Note: Quantum SHO

- ☑ **Quantum Simple Harmonic Oscillator, qSHO**

• Photons occupy an electromagnetic mode (referred as the modes in quantum optics, typically a plane wave)

- ☑ **Hamiltonian**
- ☑ **Number operator**
- ☑ **Energy Quantization (equally spacing in energy)**

•The energy in a mode is not continuous but discrete in quanta.

- ☑ **Vacuum state with zero-point energy**

•There is a zero point energy inherent to each mode, which is equivalent with fluctuations of the electromagnetic field in vacuum, due to the uncertainty principle.

- ☑ **Schrodinger picture**
- ☑ **Heisenberg picture**

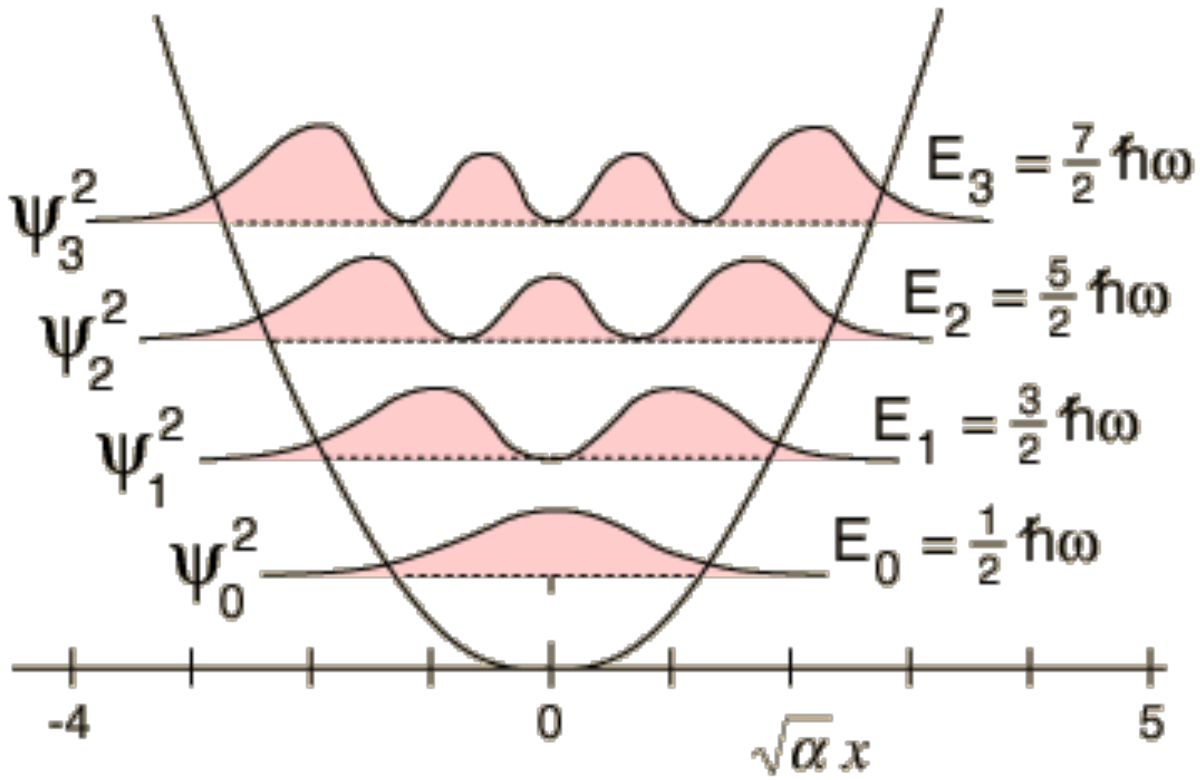
•The observables are just represented by probabilities as usual in QM.

Note: Quantum Mechanics

- ☑ Axioms
- ☑ State
- ☑ Operator
- ☑ Density Matrix

- More on States
  - □ Coherent States
  - □ Squeezed States
  - □ Uncertainty Relation ➡ Minimum Uncertainty States
  - □ Entropy
  - □ Purity
  - □ bi-particle States ➡ Entanglement (Schmidt decomposition)
  - □ Cat states

Quantum Simple Harmonic Oscillator (SHO)



$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \quad [\hat{x},\hat{p}] = i\hbar.$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \quad [\hat{a},\hat{a}^\dagger] = 1,$$

$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle, \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle, \\ E_n &= \hbar\omega(n+\frac{1}{2}). \end{aligned}$$

- Energy quantization
- Equally spacing in energy difference
- Zero-point energy $\neq 0$

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n+1, \qquad n = 0,1,2,3\ldots$$

$$E = \frac{\hbar\omega}{2}\epsilon = \hbar\omega(n+\frac{1}{2}), \qquad n = 0,1,2,3,\ldots.$$

Poisson Distribution:

$$P(n) = \frac{\bar{n}^n \exp(-\bar{n})}{n!},$$

$$\langle \hat{n} \rangle = \sum_n n P(n) = |\alpha|^2 \equiv \bar{n},$$

$$\langle \Delta \hat{n}^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 = |\alpha|^2 = \langle \hat{n} \rangle.$$

- mean = variance

Bose-Einstein Distribution:

- Boltzmann's law

$$P(n) \propto \exp[-E_n/k_BT],$$

$$P(n) = \frac{\exp[-E_n/k_BT]}{\sum_{n=0}^{\infty}\exp[-E_n/k_BT]}, \\ = \exp[-E_n/k_BT]\,(1-\exp[-\hbar\omega/k_BT]); \qquad E_n = n\,\hbar\omega$$

$$\bar{n} = \sum_{n=0}^{\infty} n\,P(n) = \frac{1}{\exp[\hbar\omega/k_BT]-1},$$

- average photon number at temperature T

$$P(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n,$$

Bose-Einstein Distribution:

- thermal state

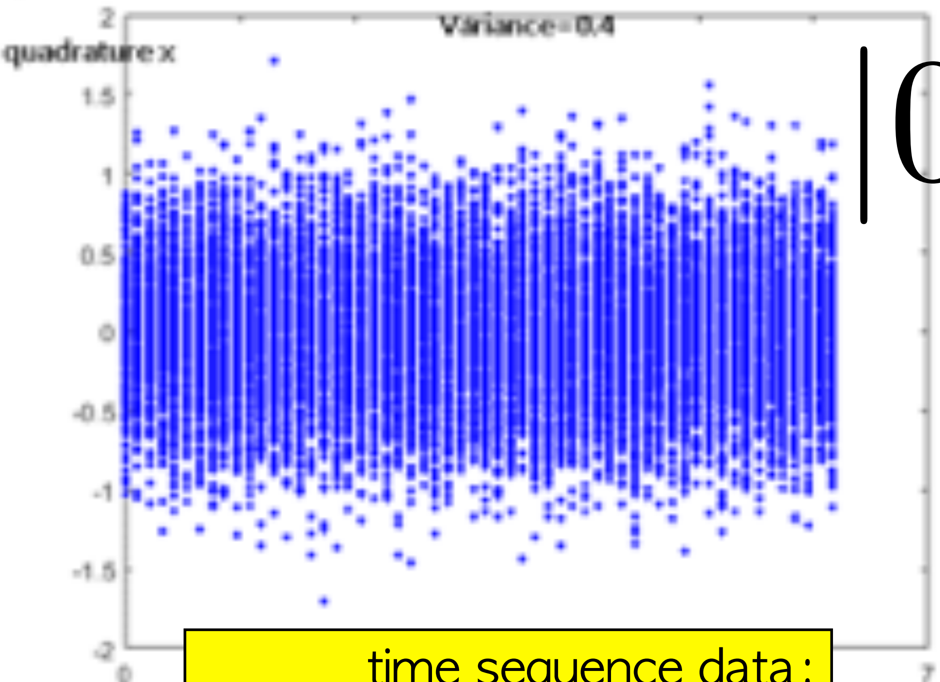
$$\rho_{th} = \sum_n = \frac{1}{\bar{n}+1}(\frac{\bar{n}}{\bar{n}+1})^n |n\rangle\langle n|.$$

$$\Delta n^2 = \bar{n} + \bar{n}^2,$$

Note: Coherent States (CS)

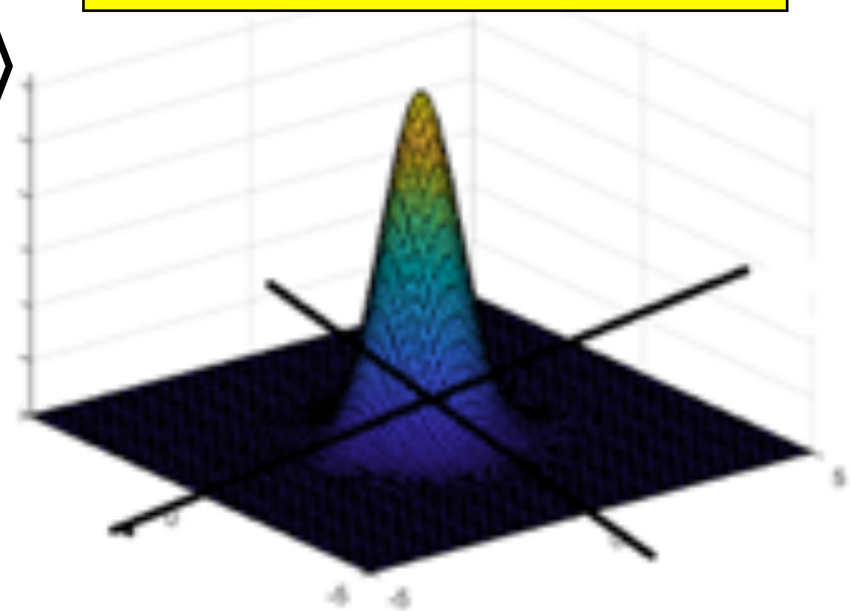
- □ Eigenstate of Annihilation operator
- □ Displacement Operator
- □ Properties of CS
- □ Representation of CS
- □ Expectation Value of E-fields
- □ Generation of CS
- More on States
- □ Minimum Uncertainty States
- □ Uncertainty Relation ➡ Minimum Uncertainty States
- □ Squeezed States
- □ CS in Phase space
- □ Max. Mixed CS
- □ Generalized CS
- □ Spin Coherent States
- □ Fermionic Coherent States

Quantum State Tomography

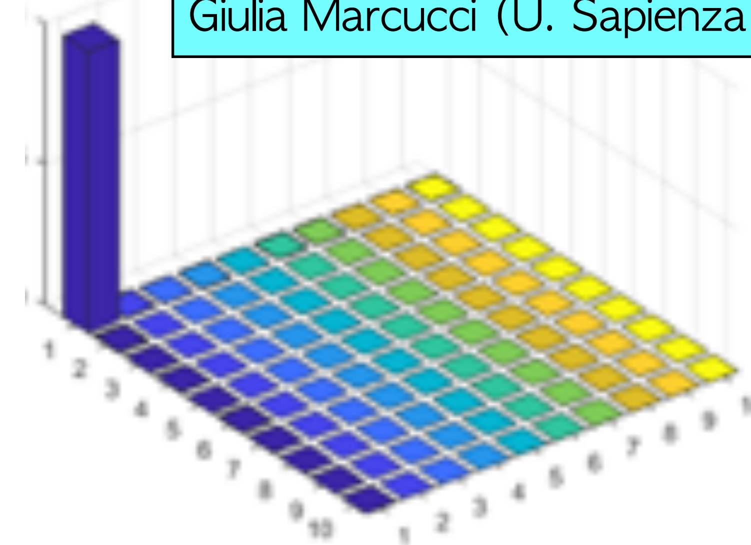


$|0\rangle$

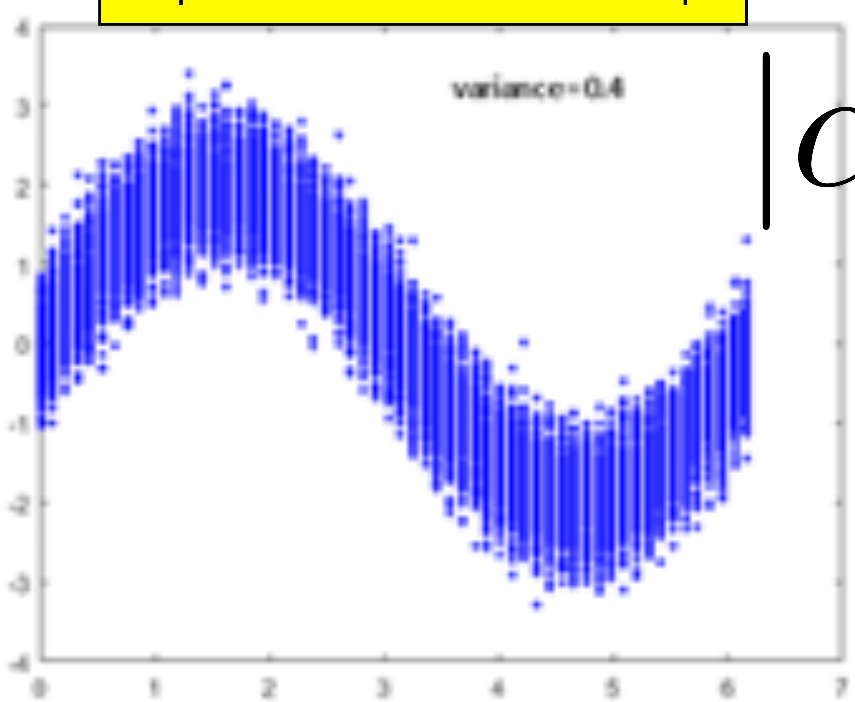
with Max. Likelihood estimation



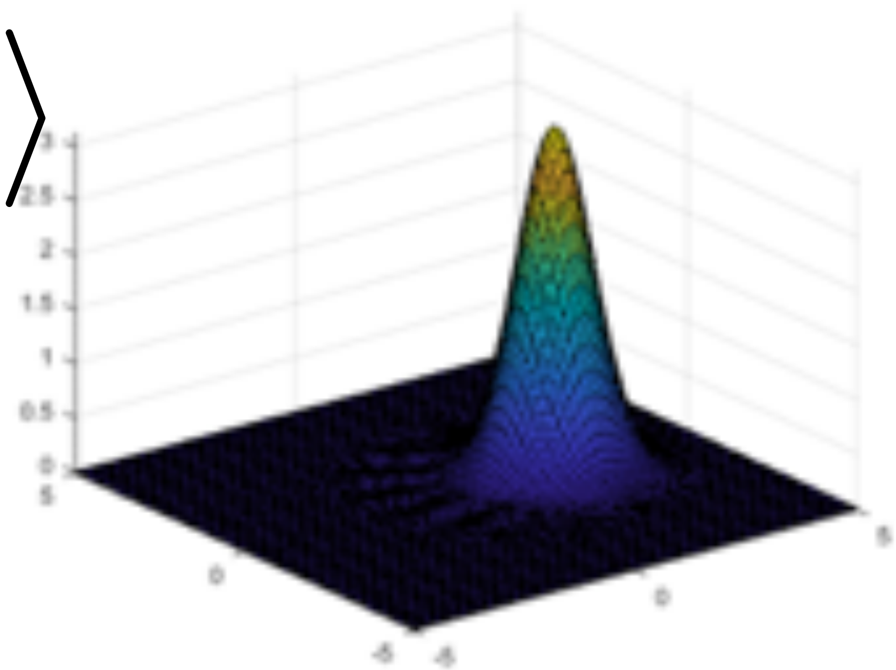
with Hsieh-Yi Hsieh;
Giulia Marcucci (U. Sapienza);



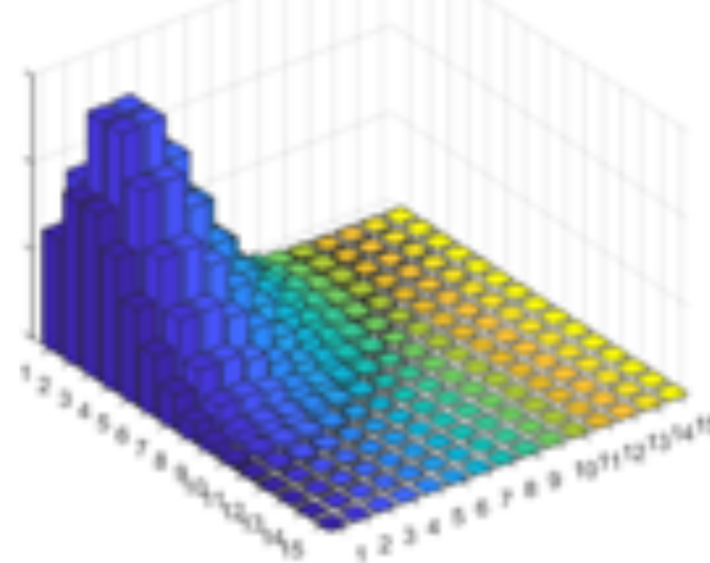
time sequence data:
Exp. data from Oscilloscope



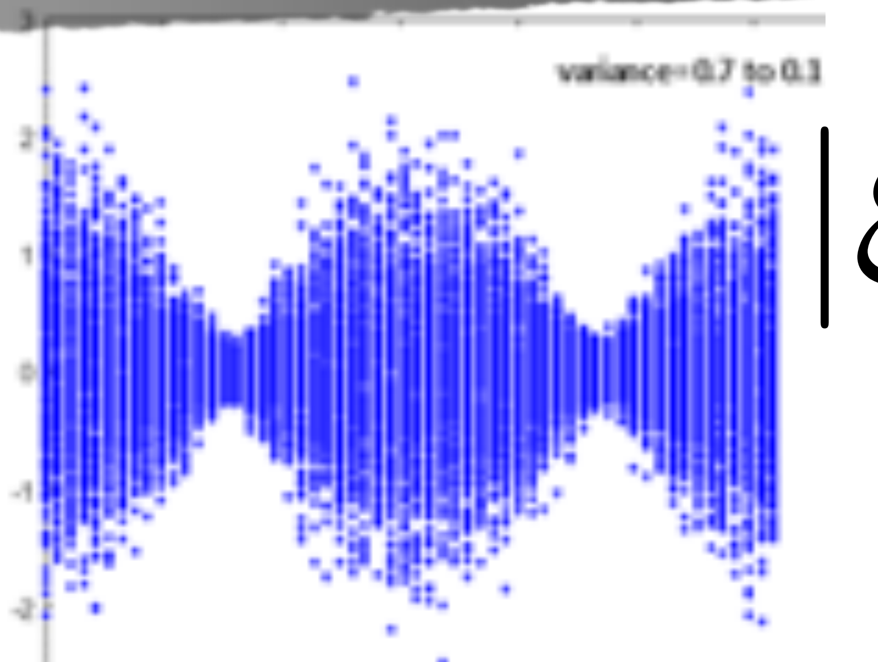
$|\alpha\rangle$



$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$



Quantum State Tomography



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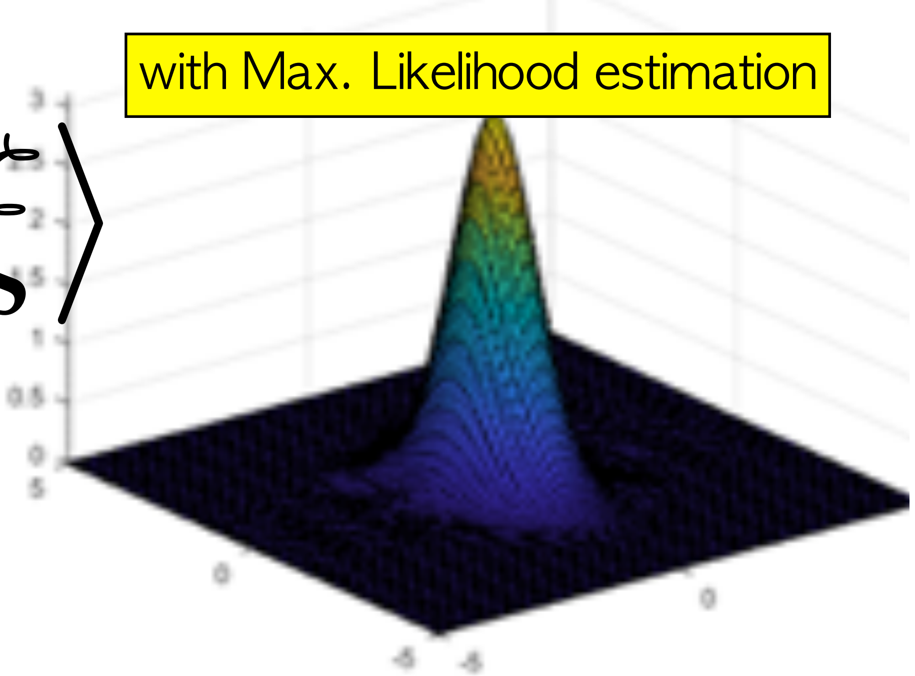
$|\xi\rangle$

with Max. Likelihood estimation



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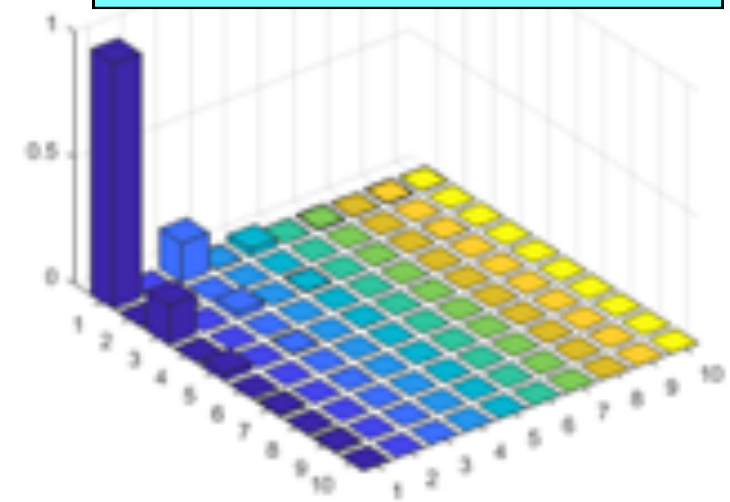
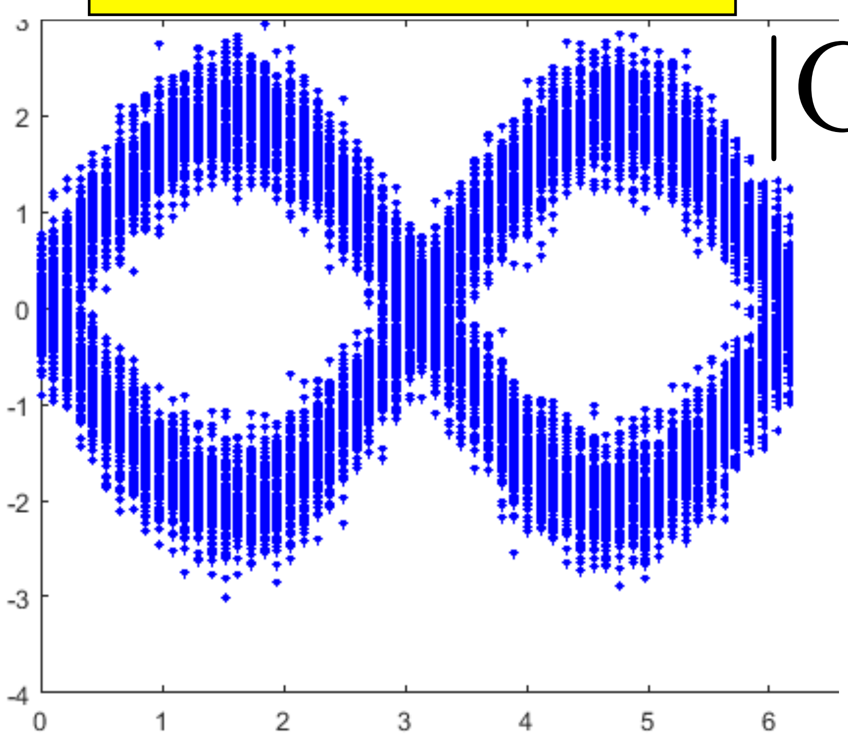
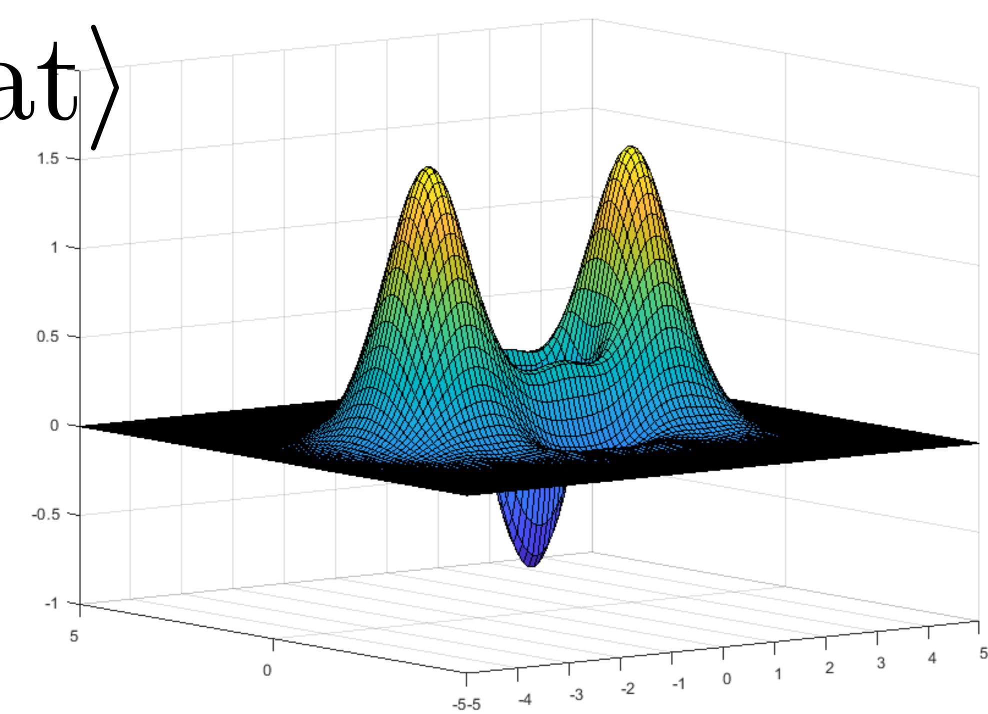




$|\text{Cat}\rangle$



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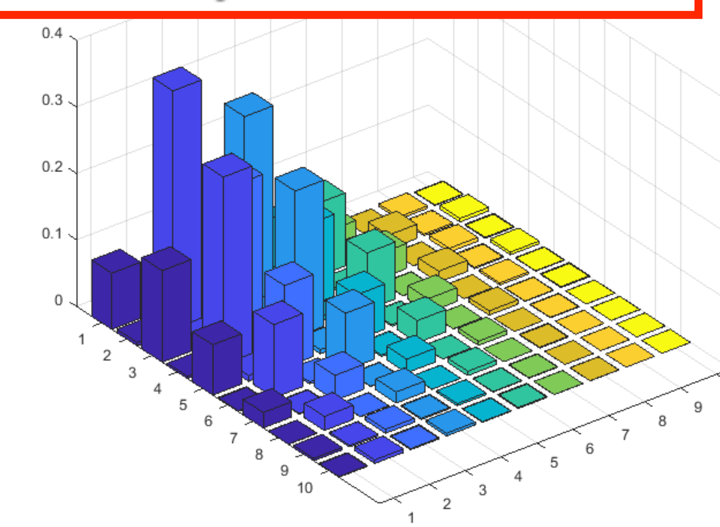

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- mean = variance

We introduce the eigenstate of annihilation operator, called the *coherent state*,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle,$$

Eigenstate of $\hat{a}$:

We introduce the eigenstate of annihilation operator, called the *coherent state*,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle,$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

- mean = variance

Displacement Operator:

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = e^{+\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle,$$

1. The probability of finding n photons in $|\alpha\rangle$ is given by a Poisson distribution.

2. The coherent state is a minimum-uncertainty states,

3. The set of all coherent states $|\alpha\rangle$ is a complete set,

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi \sum_n |n\rangle\langle n|, \quad \text{or} \quad \frac{1}{\pi}\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = 1. \tag{1}$$

4. Two coherent states corresponding to different eigenstates α and β are not orthogonal,

$$\langle\alpha|\beta\rangle = \exp(-\frac{1}{2}|\alpha|^2 + \alpha^*\beta - \frac{1}{2}|\beta|^2) = \exp(-\frac{1}{2}|\alpha-\beta|^2). \tag{2}$$

5. Coherent states are *approximately* orthogonal only in the limit of large separation of the two eigenvalues, $|\alpha - \beta| \to \infty$. Therefore, any coherent state can be expanded using other coherent state,

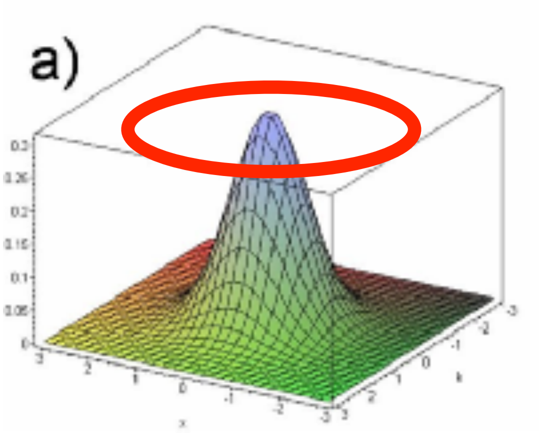
$$|\alpha\rangle = \frac{1}{\pi}\int \mathrm{d}^2\beta |\beta\rangle\langle\beta|\alpha\rangle = \frac{1}{\pi}\int \mathrm{d}^2\beta e^{-\frac{1}{2}|\beta-\alpha|^2}|\beta\rangle. \tag{3}$$

   This means that a coherent state forms an *overcomplete* set.

6. The simultaneous measurement of $\hat{a}_1$ and $\hat{a}_2$, represented by the projection operator $|\alpha\rangle\langle\alpha|$, is not an exact measurement but instead an approximate measurement with a finite measurement error.

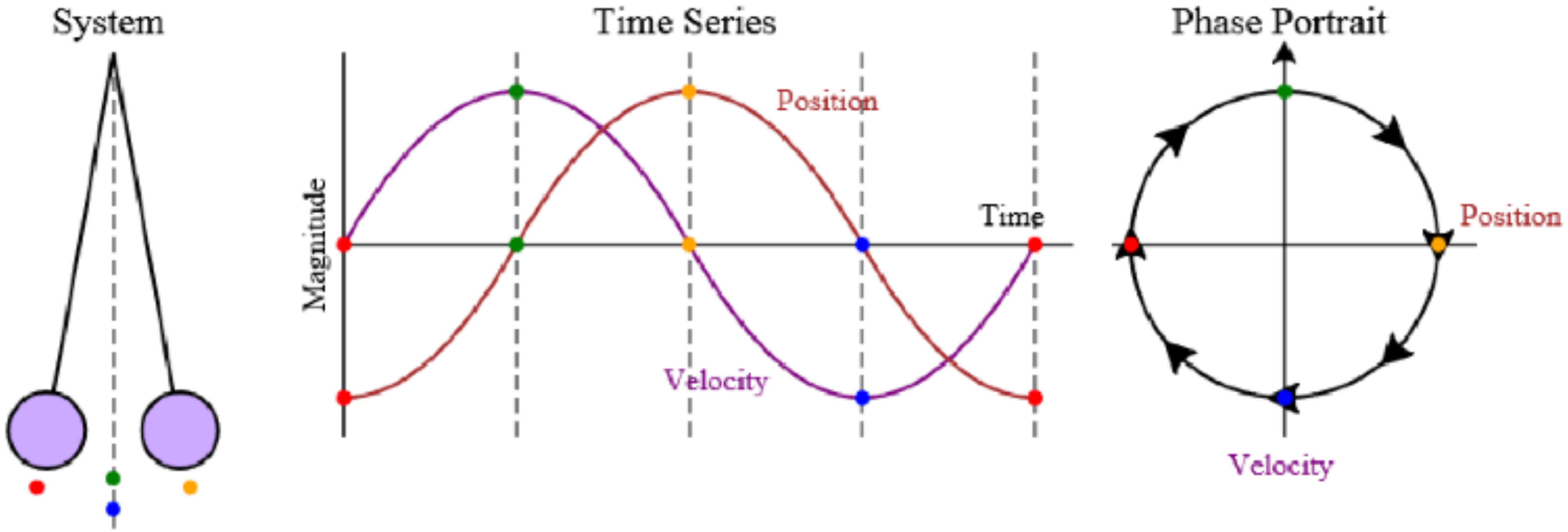
Representation:

$$\langle q|\alpha\rangle = (\frac{\omega}{\pi\hbar})^{1/4}\exp[-\frac{\omega}{2\hbar}(q-\langle q\rangle)^2 + i\frac{\langle p\rangle}{\hbar}q + i\theta],$$

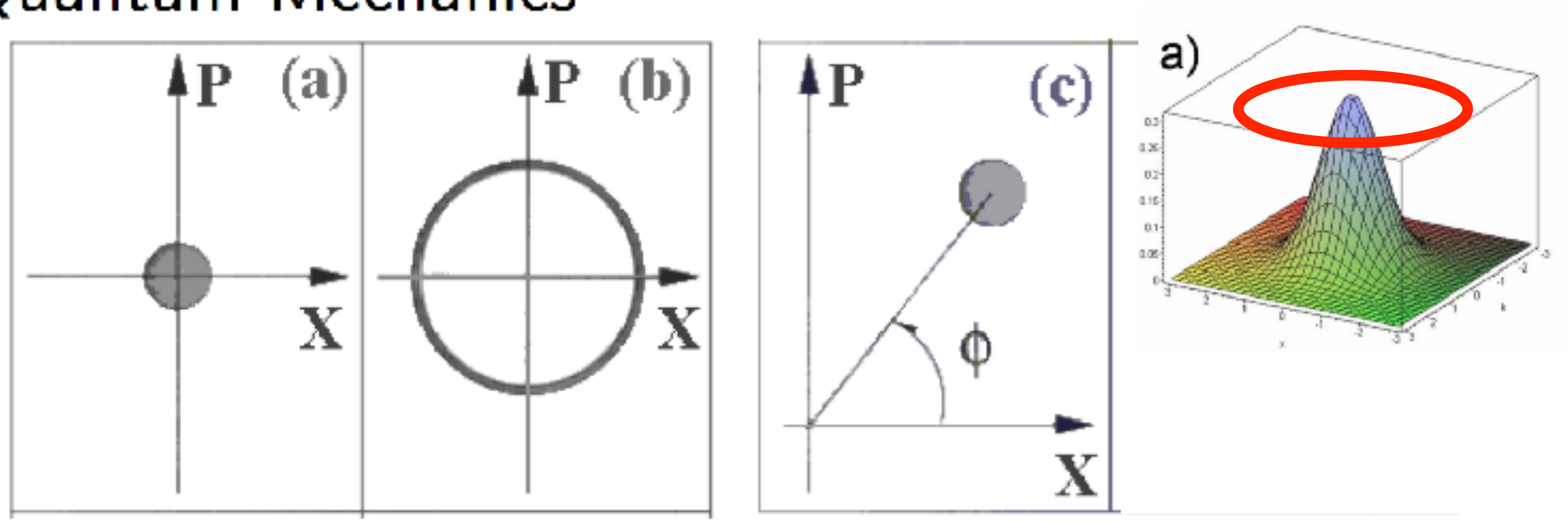


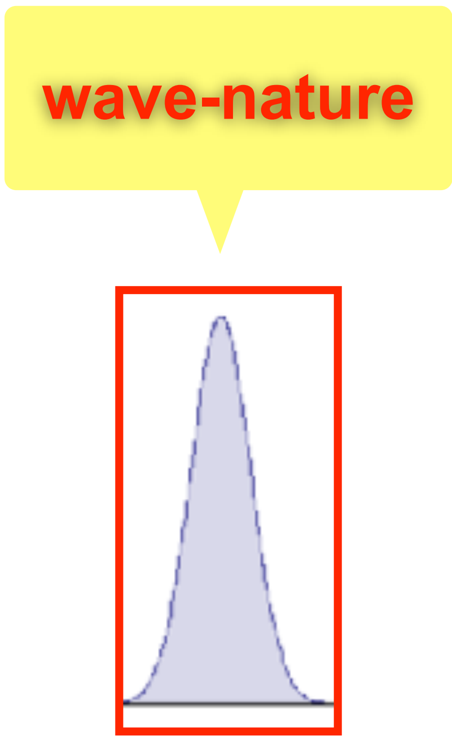
Phase space

- Classical Mechanics



- Quantum Mechanics

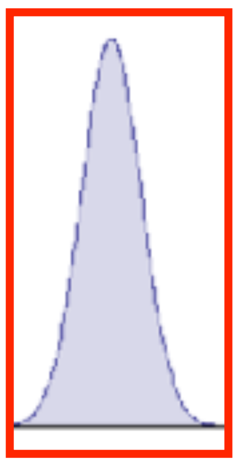


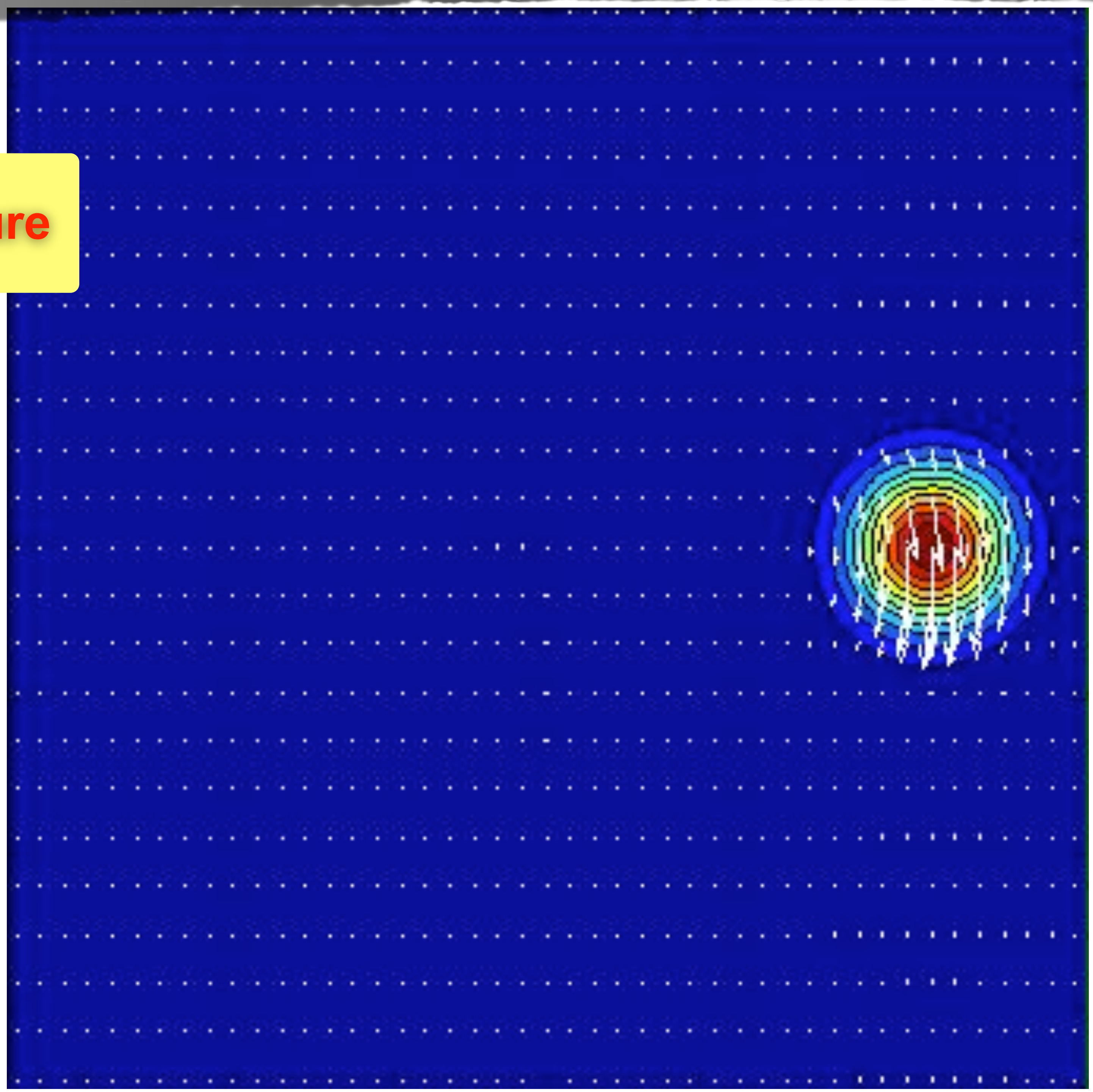


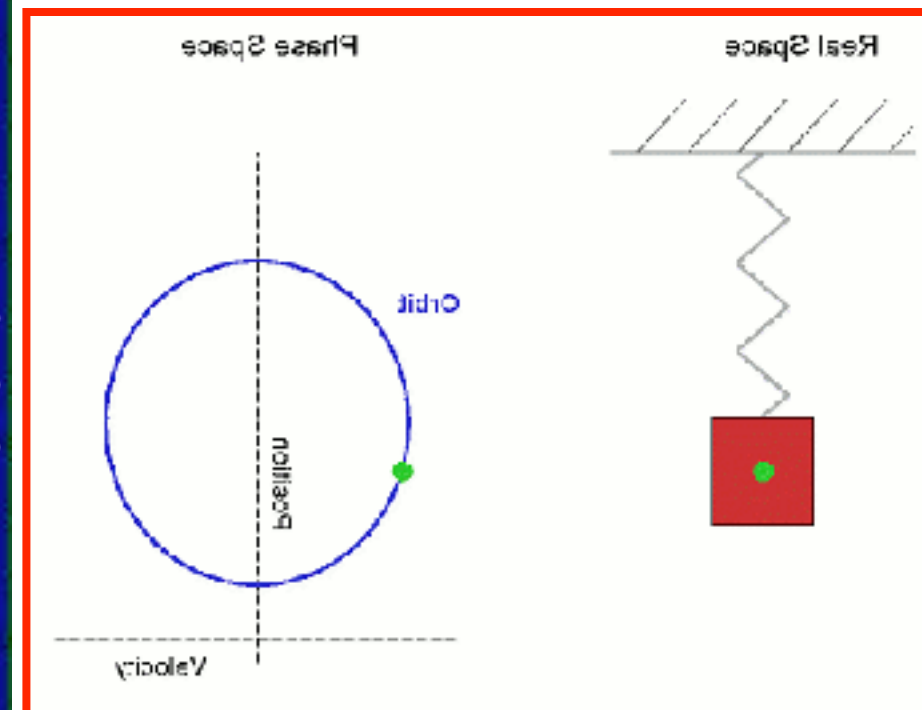
from Wiki

Coherent states

wave-nature



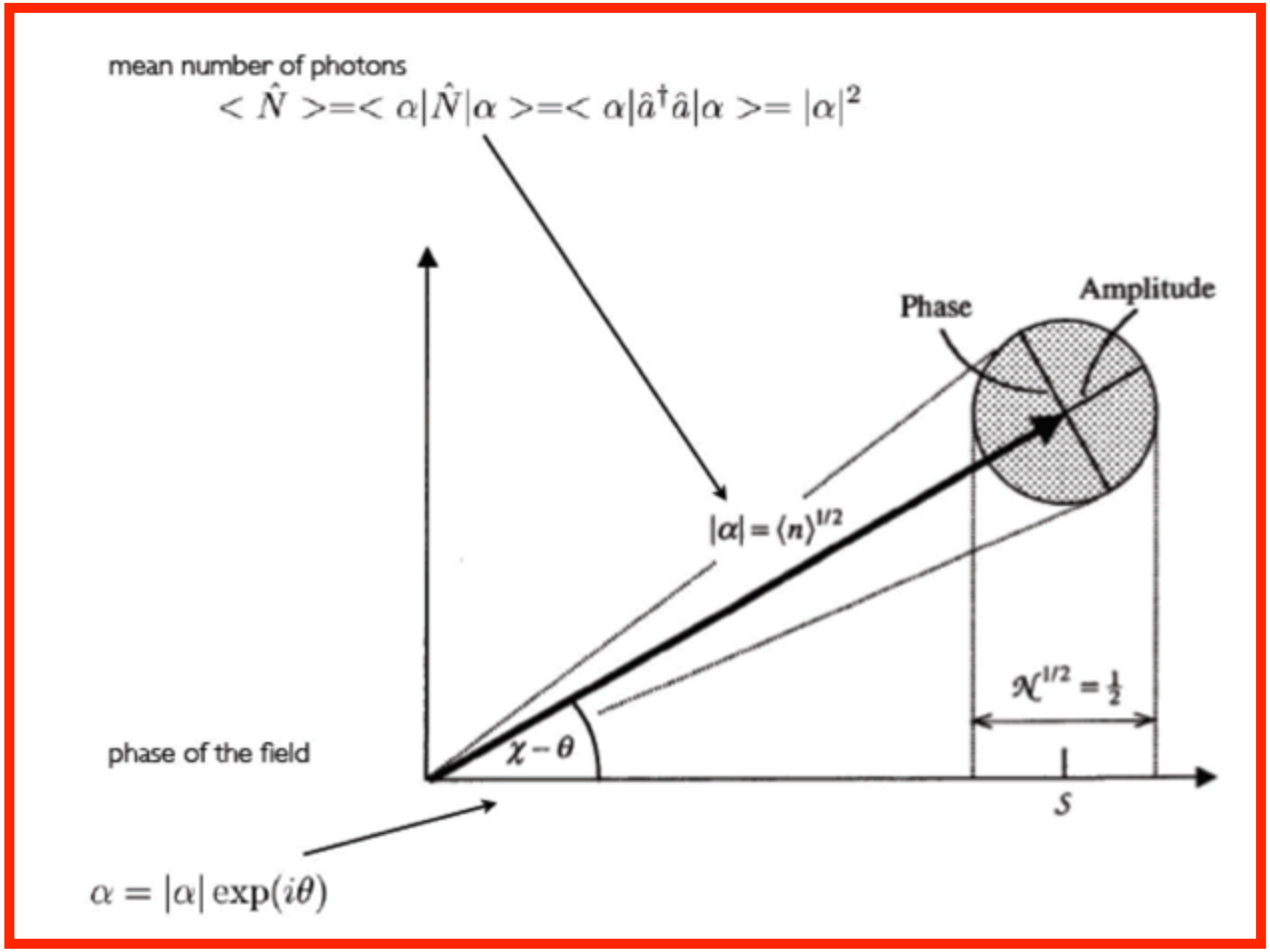




with Popo Yang

Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs, Physica Scripta for the New Focus issue: Quantum Optics and Beyond - in honour of Wolfgang Schleich.

Expectation value of E-fields:



Generation of CS: