Note: Phase Space

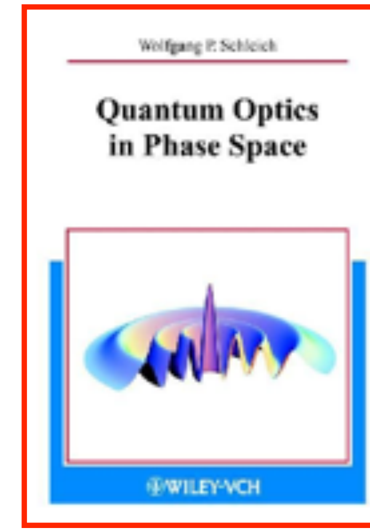
- □ Quantum Distribution Theory
- □ Ordering
- □ Normally Ordering, P-representation (Glauber-Sudarshan)
- □ Anti-Normally Ordering, Q-representation (Husimi)
- □ Symmetrically Ordering, W-representation (Wigner-Weyl)
- More on Phase Space
- □ Fokker-Planck equation
- □ Quantum Liouville equation
- □ Moyal function
- □ Homodyne detection
- □ Wigner Flow

Number (Fock) states

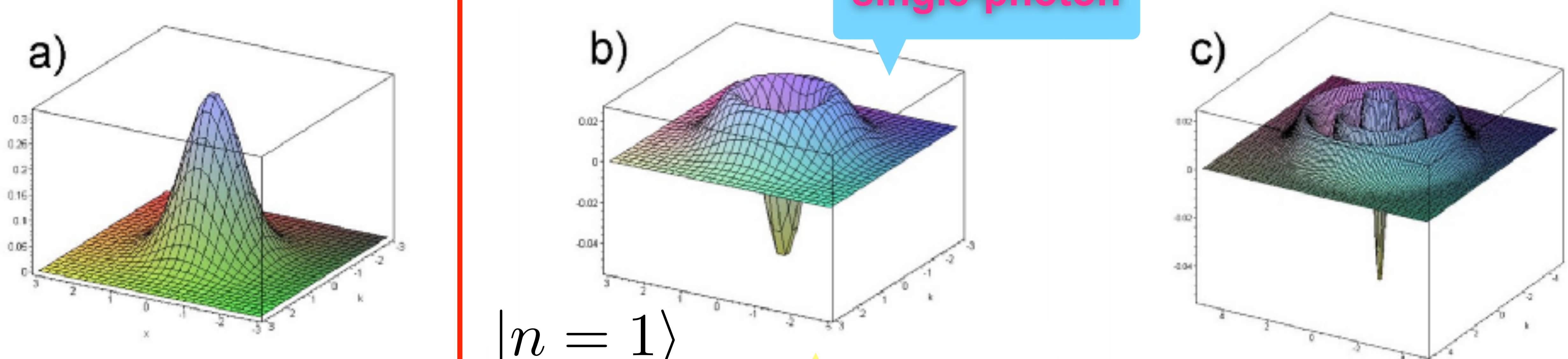
non-classical states

- Wigner quasiprobability distribution

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi \psi^*(x - \frac{\xi}{2})\psi(x + \frac{\xi}{2})e^{-ik\xi}$$



single-photon



$|0\rangle$

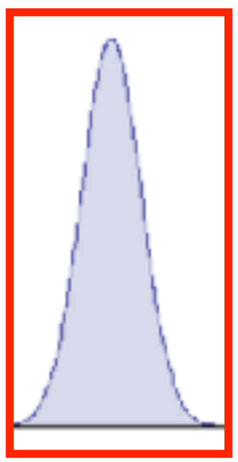
$|n = 1\rangle$

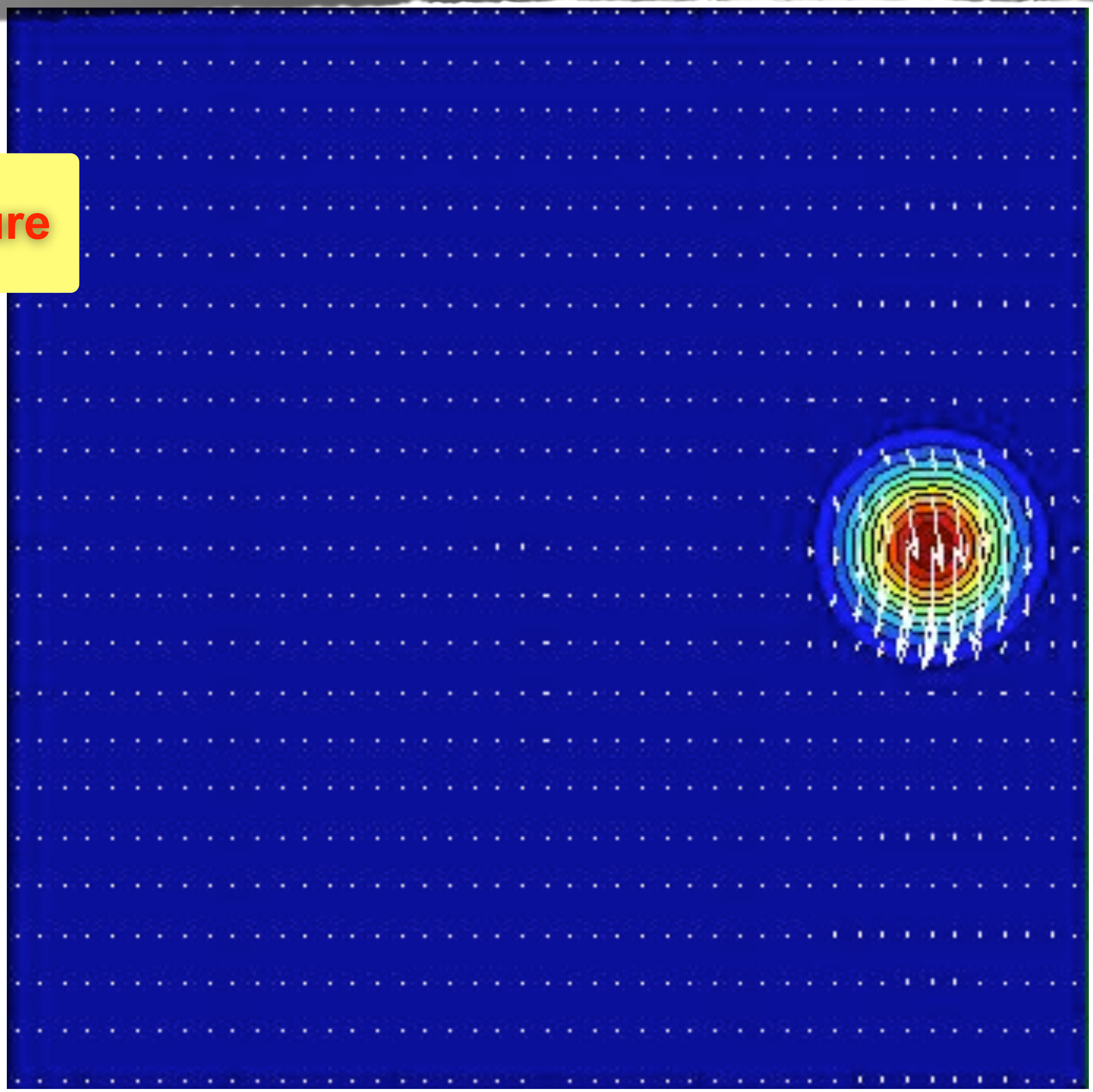
negative probability

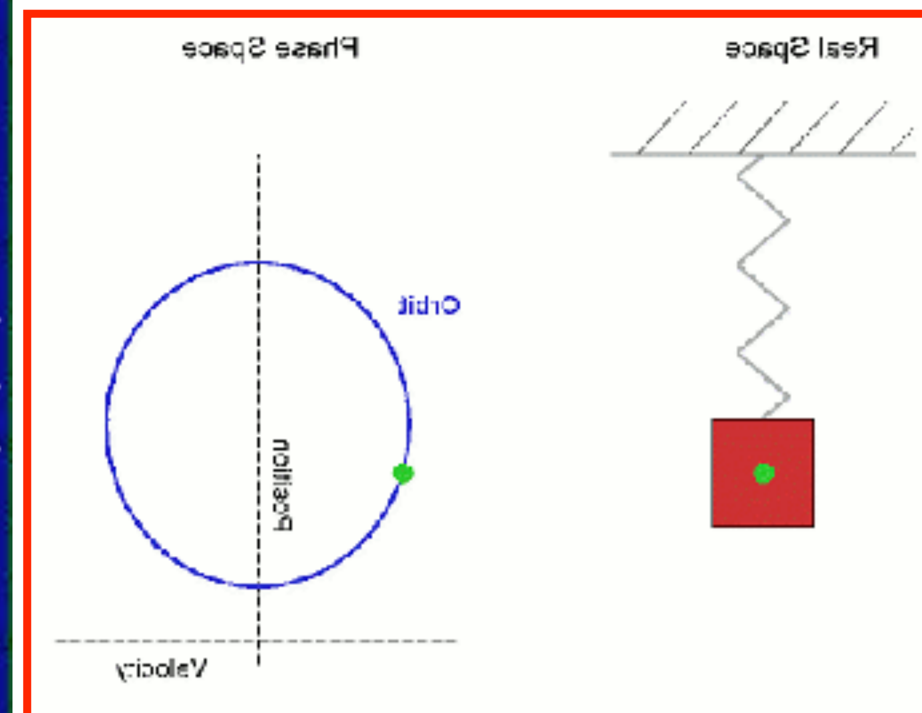
with Ludmila Praxmeyer

Coherent states

wave-nature



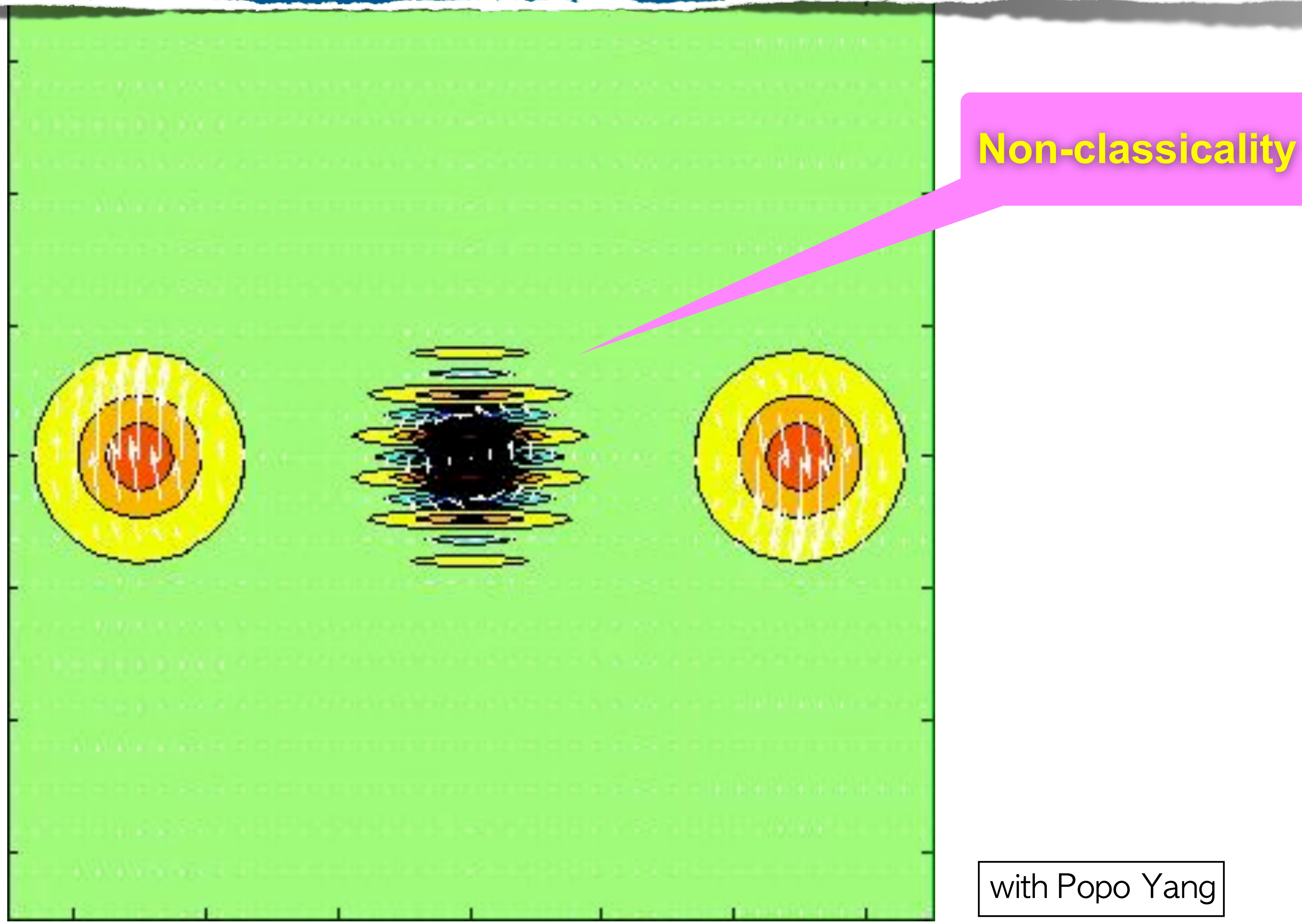




with Popo Yang

Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs, Physica Scripta for the New Focus issue: Quantum Optics and Beyond - in honour of Wolfgang Schleich.

Cat States in Phase Space



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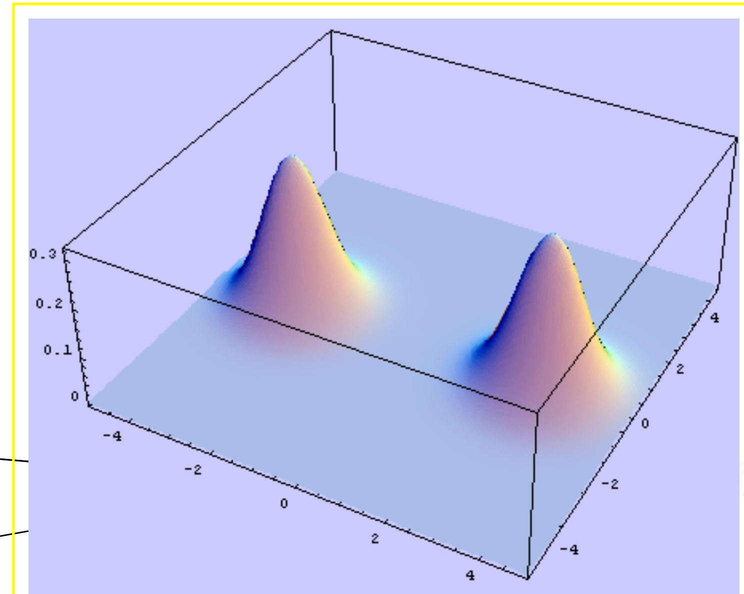
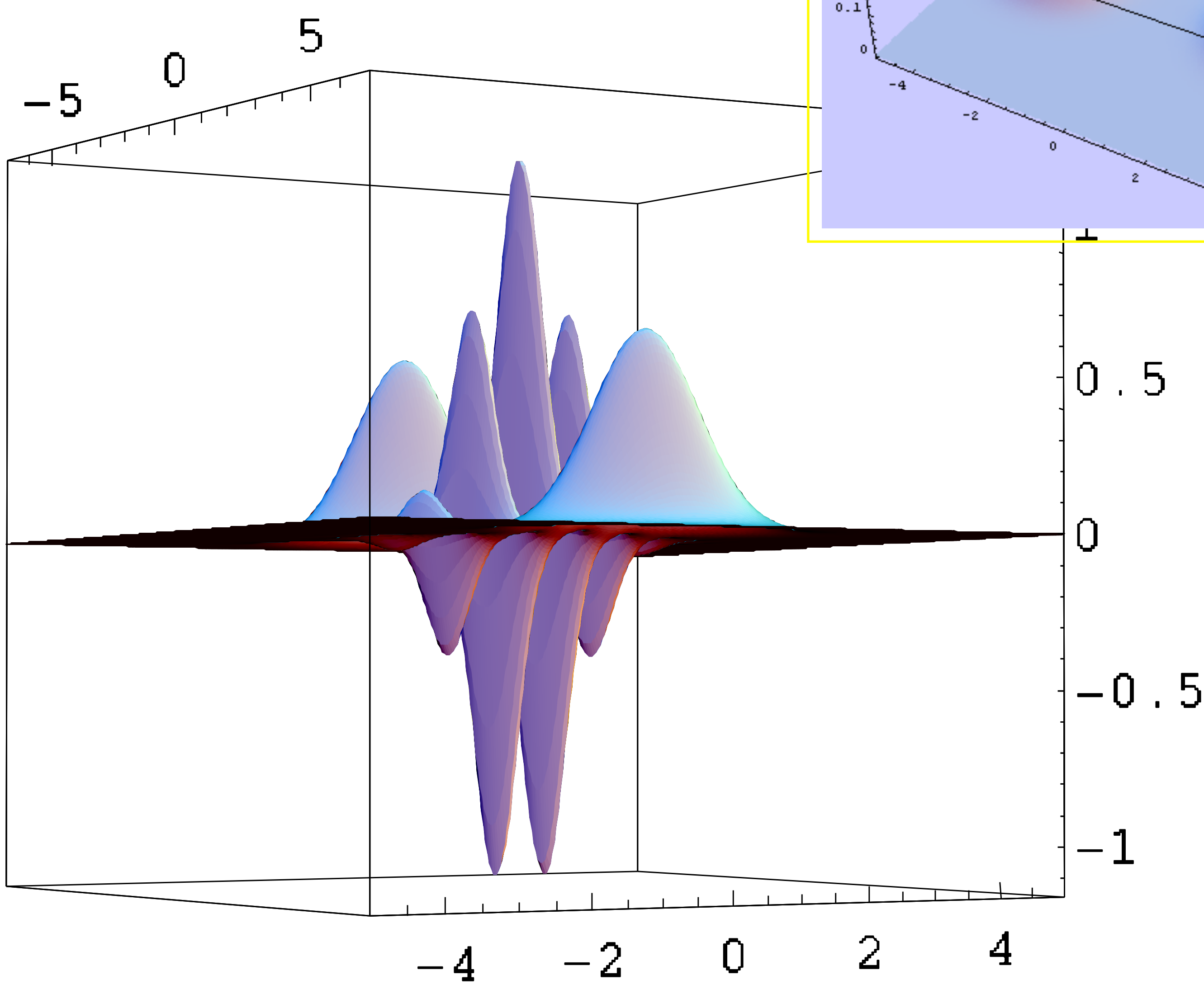
Cat States in Phase Space

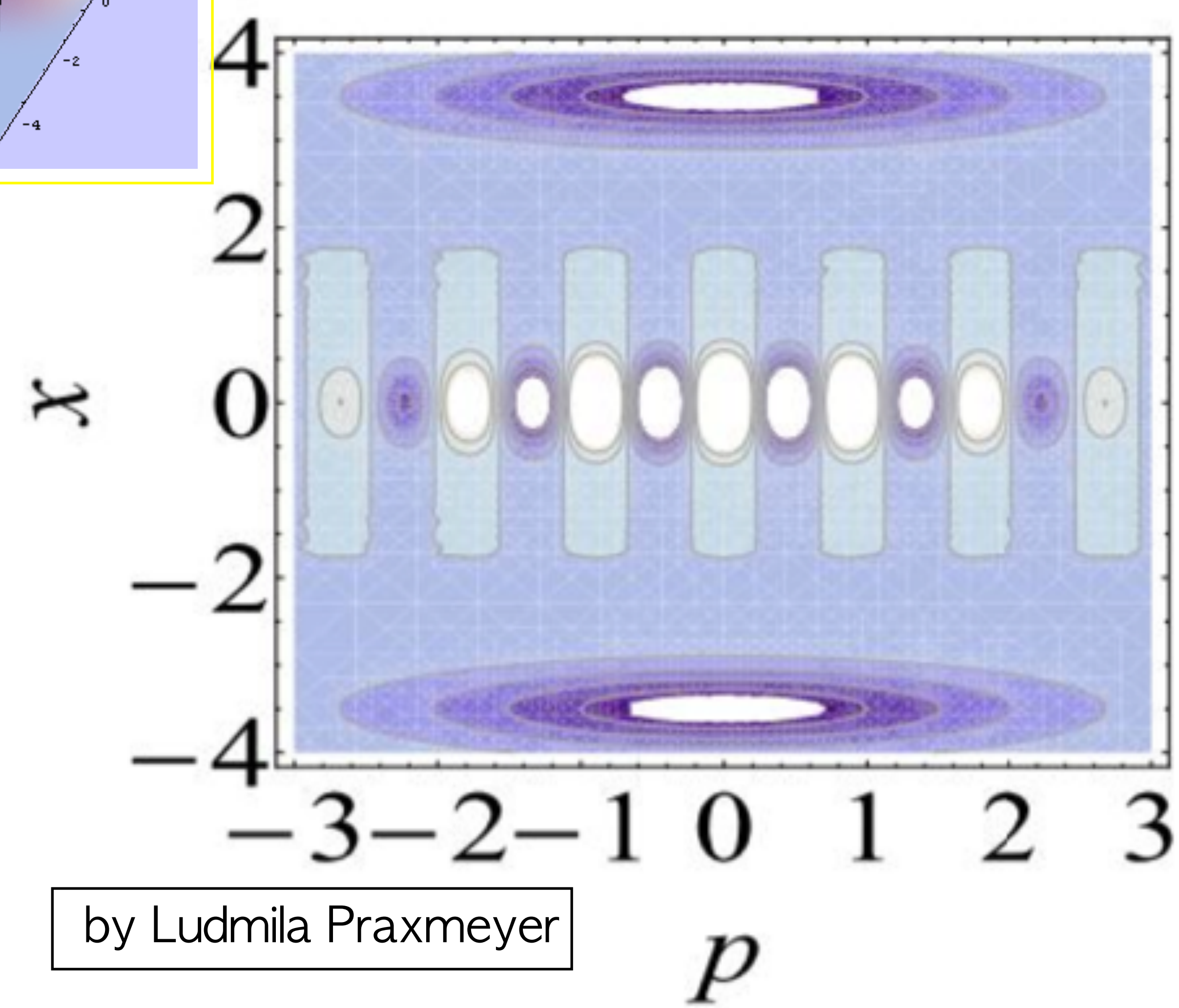
$$\psi = \mathcal{N}(|\alpha_o\rangle + |-\alpha_o\rangle)$$

SCHRÖDINGER'S CAT STATE



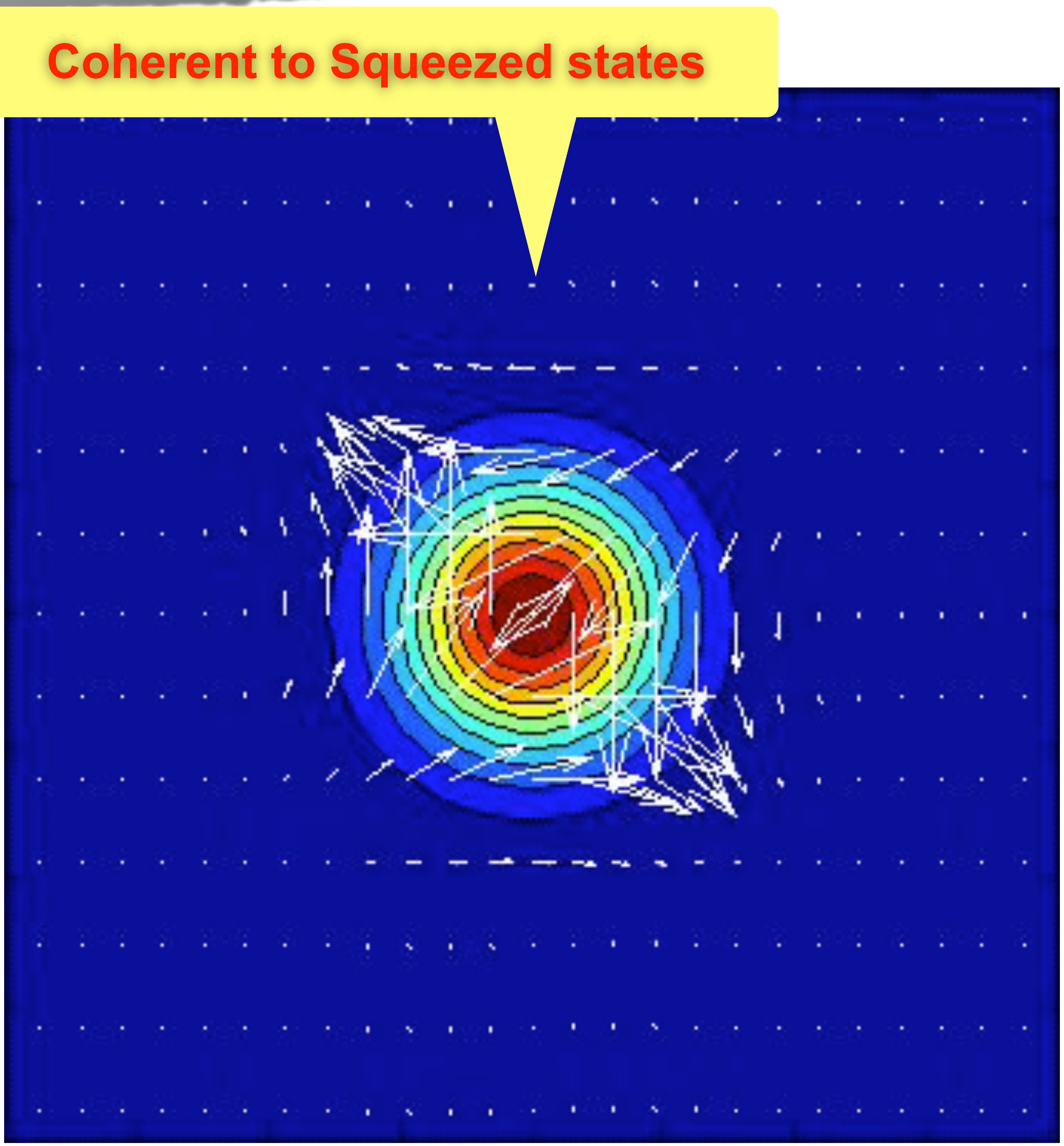
$$W_{\hat{\rho}}(\alpha) = \frac{2\mathcal{N}}{\pi}\Big[e^{-2|\alpha-\alpha_o|^2} + e^{-2|\alpha+\alpha_o|^2} + +2e^{-2|\alpha|^2}\cos\left(4(\mathrm{Re}\alpha\mathrm{Im}\alpha_o - \mathrm{Im}\alpha\mathrm{Re}\alpha_o)\right)\Big]$$

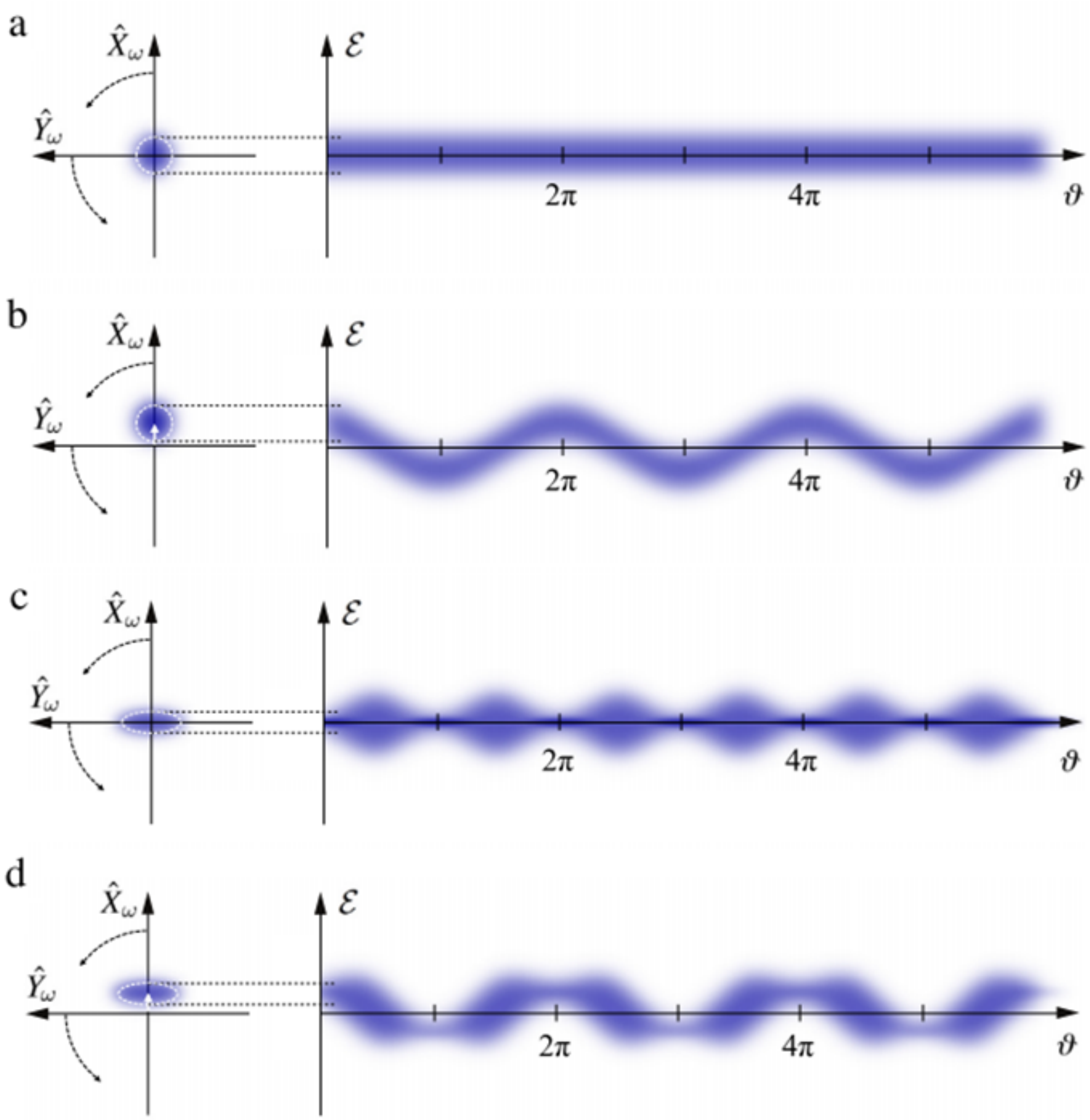




by Ludmila Praxmeyer

Squeezed States

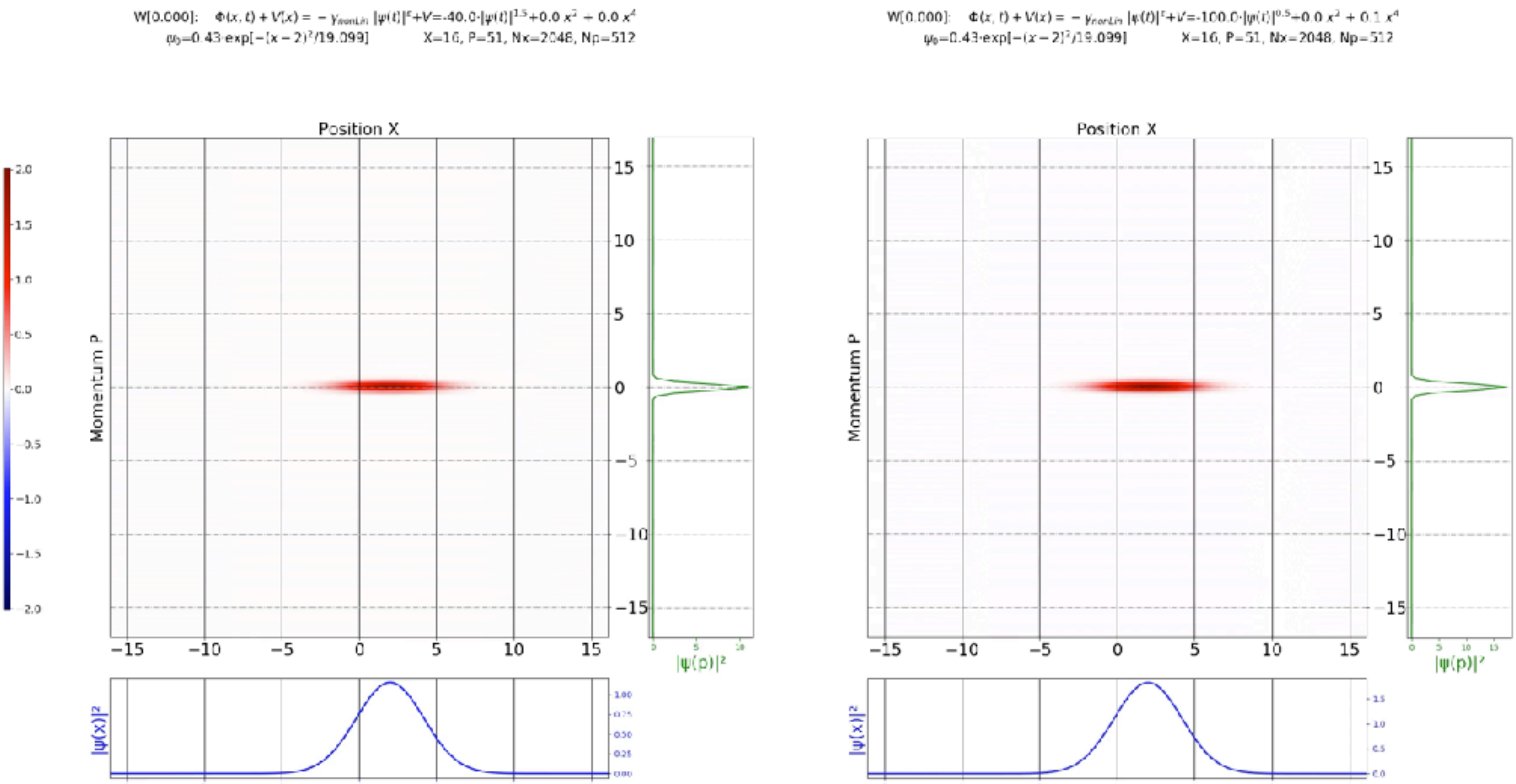




Courtesy:
Roman Schnabel (2017).

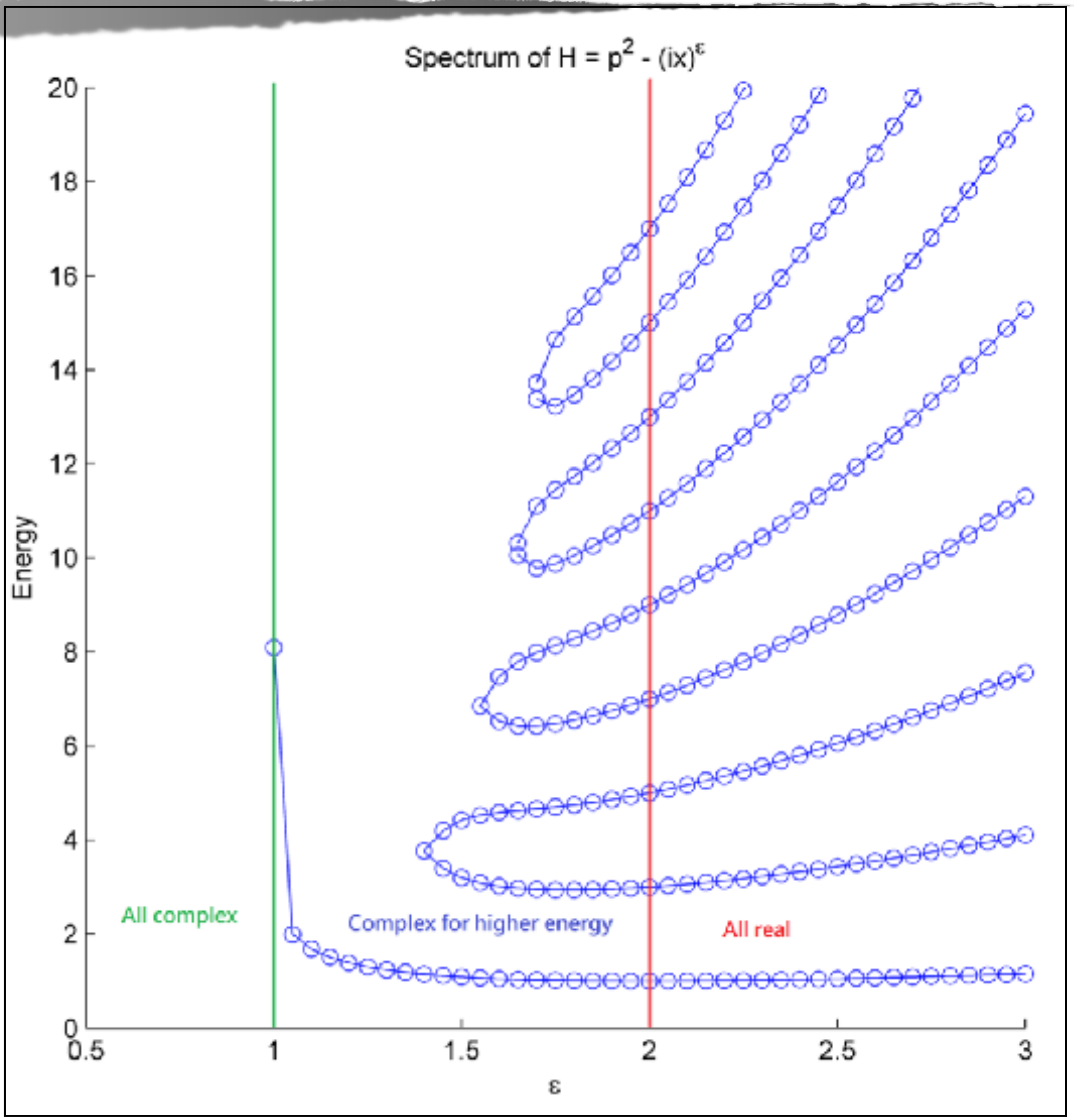
by Popo Yang

Quantum Optics in Phase Space

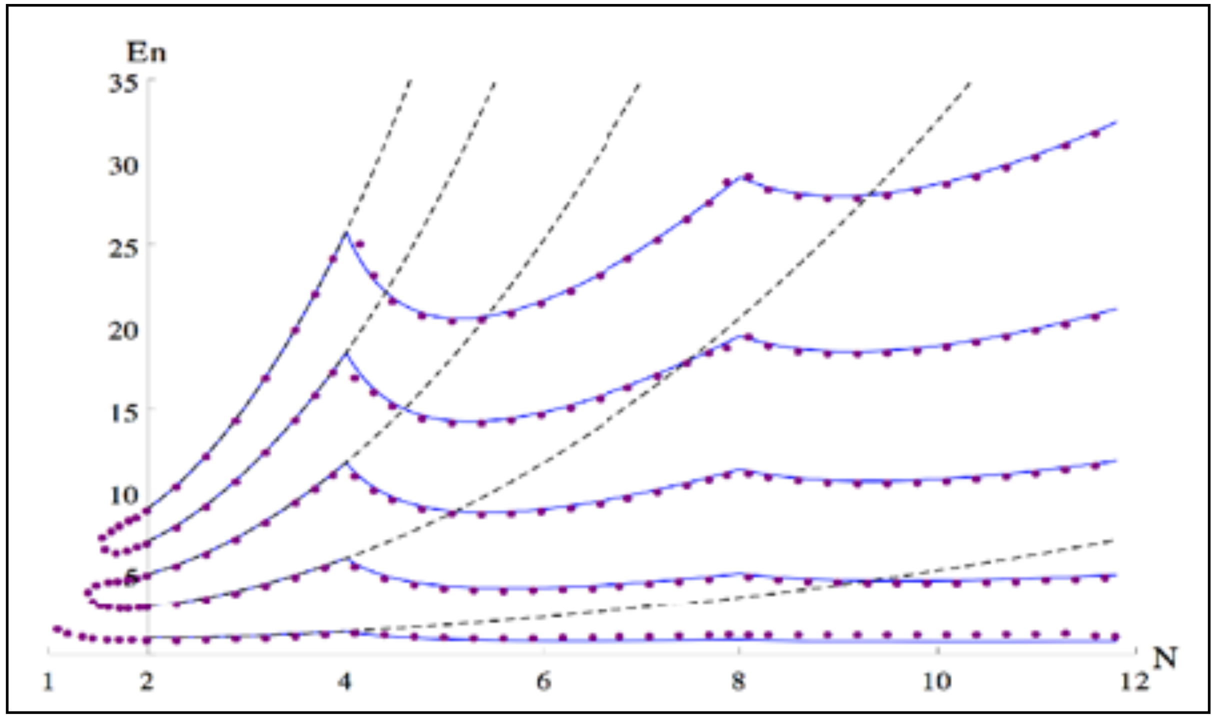


with Popo Yang,
Ole Steuernagel (U. Hertfordshire, UK)

Real Spectrum in $\mathcal{PT}$ Hamiltonian



$$\mathcal{H} = p^2 + x^2\,(ix)^\epsilon$$



$$\mathcal{H} = p^2 - (ix)^\epsilon$$

- Consider a family of differential equations parameterized by a continuous parameter $\epsilon > 0$ in the form:

$$\frac{\partial^2 \psi(x)}{\partial x^2} + V_\epsilon(x)\psi(x) + 2E\psi(x) = 0, \qquad (1)$$

- Here, let us specify the definition of $V_\epsilon(x) = -(ix)^\epsilon$, by stating explicitly the branch of logarithm :

$$V_\epsilon(x) = -(ix)^\epsilon = e^{\epsilon \log(ix)} = \begin{cases} -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) + i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x > 0; \\ 0, & \text{for } x = 0; \\ -|x|^\epsilon \left[\cos(\epsilon\frac{\pi}{2}) - i\sin(\epsilon\frac{\pi}{2})\right], & \text{for } x < 0\,. \end{cases} \qquad (2)$$

- In a Fock state basis, an analytical formula for the matrix element $a_{nm}(\epsilon) = \langle m|\mathrm{H}_\epsilon|n\rangle$ of $\mathrm{H}_\epsilon = \frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{V_\epsilon(x)}{2}$ can be constructed for any natural number n, m and positive ϵ:

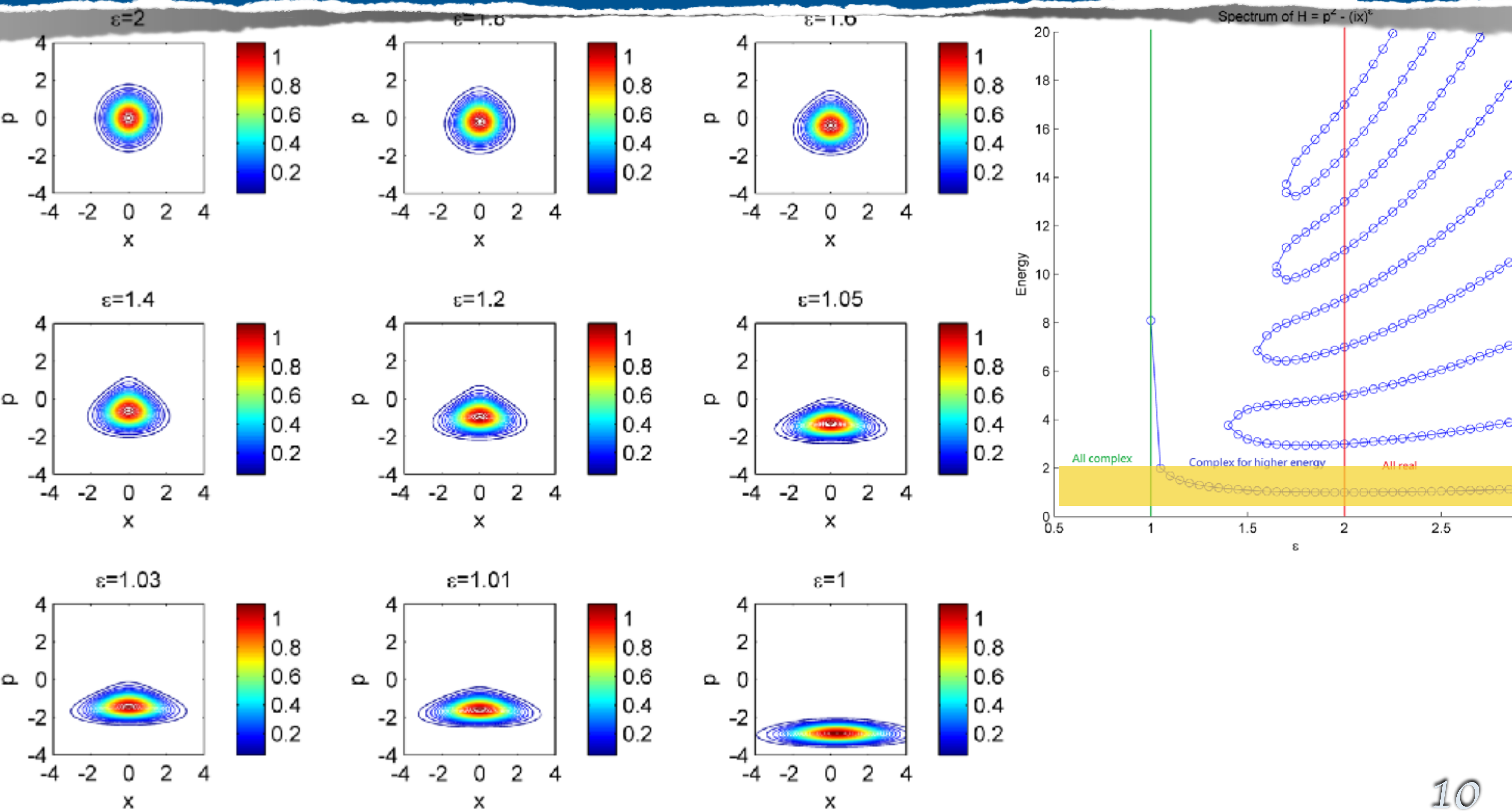
L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).

$$\begin{aligned} a_{nm}(\epsilon) \;=\;& \frac{\sqrt{n(n-1)}}{4}\,\delta_{m,n-2} + \frac{\sqrt{(n+1)(n+2)}}{4}\,\delta_{m,n+2} - \frac{2n+1}{4}\,\delta_{m,n} + \\ &+ \left[\frac{1-(-1)^{\widetilde{n}+\widetilde{m}}}{4}\cos(\epsilon\tfrac{\pi}{2}) + \frac{1+(-1)^{\widetilde{n}+\widetilde{m}}}{4}\, i\sin(\epsilon\tfrac{\pi}{2})\right] \frac{(-1)^{\lfloor\frac{n}{2}\rfloor+\lfloor\frac{m}{2}\rfloor} 2^{\widetilde{n}+\widetilde{m}} n!m!}{\lfloor\frac{n}{2}\rfloor!\,\lfloor\frac{m}{2}\rfloor!} \times \\ &\times \Gamma\left(\tfrac{1+\epsilon+\widetilde{n}+\widetilde{m}}{2}\right) F_A\left(\tfrac{1+\epsilon+\widetilde{n}+\widetilde{m}}{2}; -\lfloor\tfrac{n}{2}\rfloor, -\lfloor\tfrac{m}{2}\rfloor; \tfrac{2\widetilde{n}+1}{2}, \tfrac{2\widetilde{m}+1}{2}; 1, 1\right)\delta_{m,n} \end{aligned} \qquad (3)$$

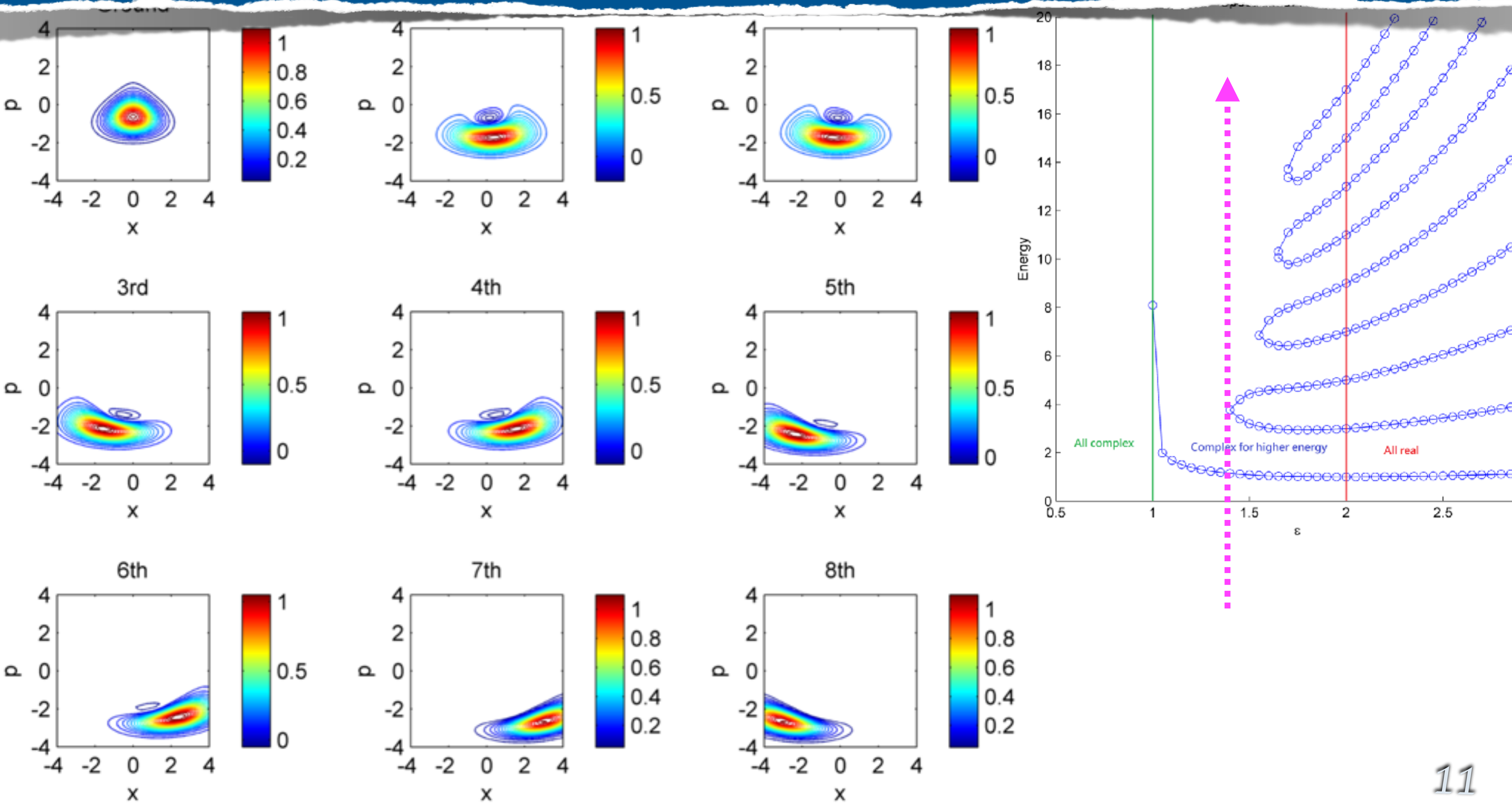
where Γ is an Euler gamma function; F_A is a Lauricella hypergeometric function; symbol $\lfloor\ \rfloor$ denotes a floor function: $\lfloor k\rfloor$ is the largest integer not greater than k; character tilde $\widetilde{\ }$ denotes a binary parity function: $\widetilde{k}$ is 0 for an even k and 1 for an odd k.

- By using truncated Fock state basis, we diagonalize the matrix $M_{nn}(\epsilon)$ numerically, having truncated the basis to the first 31, 51, or 71 elements.

$\mathcal{PT}$ in Phase space: Ground states



$\mathcal{PT}$ in Phase space: at the Exceptional Point



Phase space: Wigner flow

- The time evolution of Wigner distribution can be cast in the form of a flow field $J(x, p; t)$ describes the flow of Wigner's quasiprobability density

  $J_x = \frac{p}{m} W(x, p, t)$

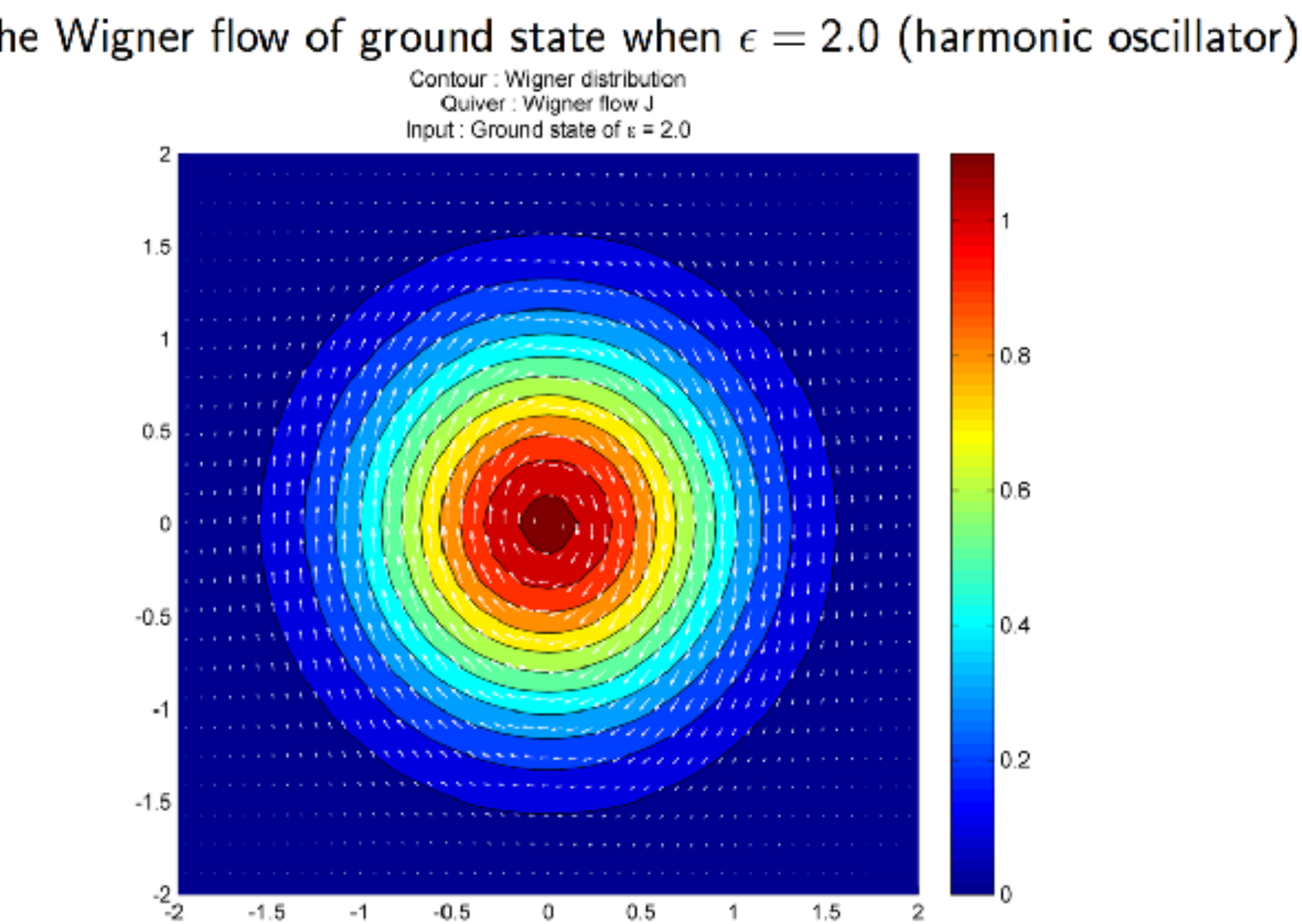
  $J_p = \int d\xi e^{\frac{i\xi p}{\hbar}} \Psi^*(x + \frac{\xi}{2}, t)\Psi(x - \frac{\xi}{2}, t)[\frac{V(x-\frac{\xi}{2}) - V(x)}{\xi} - \frac{V^*(x+\frac{\xi}{2}) - V^*(x)}{\xi}]$
- Continuity equation for Hermitian Hamiltonian

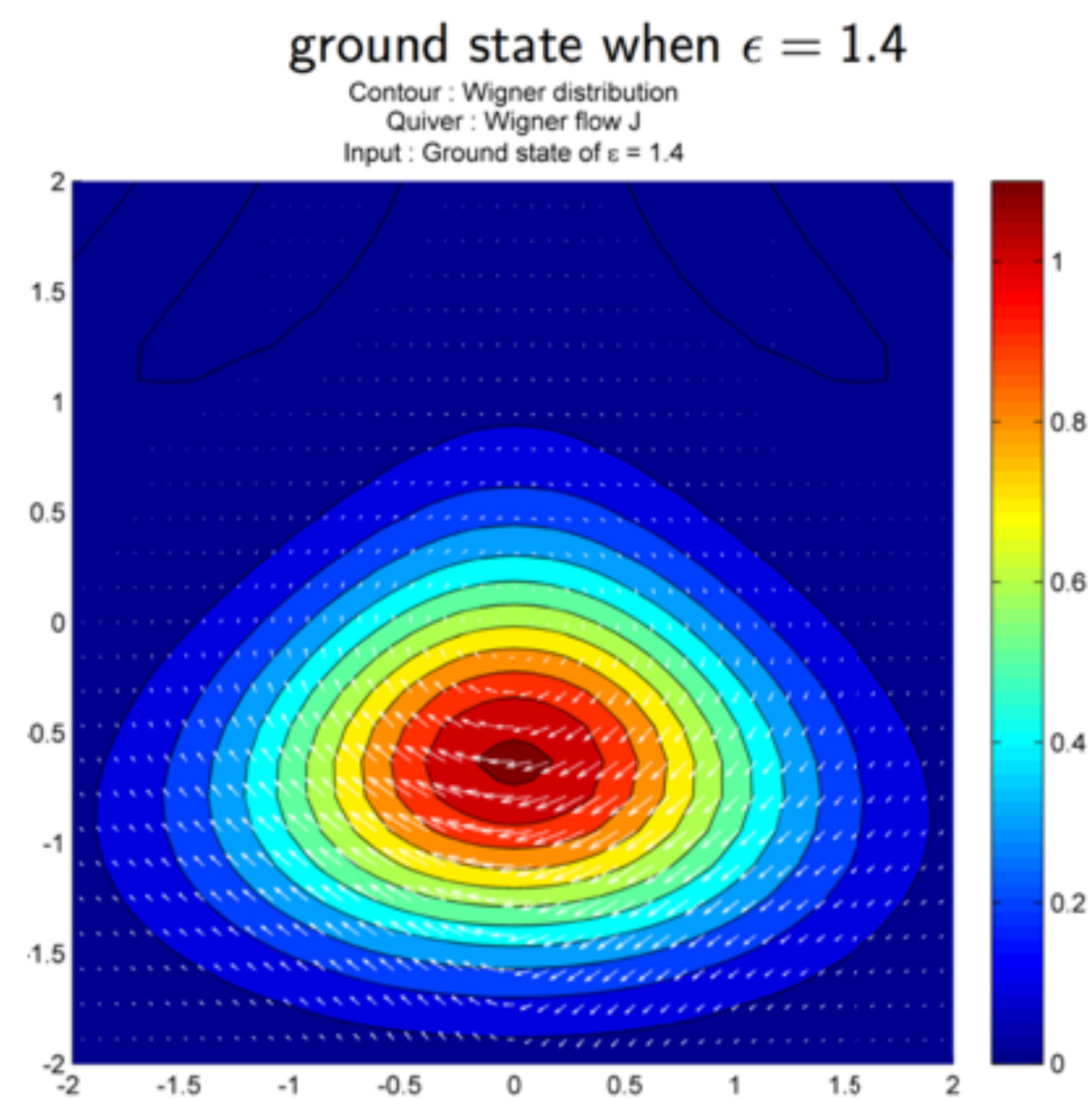
  $\frac{\partial}{\partial t} W(x, p; t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = 0$
- Continuity equation for Hermitian non-Hamiltonian

  $\frac{\partial}{\partial t} W(x, p, t) + \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial p} J_p = \frac{i}{\hbar}[V^*(x, t) - V(x, t)] W(x, p, t)$

O. Steuernagel, D. Kakofengitis, and G. Ritter, Phys. Rev. Lett. 110, 030401 (2013).

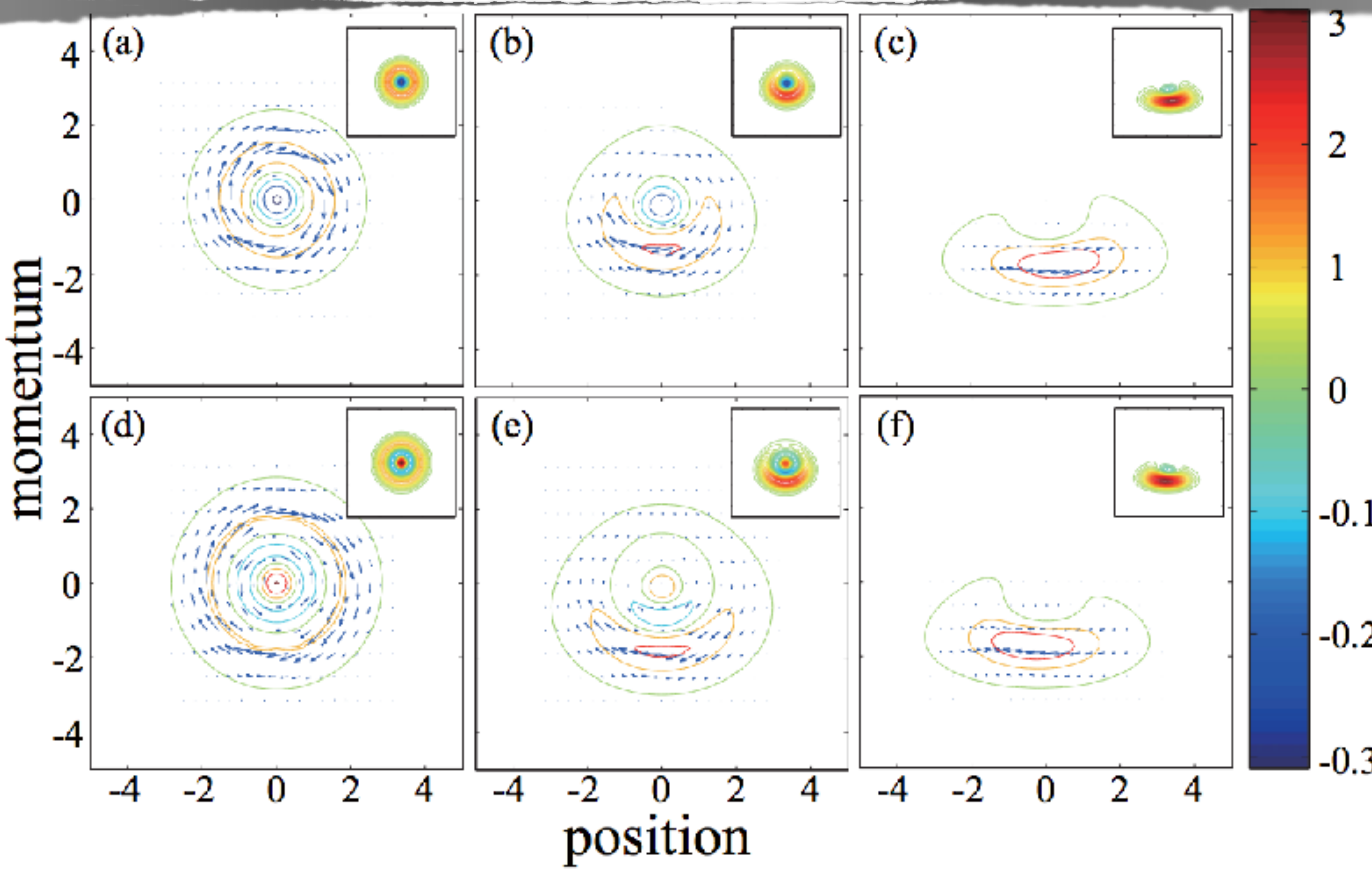
Wigner flow of $\mathcal{PT}$: Ground states





L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).

Wigner flow of $\mathcal{PT}$: 1st/2nd Excited states



L. Praxmey, Popo Yang, and RKL, Phys. Rev. A 93, 042122 (2016).