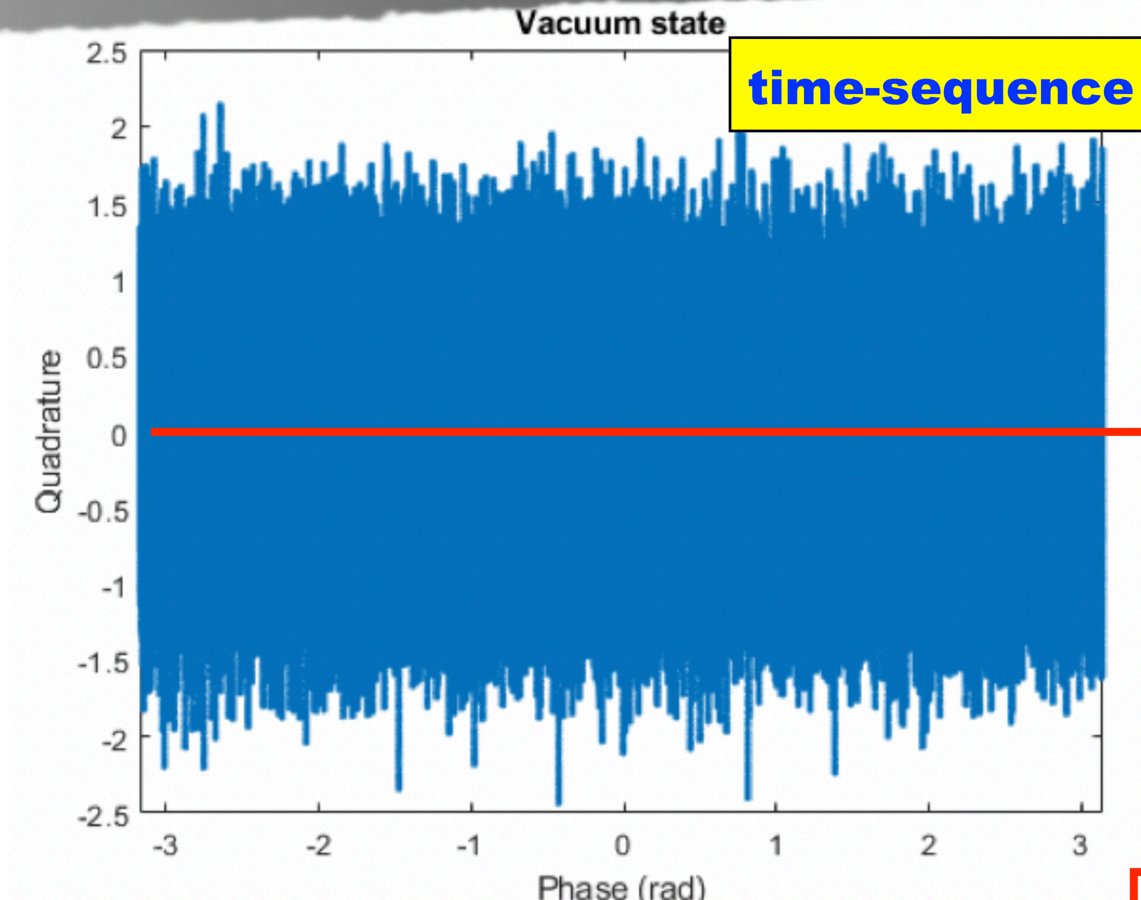
Vacuum State: l0>

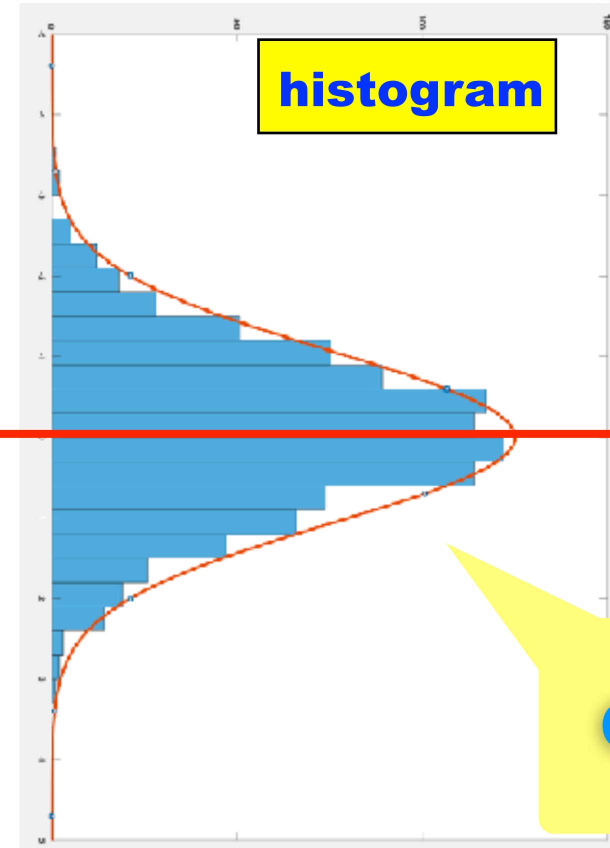


Gaussian wave-package

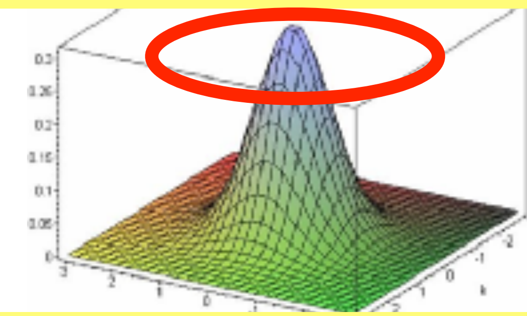
with Zero Mean

Vacuum State: |0>



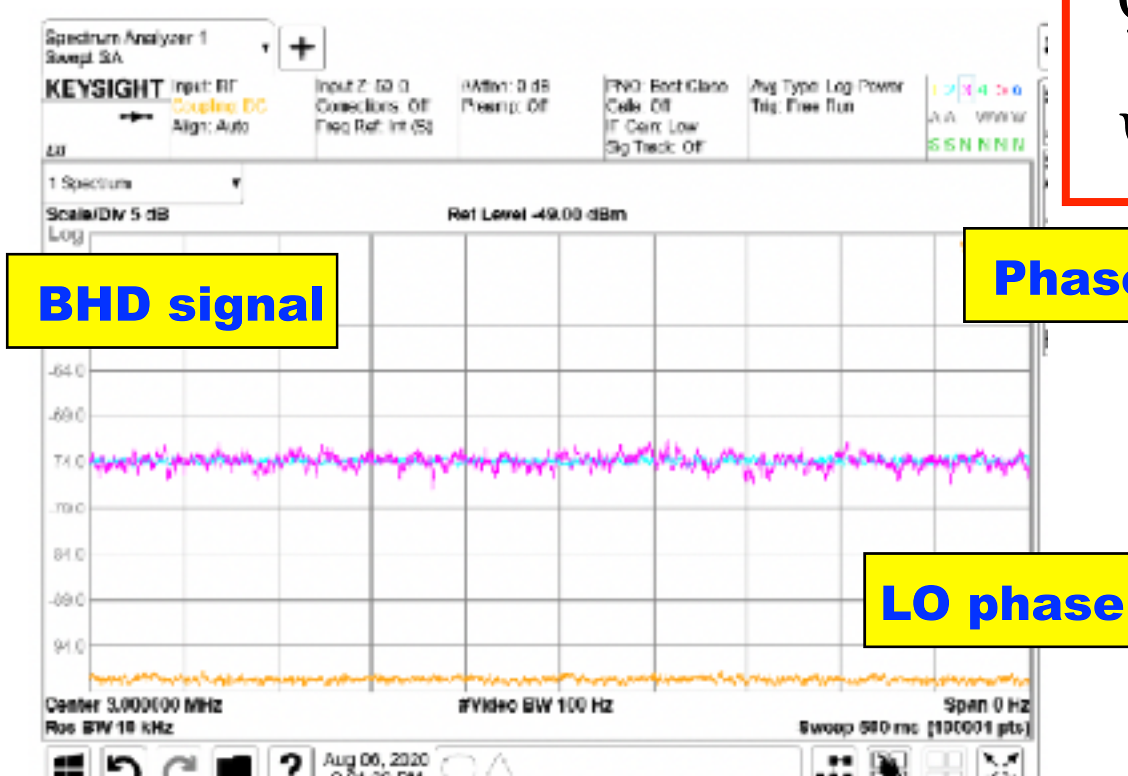


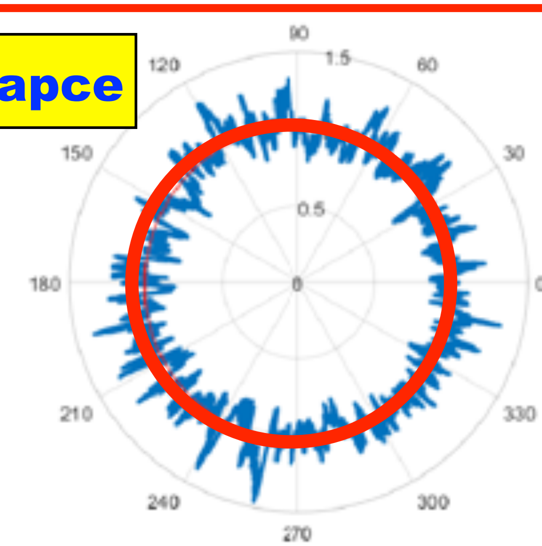
with Zero Mean

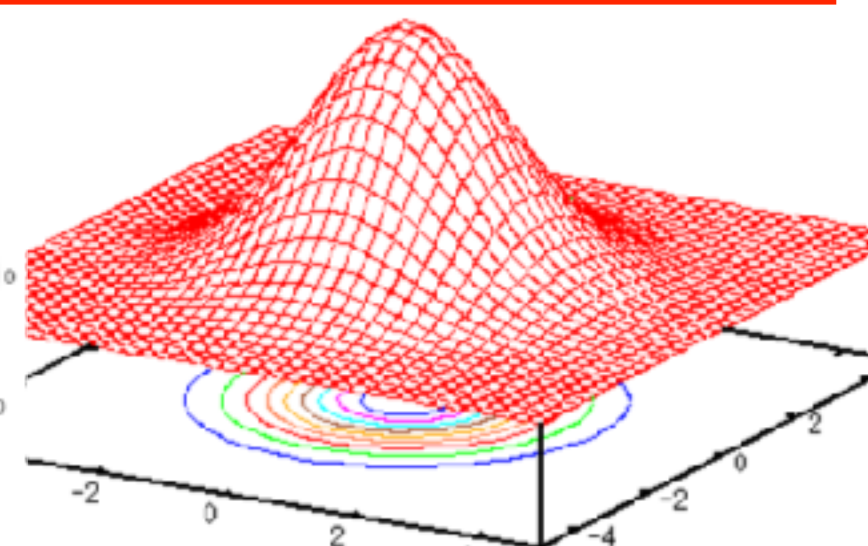


Gaussian wave-package

$$\Psi(x) \quad = \quad < x|0 >= C\, exp[-x^2/\Delta x^2]$$
$$\tilde{\Psi}(p) \quad = \quad < p|0 >= C\, exp[-\Delta x^2\, p^2]$$







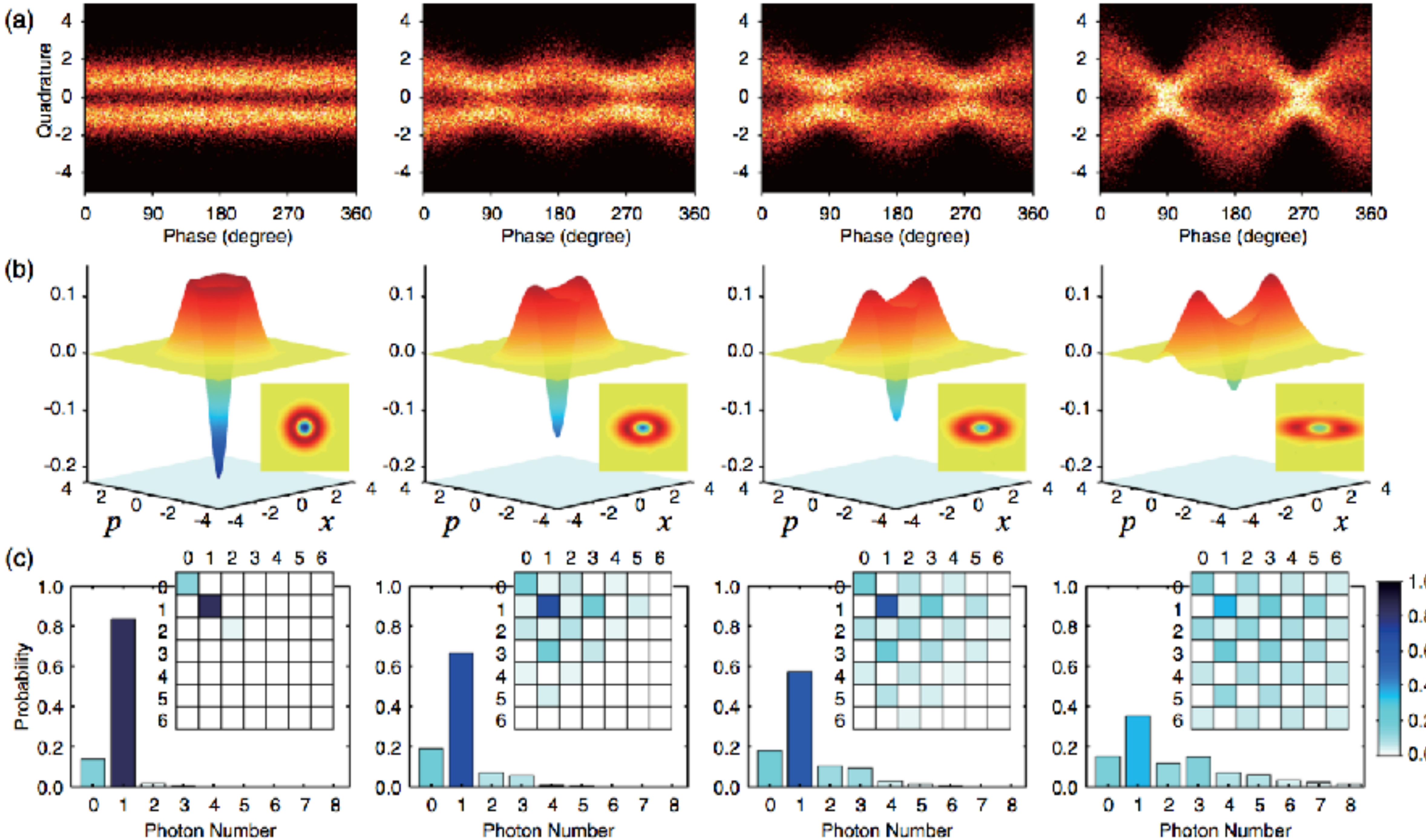


FIG. 2 (color online). Experimental quantum states for the conversion from particle to wave. The leftmost column shows the input single-photon state, while the other three columns show the output states for a squeezing parameter γ of 0.26, 0.37, and 0.67, from left to right. (a) Quadrature distributions over a period. (b) Wigner functions. (c) Photon number distributions and photon number representation of density matrices. The minimum value of -0.22 for the input Wigner function becomes, respectively, -0.15, -0.12, and -0.06, after the conversion.

Y. Miwa et al., PRL 113, 013601 (2014).

Quantum Mechanics: Heisenberg's picture

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}).$$

$$\boxed{\begin{aligned} \hat{a} &= \frac{1}{\sqrt{2m\hbar\omega}}[m\omega\hat{x} + i\hat{p}], \\ \hat{a}^\dagger &= \frac{1}{\sqrt{2m\hbar\omega}}[m\omega\hat{x} - i\hat{p}]. \end{aligned}}$$

$$[\hat{a}, \hat{a}^\dagger] = 1,$$

Heisenberg's equation: The dynamics of the operator is governed by the Heisenberg's equation:

$$\frac{d}{dt}\hat{O} = \frac{1}{i\hbar}[\hat{O}, \hat{H}].$$

For the annihilation operator of SHO, $\hat{a}$, we have

$$\frac{d}{dt}\hat{a} = \frac{1}{i\hbar}[\hat{a}, \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})] = -i\omega\hat{a},$$

with the solution

$$\hat{a}(t) = \hat{a}(t=0)\exp[-i\omega t].$$

Quantum Mechanics: Heisenberg's picture

1. $\hat{a}$ and $\hat{a}^\dagger$ are *NOT* Hermitian operators, i.e., $\hat{a} \neq \hat{a}^\dagger$, for which no real eigenvalue are generated.

2. Number operator: An Hermitian operator can be defined as $\hat{N} \equiv \hat{a}^\dagger \hat{a}$.

3. The commutation relations with the SHO Hamiltonian are:

$$\begin{aligned} [\hat{H}, \hat{a}] &= -\hbar\omega\hat{a}, \\ [\hat{H}, \hat{a}^\dagger] &= \hbar\omega\hat{a}^\dagger, \end{aligned}$$

4. Raising (step-up) and Lowering (step-down) operators: Consider the eigen-state of SHO Hamiltonian, $\hat{H}|\phi\rangle = E|\phi\rangle$, then

$$\boxed{\begin{aligned} \hat{H}\,\hat{a}|\phi\rangle &= (E - \hbar\omega)\hat{a}|\phi\rangle, \\ \hat{H}\,\hat{a}^\dagger|\phi\rangle &= (E + \hbar\omega)\hat{a}^\dagger|\phi\rangle, \end{aligned}}$$

where $\hat{a}|\phi\rangle$ and $\hat{a}^\dagger|\phi\rangle$ are also eigen-states of SHO, but with the eigen-values $E - \hbar\omega$ and $E + \hbar\omega$, respectively. $|\phi\rangle$ is a *discrete* states, denoted as $|n\rangle$ in the following.

Quantum Mechanics: Heisenberg's picture

5. Ground state:

$$\langle\phi|\hat{H}|\phi\rangle \quad = \quad \hbar\omega\langle\phi|\hat{a}^\dagger\hat{a} + \frac{1}{2}|\phi\rangle, \geq \frac{\hbar\omega}{2}.$$

Based on above, we denote the ground state as $|n = 0\rangle = |0\rangle$, with the energy $E_0 = \frac{1}{2}\hbar\omega$, which is the eigen-state of

$$\boxed{\hat{a}|0\rangle = 0, \qquad \text{the lowest energy state.}}$$

6. Exited state:

$$|n\rangle = (\hat{a}^\dagger)^n|0\rangle,$$

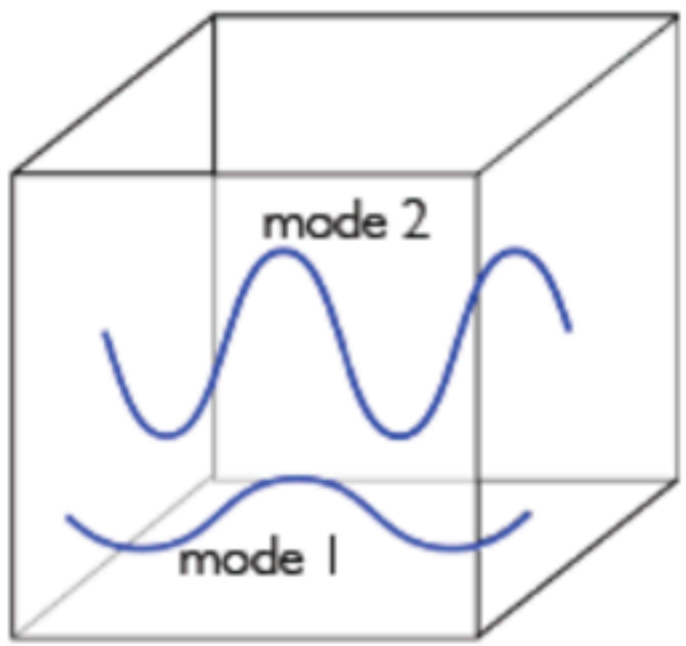
with the energy

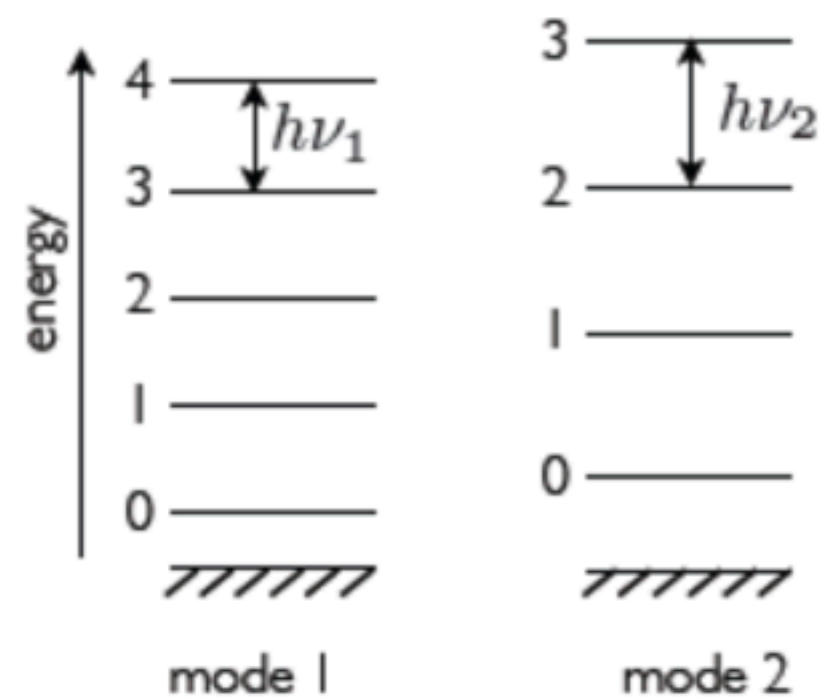
$$E_n = \hbar\omega(n + \frac{1}{2}).$$

7. Normalization constants:

$$\begin{aligned}
\hat{N}|n\rangle &= n|n\rangle, \\
\hat{a}|n\rangle &= C_n|n-1\rangle = \sqrt{n}|n-1\rangle, \\
\hat{a}^\dagger|n\rangle &= C_{n+1}|n+1\rangle = \sqrt{n+1}|n+1\rangle.
\end{aligned}$$

Quantized EM fields





- Energy quantization: nhν
- Equally spacing in energy difference
- Zero-point energy $\neq 0$, hν/2
- Characterized by mode (frequency, space mode, temporal mode, …)

More on Quantum SHO

PRL 112, 130404 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Local $\mathcal{PT}$ Symmetry Violates the No-Signaling Principle

Yi-Chan Lee,[1,2,*] Min-Hsiu Hsieh,[2] Steven T. Flammia,[3] and Ray-Kuang Lee[1,4]

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Synopsis: Reflecting on an Alternative Quantum Theory

APS/Alan Stonebraker

Local PT Symmetry Violates the No-Signaling Principle
Yi-Chan Lee, Min-Hsiu Hsieh, Steven T. Flammia, and Ray-Kuang Lee
Phys. Rev. Lett. 112, 130404 (2014)
Published April 3, 2014

PHYSICAL REVIEW LETTERS 123, 080404 (2019)

Simulating Broken $\mathcal{PT}$-Symmetric Hamiltonian Systems by Weak Measurement

Minyi Huang,[1,*] Ray-Kuang Lee,[2,3,4,†] Lijian Zhang,[5,‡] Shao-Ming Fei,[6,7,§] and Junde Wu[1,∥]

Parity-Time Hamiltonian

- Generalized Harmonic Oscillator (continuous):

$$\boxed{\mathcal{H} = p^2 + x^2\,(ix)^\epsilon}$$

- Spin-$\frac{1}{2}$ or two-mode coupler (discrete):

$$\boxed{H = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix}}$$

- Consider a family of differential equations parameterized by a continuous parameter $\epsilon > 0$ in the form:

$$\frac{\partial^2 \psi(x)}{\partial x^2} + V_\epsilon(x)\psi(x) + 2E\psi(x) = 0, \tag{1}$$

- Here, let us specify the definition of $V_\epsilon(x) = -(ix)^\epsilon$, by stating explicitly the branch of logarithm :

$$\boxed{V_\epsilon(x) = -(ix)^\epsilon = e^{\epsilon \log(ix)} = \begin{cases} -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) + i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x > 0; \\ 0, & \text{for } x = 0; \\ -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) - i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x < 0\,. \end{cases} \quad (2)}$$

- In a Fock state basis, an analytical formula for the matrix element $a_{nm}(\epsilon) = \langle m|\mathrm{H}_\epsilon|n\rangle$ of $\mathrm{H}_\epsilon = \frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{V_\epsilon(x)}{2}$ can be constructed for any natural number n, m and positive ϵ:

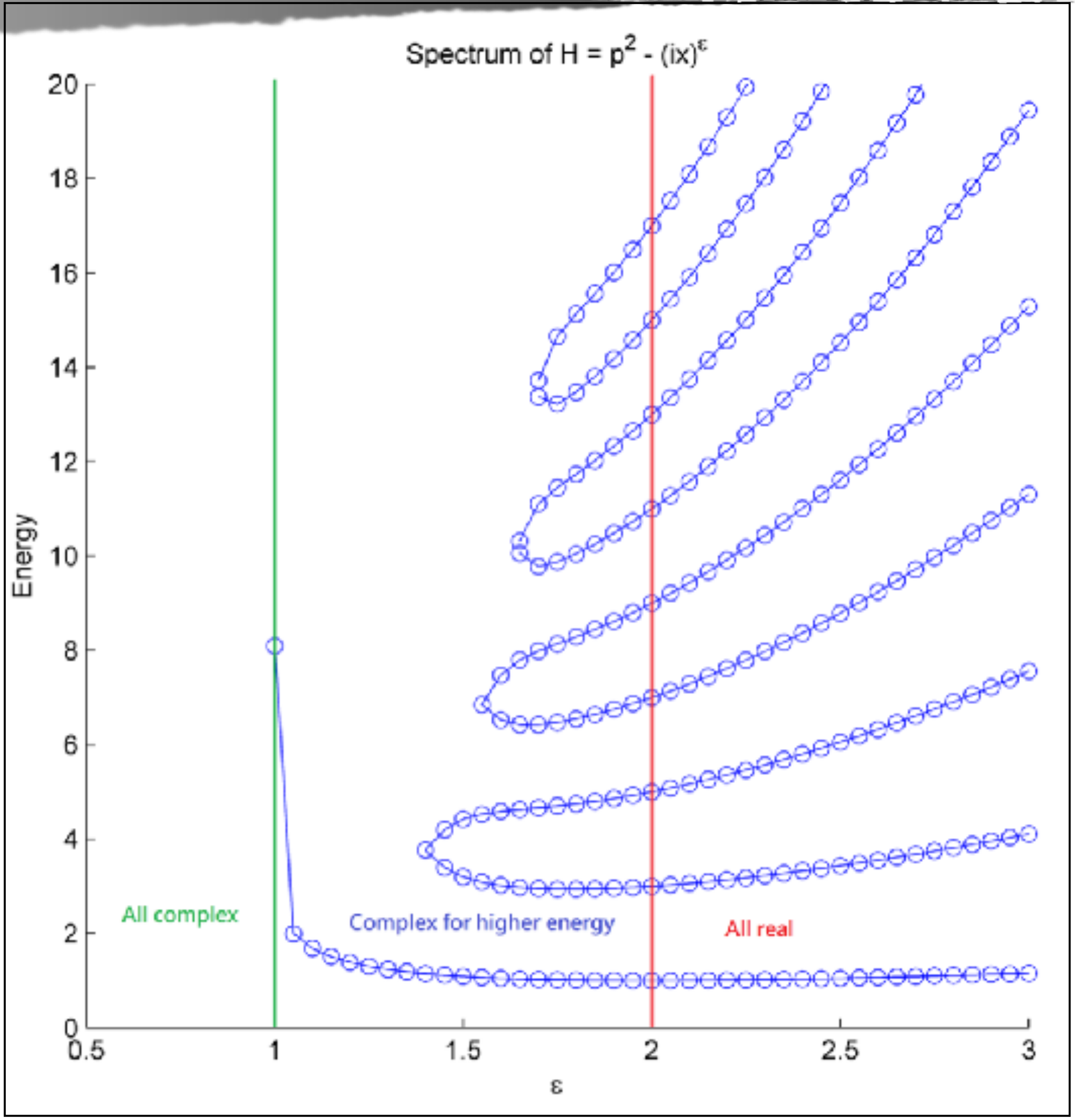
L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).

$$\begin{aligned} a_{nm}(\epsilon) \;&=\; \frac{\sqrt{n(n-1)}}{4}\,\delta_{m,n-2} + \frac{\sqrt{(n+1)(n+2)}}{4}\,\delta_{m,n+2} - \frac{2n+1}{4}\,\delta_{m,n} + \\ &+\; \left[\frac{1-(-1)^{\widetilde{n}+\widetilde{m}}}{4}\cos(\epsilon\tfrac{\pi}{2}) + \frac{1+(-1)^{\widetilde{n}+\widetilde{m}}}{4}\,i\sin(\epsilon\tfrac{\pi}{2})\right]\frac{(-1)^{\lfloor\frac{n}{2}\rfloor+\lfloor\frac{m}{2}\rfloor}2^{\widetilde{n}+\widetilde{m}}n!m!}{\lfloor\frac{n}{2}\rfloor!\,\lfloor\frac{m}{2}\rfloor!} \times \\ &\times\,\Gamma\!\left(\tfrac{1+\epsilon+\widetilde{n}+\widetilde{m}}{2}\right) F_A\!\left(\tfrac{1+\epsilon+\widetilde{n}+\widetilde{m}}{2}; -\lfloor\tfrac{n}{2}\rfloor, -\lfloor\tfrac{m}{2}\rfloor; \tfrac{2\widetilde{n}+1}{2}, \tfrac{2\widetilde{m}+1}{2}; 1, 1\right)\delta_{m,n} \end{aligned} \tag{3}$$

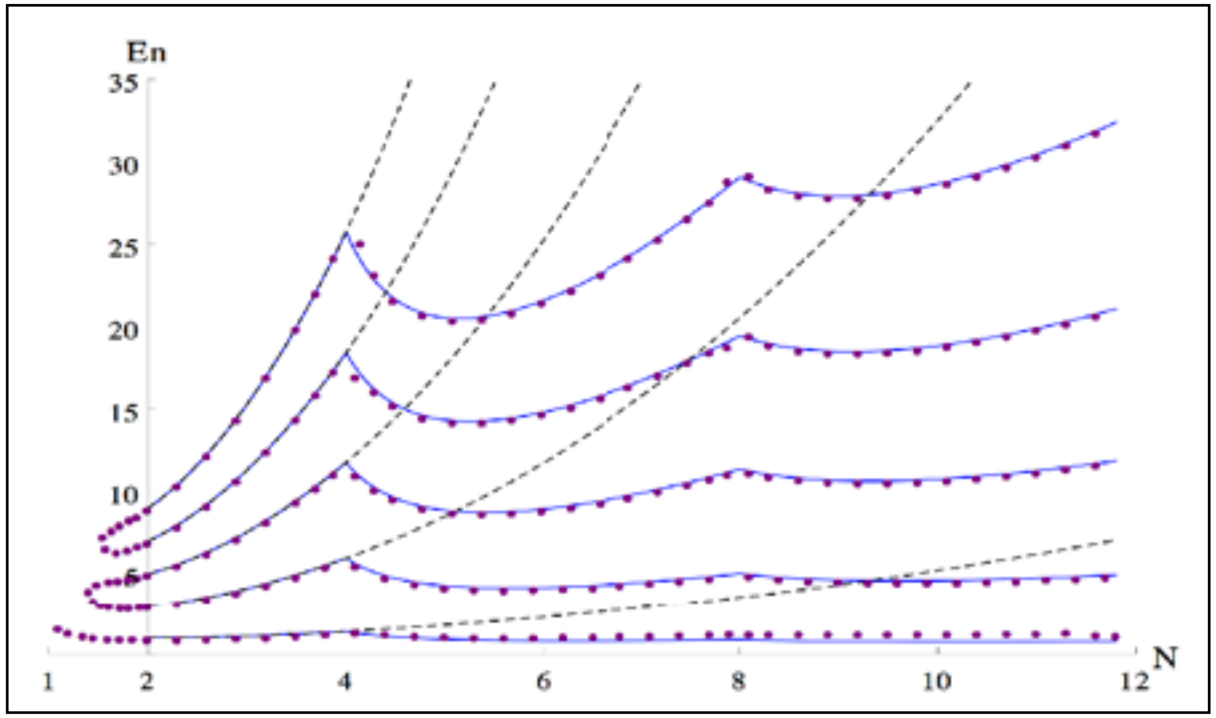
where Γ is an Euler gamma function; F_A is a Lauricella hypergeometric function; symbol $\lfloor\;\rfloor$ denotes a floor function: $\lfloor k\rfloor$ is the largest integer not greater than k; character tilde $\widetilde{\ }$ denotes a binary parity function: $\widetilde{k}$ is 0 for an even k and 1 for an odd k.

- By using truncated Fock state basis, we diagonalize the matrix $M_{nn}(\epsilon)$ numerically, having truncated the basis to the first 31, 51, or 71 elements.

Real Spectrum in $\mathcal{PT}$ Hamiltonian

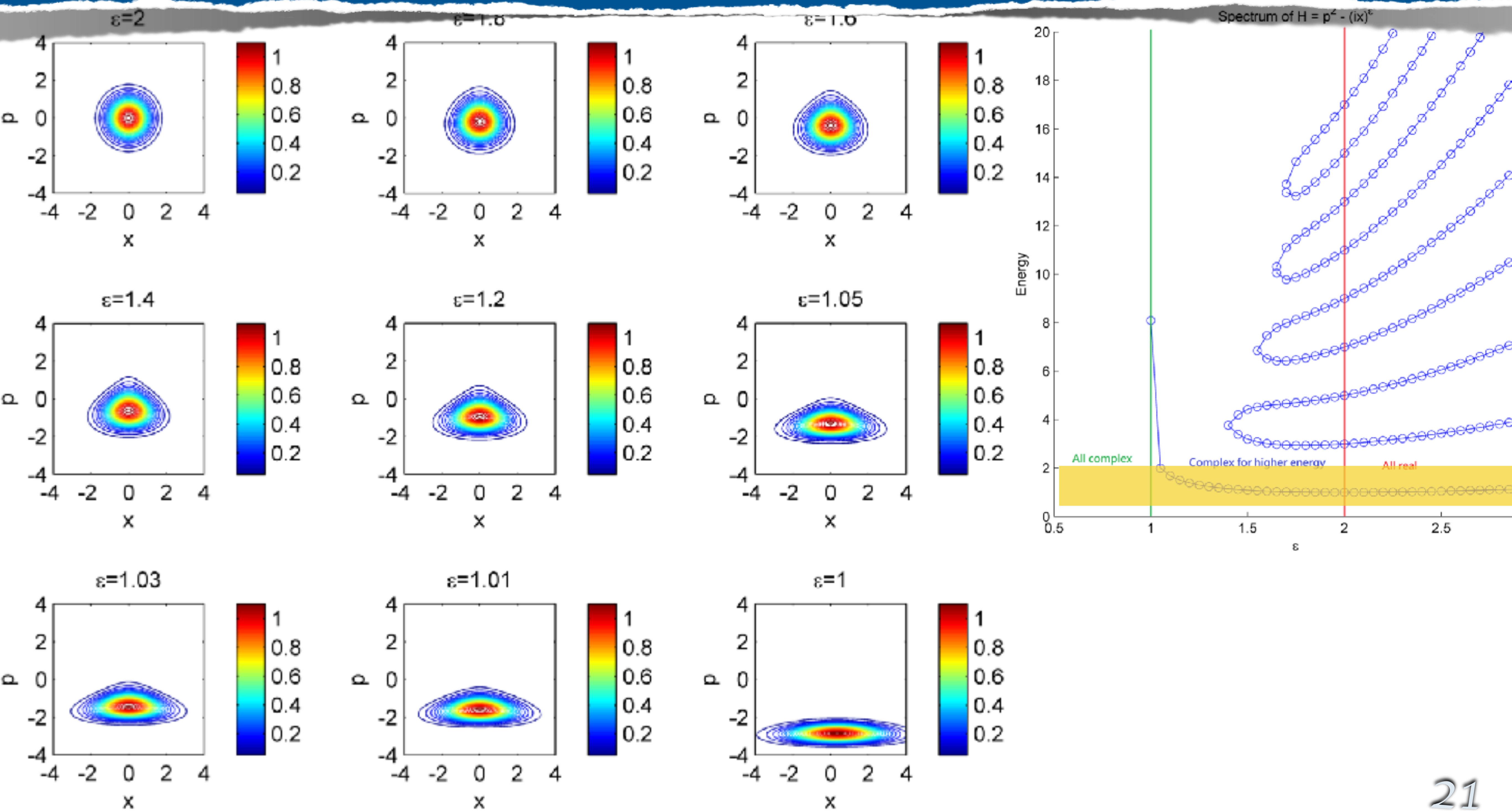


$$\mathcal{H} = p^2 + x^2\,(ix)^\epsilon$$

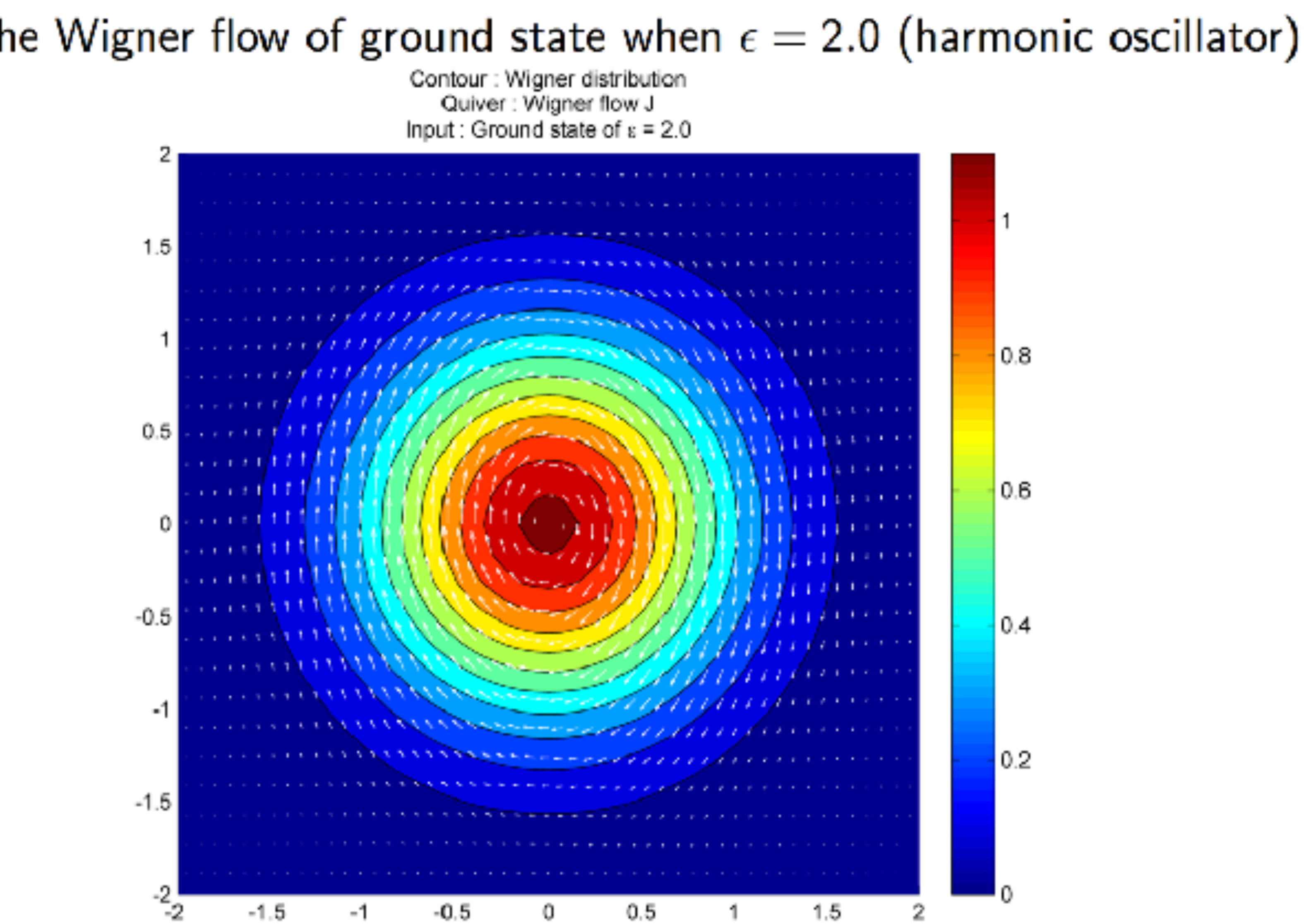


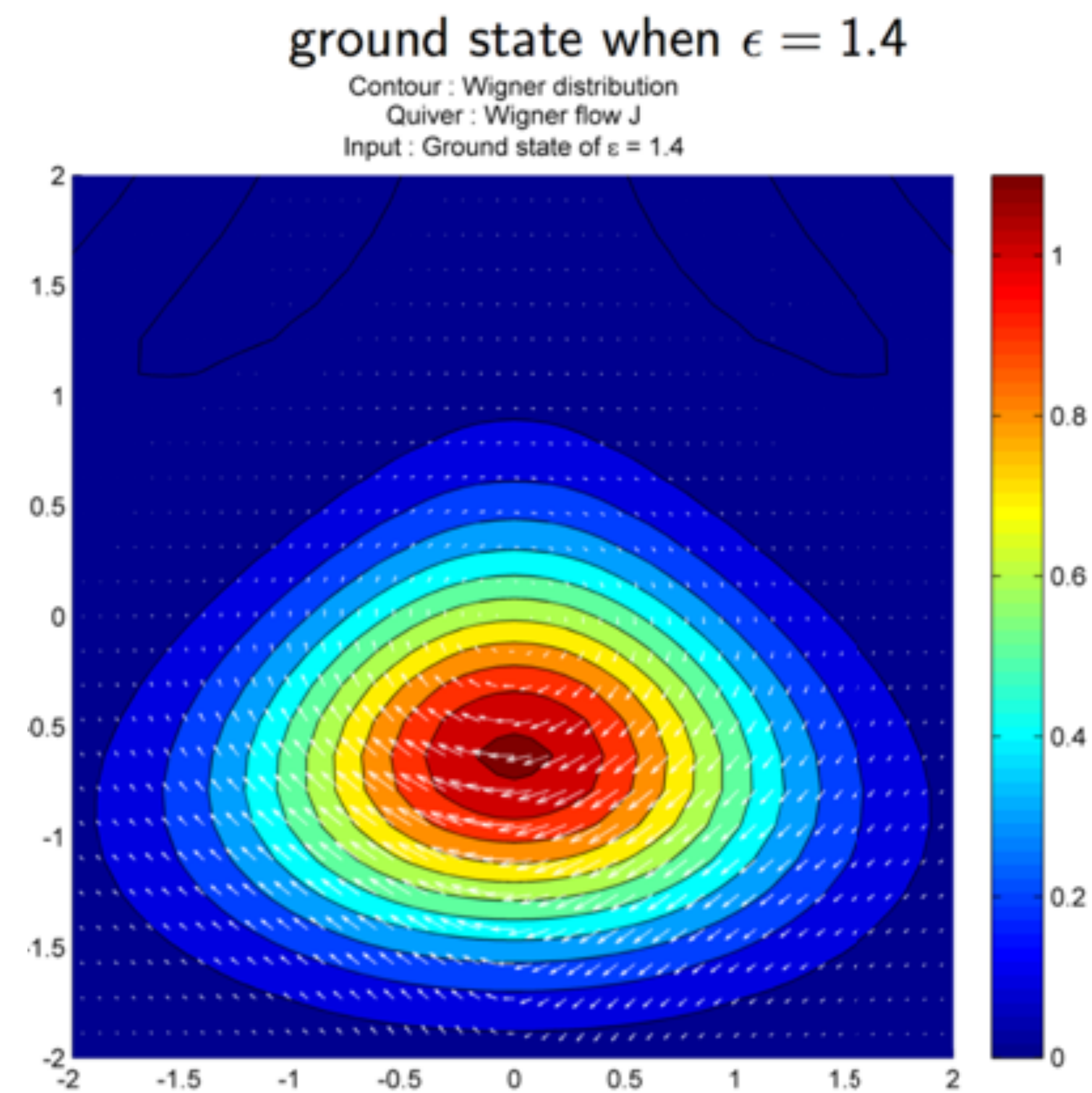
$$\mathcal{H} = p^2 - (ix)^\epsilon$$

$\mathcal{PT}$ in Phase space: Ground states



Wigner flow of $\mathcal{PT}$: Ground states





L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).