# Note: Coherent States (CS)

- ☑ Eigenstate of Annihilation operator
- ☑ Displacement Operator
- ☑ Properties of CS
- ☑ Representation of CS
- ☑ Expectation Value of E-fields
- □ Generation of CS

- More on States
  - □ Minimum Uncertainty States
  - □ Uncertainty Relation ➡ Minimum Uncertainty States
  - □ Squeezed States
  - □ CS in Phase space
  - □ Max. Mixed CS
  - □ Generalized CS
  - □ Spin Coherent States
  - □ Fermionic Coherent States

# Bose-Einstein Distribution:

- Boltzmann's law

$$P(n) \propto \exp[-E_n/k_B T],$$

$$\begin{aligned} P(n) &= \frac{\exp[-E_n/k_B T]}{\sum_{n=0}^{\infty} \exp[-E_n/k_B T]}, \\ &= \exp[-E_n/k_B T]\,(1 - \exp[-\hbar\omega/k_B T]); \qquad E_n = n\,\hbar\omega \end{aligned}$$

$$\bar{n} = \sum_{n=0}^{\infty} n\,P(n) = \frac{1}{\exp[\hbar\omega/k_B T] - 1},$$

- average photon number at temperature T

$$P(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n,$$

$$\Delta n^2 = \bar{n} + \bar{n}^2,$$

# Quantum Mechanics: von Neuman entropy

How can we discriminate pure from mixed states, or more generally, characterize the purity of a state? One option is the *von Neumann entropy*, i.e.,

$$S = -k_B \, \mathrm{tr}[\hat{\rho} \ln \hat{\rho}],$$

where $k_B$ denotes the Boltzmann constant.

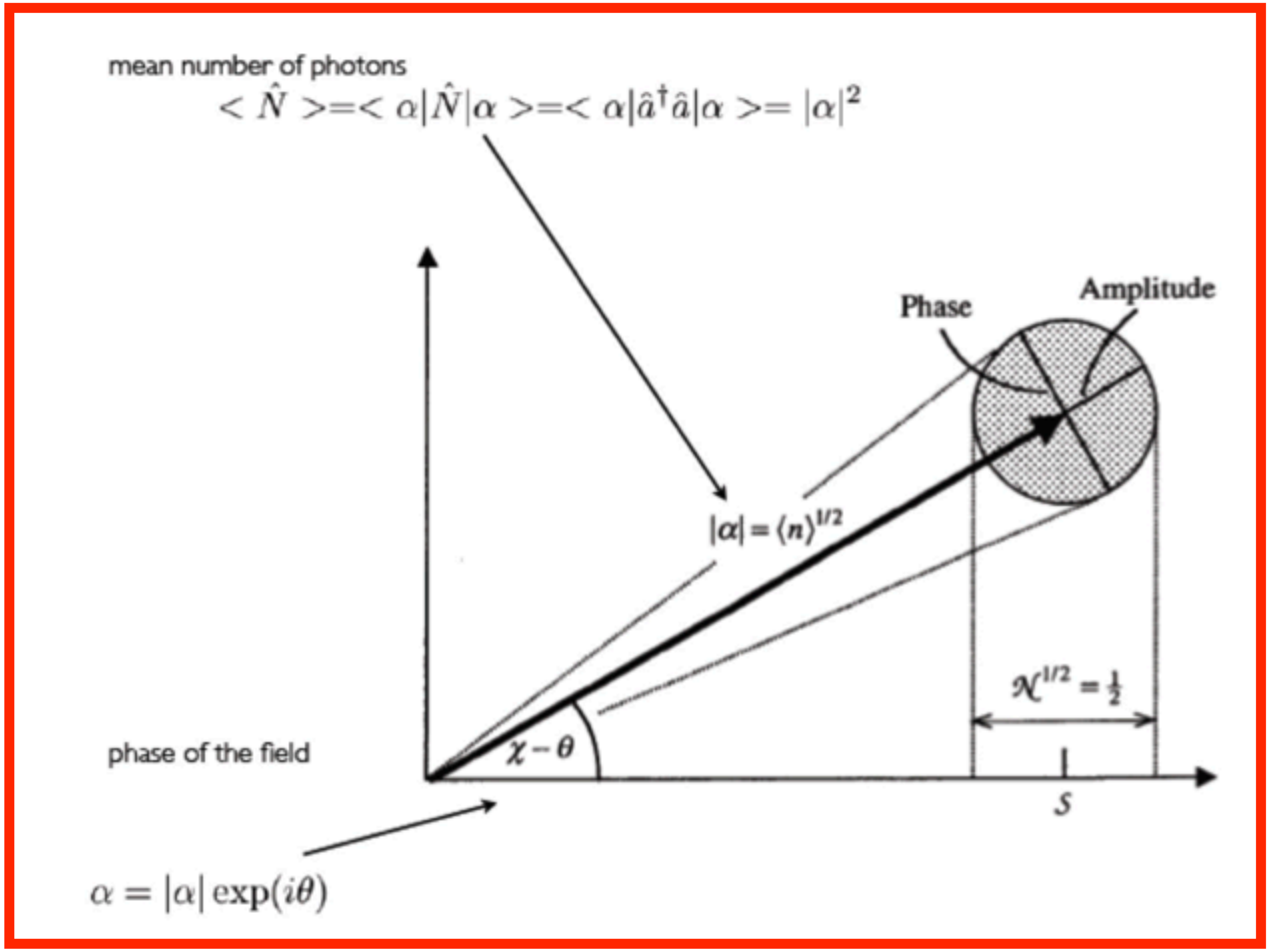
- $S(\rho)$ is zero if and only if $\rho$ represents a pure state.
- $S(\rho)$ is maximal and equal to $\ln N$ for a maximally mixed state, $N$ being the dimension of the Hilbert space.
- $S(\rho)$ is invariant under changes in the basis of $\rho$, that is, $S(\rho) = S(\hat{U}\rho\hat{U}^\dagger)$, with $\hat{U}$ a unitary transformation.
- $S(\rho)$ is additive for independent systems. Given two density matrices $\rho_A$, $\rho_B$ describing independent systems A and B, we have

$$S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B).$$

# Thermal states with the Maximal Entropy

$$\mathcal{S} = -k_B \sum_n \rho_n \ln \rho_n - \mu_1 (\sum_n \rho_n - 1) - \mu_2 (\sum_n \rho_n E_n - E),$$

# Expectation value of E-fields:

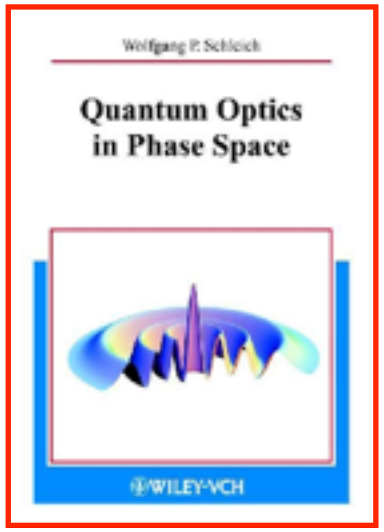


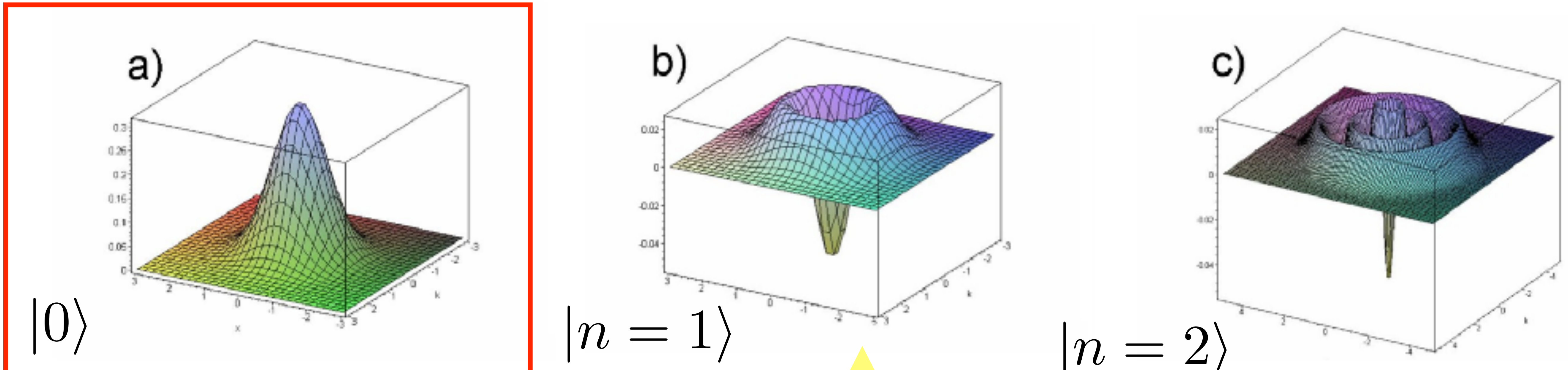
# Number (Fock) states

**Non-classical states**

- Wigner quasiprobability distribution

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi \psi^*(x - \frac{\xi}{2})\psi(x + \frac{\xi}{2})e^{-ik\xi}$$




**negative probability**

with Ludmila Praxmeyer

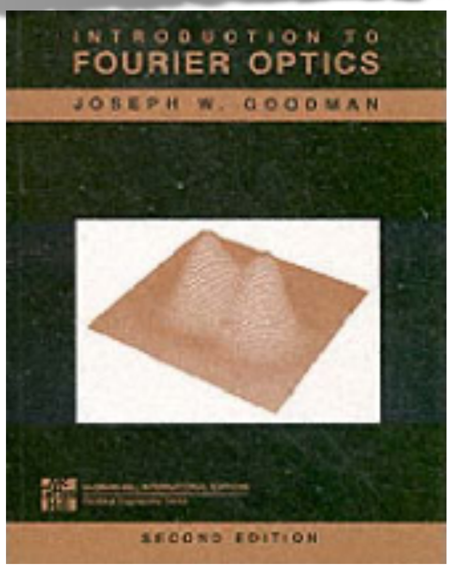
# Note: Minimum Uncertainty States (MUS)

- □ Heisenberg's Uncertainty Relation
- □ Minimum Uncertainty States (MUS)
- □ Gaussian States
- □ Free-particle expansion
- □ Squeezed States
- More on Uncertainty Relations
- □ Intelligent States
- □ Robertson–Schrodinger uncertainty relations
- □ Quantum entropic uncertainty principle
- □ Quantum Metrology
- □ Heisenberg limit ➡ Quantum Cramer-Rao bound
- □ Quantum Non-Demolition (QND) Measurement

# From Scratch !!

- **How much do you known about Uncertainty Relation?**

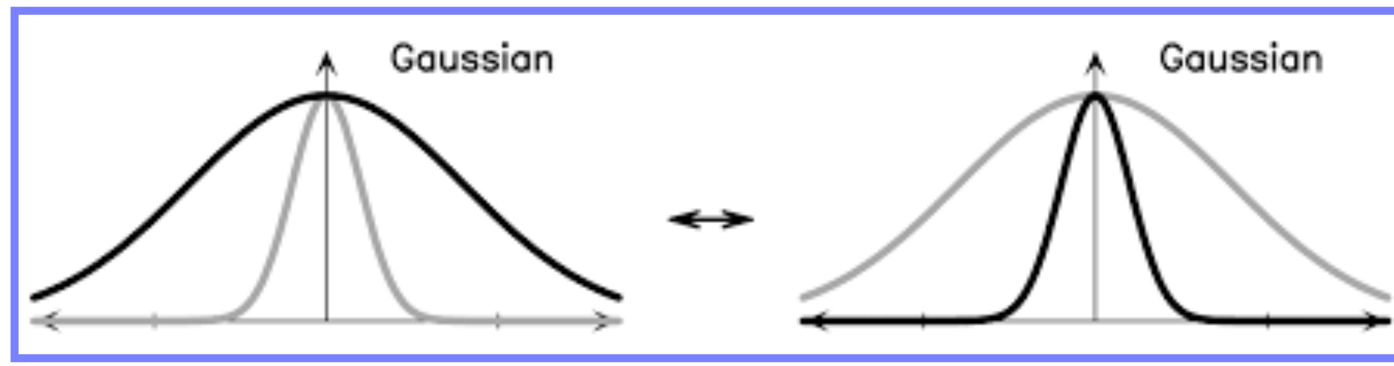
# Heisenberg's Uncertainty Principle

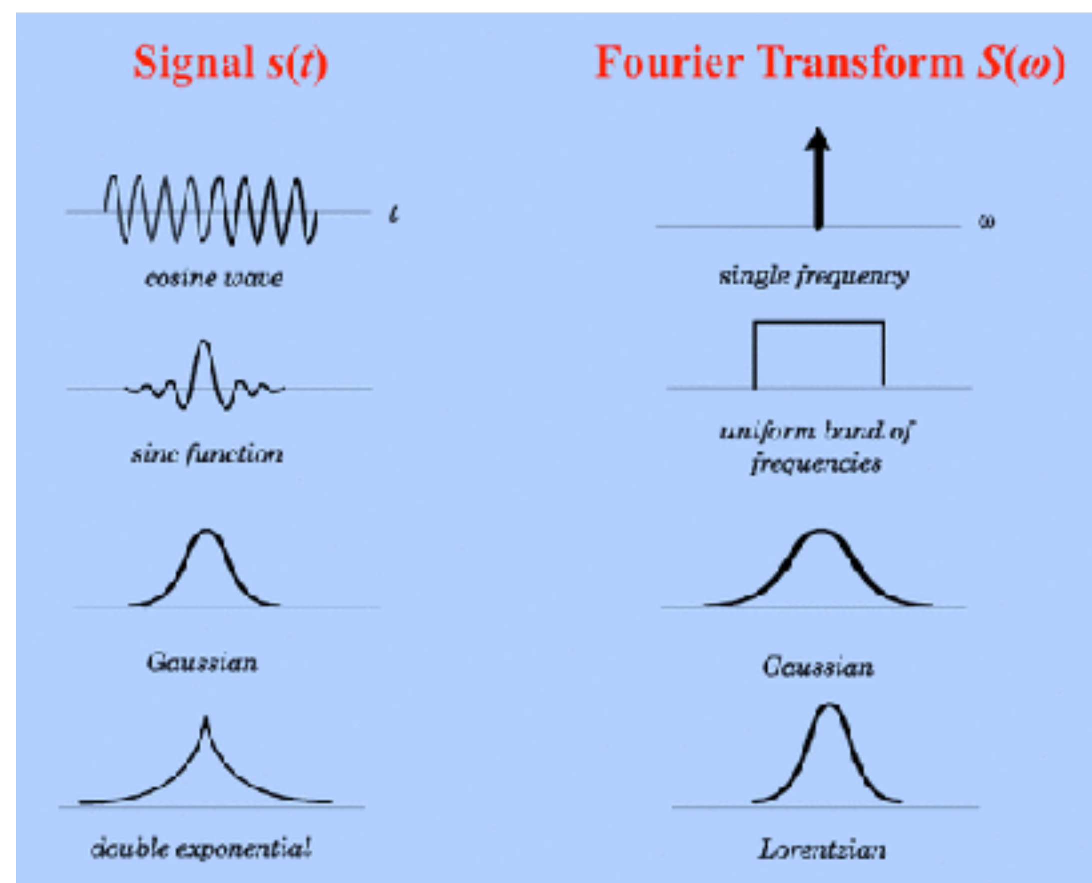



Joseph Fourier (1768-1830)

Werner Heisenberg (1901-1976)

Fourier-Transform Limit




uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in position

波函數

wave function

# Uncertainty Relation:

- Non-commuting observable do not admit common eigenvectors.

- Non-commuting observables can not have definite values simultaneously.

- Simultaneous measurement of non-commuting observables to an arbitrary degree of accuracy is thus incompatible.

# Uncertainty Relation:

For any two non-commuting observables, $[\hat{A}, \hat{B}] = i\hat{C}$, we have the *uncertainty relation*:

$$\Delta A^2 \Delta B^2 \geq \frac{1}{4}[\langle \hat{F} \rangle^2 + \langle \hat{C} \rangle^2],$$

where

$$\hat{F} = \hat{A}\hat{B} + \hat{B}\hat{A} - 2\langle \hat{A} \rangle \langle \hat{B} \rangle,$$

where the operator $\hat{F}$ is a measure of correlations between $\hat{A}$ and$\hat{B}$.

# Uncertainty Relation:

A *Minimum Uncertainty State* (MUS), $|\psi\rangle$, satisfies

$$[\hat{A} + i\lambda\hat{B}]|\psi\rangle = [\langle\hat{A}\rangle + i\lambda\langle\hat{B}\rangle]|\psi\rangle = z|\psi\rangle,$$

where $z$ is a complex number.