Note: Quantum SHO

- ☑ **Quantum Simple Harmonic Oscillator, qSHO**

• Photons occupy an electromagnetic mode (referred as the modes in quantum optics, typically a plane wave)

- ☑ **Hamiltonian**
- ☑ **Number operator**
- ☑ **Energy Quantization (equally spacing in energy)**

•The energy in a mode is not continuous but discrete in quanta.

- ☑ **Vacuum state with zero-point energy**

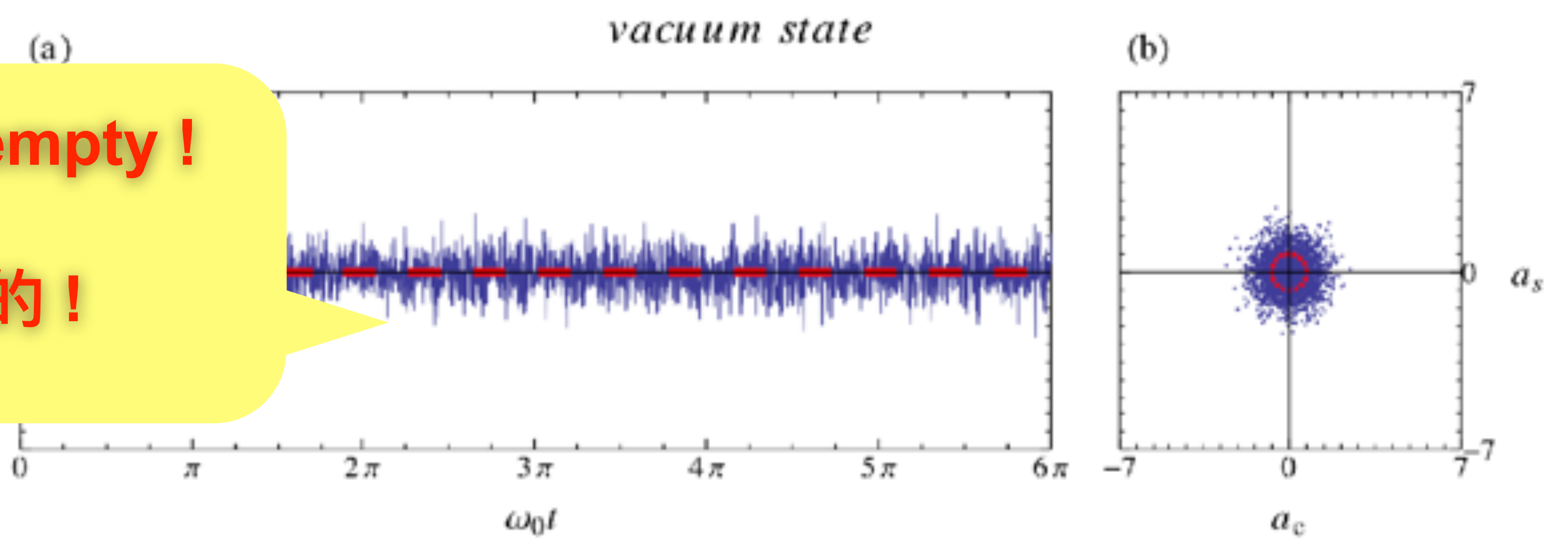
•There is a zero point energy inherent to each mode, which is equivalent with fluctuations of the electromagnetic field in vacuum, due to the uncertainty principle.

- ☑ **Schrodinger picture**
- ☑ **Heisenberg picture**

•The observables are just represented by probabilities as usual in QM.

Vacuum is NOT empty !

真空並不是空的！



Vacuum, which is not just nothing, it is full of energy.

- □ Spontaneous emission: timulated by the vacuum fluctuation
- □ Purcell effect: modify vacuum fluctuations by resonators
- □ The electron does not crash into the core in the atomic structure, due to vacuum fluctuation of the electromagnetic field.
- □ Casimir effect: two charged metal plates repel, or attract, each other due to the potential force induced by the vacuum.
- □ Lamb shift: the energy level difference between $2S_{1/2}$ and $2P_{1/2}$ in hydrogen.
- □ Gravity is not a fundamental force but a side effect matter modifies the vacuum fluctuations, by Sakharov.
- □ Dark energy:
- □ Comic inflation:

What is Quantum !?

Quantum Mechanics

Non-Classicality

Bell's inequality
Non-locality

Energy Quanta

Schrödinger's Cat
Realism

Steering

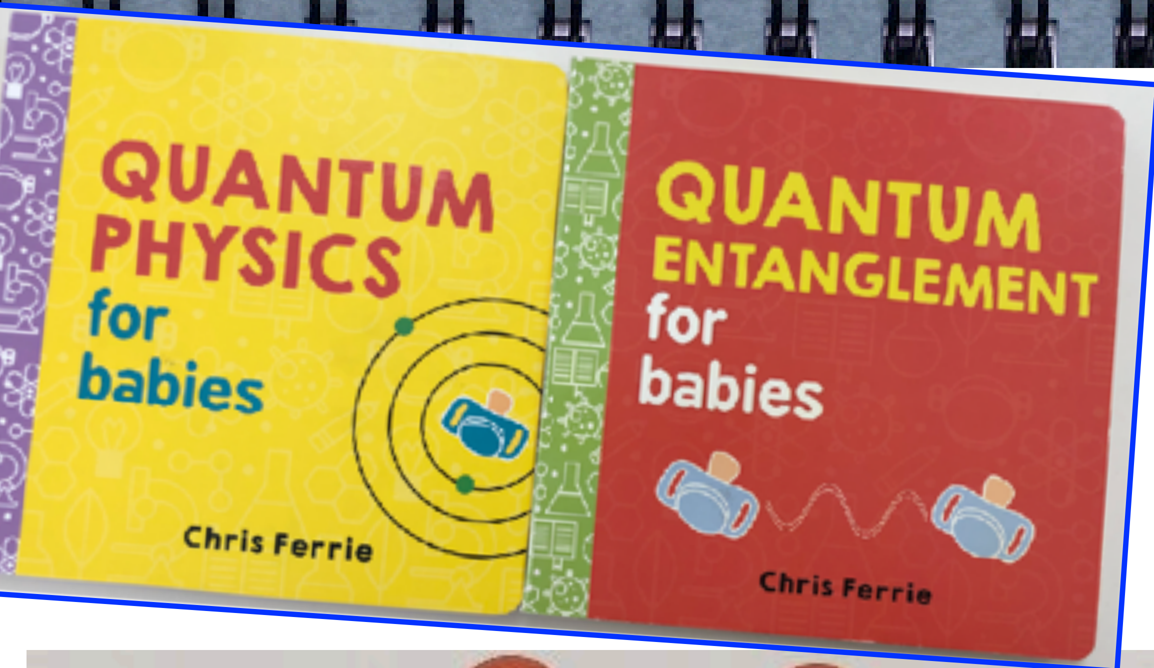
Entanglement
(EPR-pair)

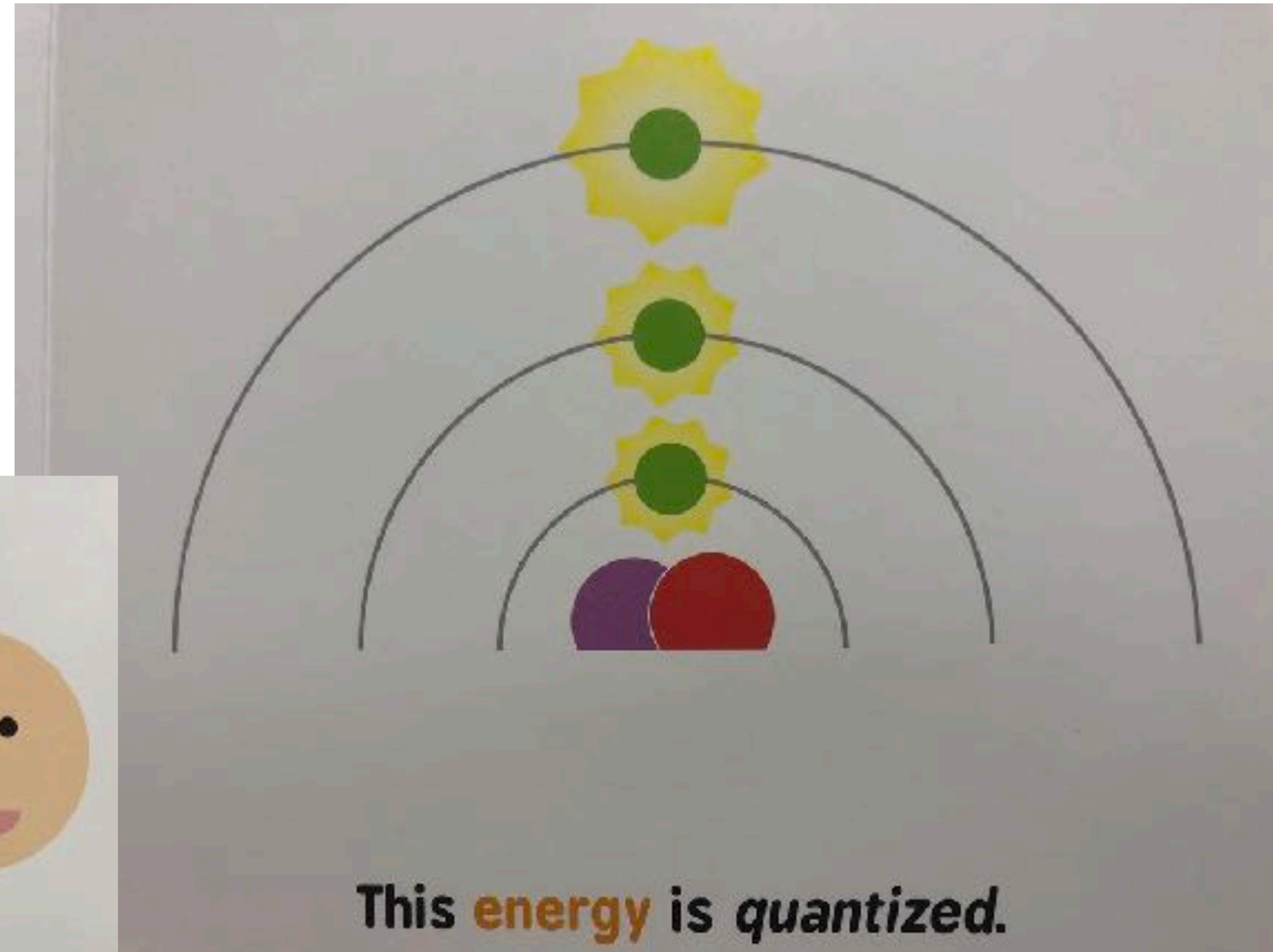
Wavefunction

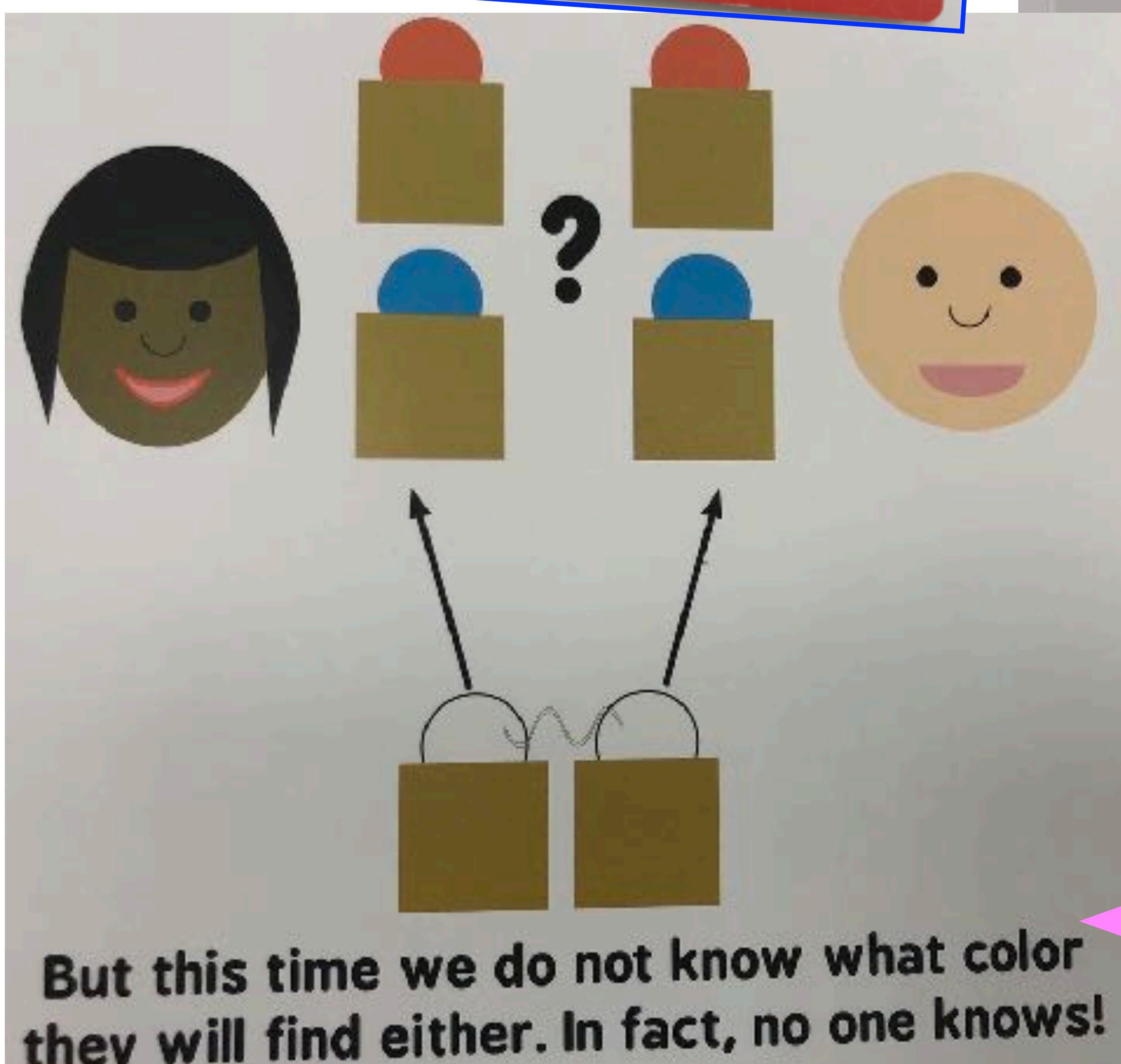
Quantum Optics, Quantum Technologies,
Q-Sensors, Q-Simulators, Q-Communication, Q-Computers !!

There are MANY aspects of Quantum !

Mark Zuckerberg is already reading his baby books about quantum physics







Entanglement for Baby !

Syllabus:

Date	Topic	To Know	To Think
week 1 (3/2, 3/9)	Quantum SHO	☐ Fock states, $\|n\rangle$ ☐ creation operator, $\hat{a}^\dagger$	☐ single-photon detection ☐ Wave-Particle Duality ☐ photon-number resolving ☐
		☐ Vacuum state ☐ Quantum Fluctuations	☐ Shot Noise Limit ☐ Casimir Force ☐
week 2 (3/12, 3/16, ~~3/19~~)	Quantum Mechanics	☐ Schrödinger picture ☐ Heisenberg picture ☐ Interaction picture	☐ Uncertainty Relation ☐ Probability Interpretation ☐ Measurement problem ☐ Non-locality ☐ Macrorealism ☐

From Scratch !!

- How much do you known about Quantum Mechanics ?

Anyone who is not shocked by quantum theory has not understood it.

Focus: What's Wrong with Quantum Mechanics?

September 23, 2005 • *Phys. Rev. Focus* 16, 10

In 1935 Einstein and his co-authors claimed to show that quantum mechanics led to logical contradictions. The objections exposed the theory's strangest predictions.

P. Ehrenfest, courtesy AIP Emilio Segrè Visual Archives

Quantum Opponents. In 1935 Albert Einstein and two colleagues published an attack on the new quantum mechanics, saying that it led to contradictions. Niels Bohr (left) defended the theory.

Print

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky, and N. Rosen

Phys. Rev. 47, 777 (1935)

Published May 15, 1935

Note: Quantum Mechanics

- □ Axioms
- □ State
- □ Operator
- □ Density Matrix

- More on States
  - □ Coherent States
  - □ Squeezed States
  - □ Uncertainty Relation ➡ Minimum Uncertainty States
  - □ Entropy
  - □ Purity
  - □ bi-particle States ➡ Entanglement (Schmidt decomposition)
  - □ Cat states

Axioms of Quantum Mechanics

1. **State:** The properties of a quantum system are completely defined by specification of its state vector $|\Psi\rangle$. The state vector is an element of a complex Hilbert space $\mathcal{H}$ called the space of states.

2. **Observable:** With every physical property $\hat{A}$ (energy, position, momentum, angular momentum, ...) there exists an associated linear, *Hermitian operator* $\hat{A}$ (usually called observable), which acts in the space of states $\mathcal{H}$. The eigenvalues of the operator are the possible values of the physical properties.

3. **Probability:**

   (a) If $|\Psi\rangle$ is the vector representing the state of a system and if $|\Phi\rangle$represents another physical state, there exists a probability $p(|\Psi\rangle, |\Phi\rangle)$ of finding $|\Psi\rangle$ in state $|\Phi\rangle$, which is given by the squared modulus of the scalar product on $\mathcal{H}$: : $p(|\Psi\rangle, |\Phi\rangle) = |\langle\Psi|\Phi\rangle|^2$ (Born Rule).

   (b) If $\mathcal{A}$ is an observable with eigenvalues a_k and eigenvectors $|k\rangle$, $\hat{A}|k\rangle = a_k|k\rangle$, given a system in the state $|\Psi\rangle$, the probability of obtaining a_k as the outcome of the measurement of $\hat{A}$ is $p(a_k) = |\langle k|\Psi\rangle|$. After the measurement the system is left in the state projected on the subspace of the eigenvalue a_k (Wave function collapse).

4. **Time evolution:** The evolution of a closed system is *unitary.* The state vector $|\Psi(t)\rangle$ at time t is derived from the state vector $|\Psi(t_0)\rangle$ at time t_0 by applying a unitary operator $\hat{U}(t, t_0)$, called the evolution operator: $|\Psi(t)\rangle = \hat{U}(t, t_0)|\Psi(t_0)\rangle$.

Quantum State Tomography

Non-Hermitian QM

Quantum Measurement (weak measurement)

Decoherence

Arrow of Time

Entangled-History

PRL **112,** 130404 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Local $\mathcal{PT}$ Symmetry Violates the No-Signaling Principle

Yi-Chan Lee,[1,2,*] Min-Hsiu Hsieh,[2] Steven T. Flammia,[3] and Ray-Kuang Lee[1,4]

Physics
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Synopsis: Reflecting on an Alternative Quantum Theory

APS/Alan Stonebraker

Local PT Symmetry Violates the No-Signaling Principle
Yi-Chan Lee, Min-Hsiu Hsieh, Steven T. Flammia, and Ray-Kuang Lee
Phys. Rev. Lett. **112**, 130404 (2014)
Published April 3, 2014

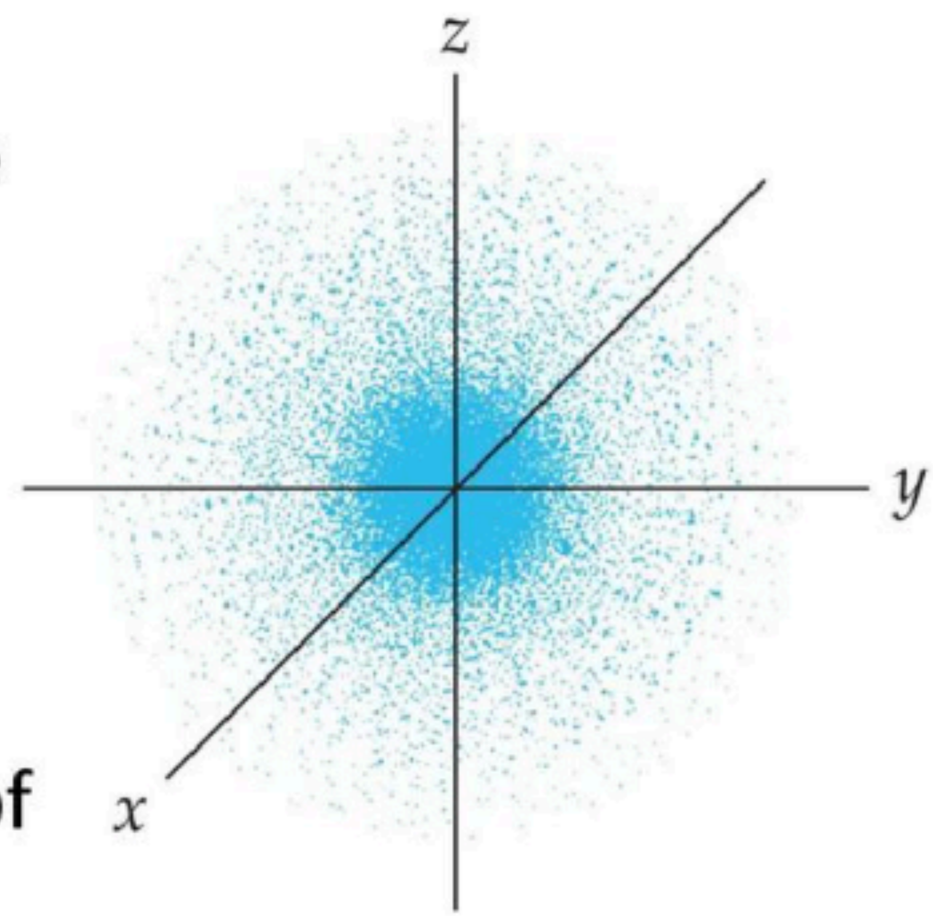
PHYSICAL REVIEW LETTERS **123,** 080404 (2019)

Simulating Broken $\mathcal{PT}$-Symmetric Hamiltonian Systems by Weak Measurement

Minyi Huang,[1,*] Ray-Kuang Lee,[2,3,4,†] Lijian Zhang,[5,‡] Shao-Ming Fei,[6,7,§] and Junde Wu[1,‖]

Quantum Mechanics: States

- The wave equation is designated with a lower case Greek *psi* (ψ).
- The square of the wave equation, ψ^2, gives a probability density map of where an electron has a certain statistical likelihood of being at any given instant in time.



- Pure state
- Mixed state

- State properties:
  1. quantum state: $|\Psi\rangle = \sum_i \alpha_i |\psi_i\rangle$,
  2. completeness: $\sum_i |\psi_i\rangle\langle\psi_i| = I$, or $\int dx |x\rangle\langle x|$.
  3. probability interpretation (projection): $\Psi(x) = \langle x|\Psi\rangle$,

Quantum Mechanics: Operators

- Operators:
  1. operator: $\hat{A}|\Psi\rangle = |\Phi\rangle$,
  2. representation: $\langle\phi|\hat{A}|\psi\rangle$,
  3. adjoint of $\hat{A}$: $\langle\phi|\hat{A}|\psi\rangle = \langle\psi|\hat{A}^\dagger|\phi\rangle^*$,
  4. Hermitian operator: $\hat{H} = \hat{H}^\dagger$, *self-adjoint.*
  5. unitary operator: $\hat{U}\hat{U}^\dagger = \hat{U}^\dagger\hat{U} = I$.
  6. $\hat{U}$ can be represented as $\hat{U} = \exp(i\hat{H})$ if $\hat{H}$ is Hermitian.
  7. normal operator: $[\hat{A}, \hat{A}^\dagger] = 0$, the eigenstates of only a normal operator are *orthonormal.*
  8. hermitian and unitary operators are normal operators.
  9. The sum of the diagonal elements $\langle\phi|\hat{A}|\psi\rangle$ is call the *trace* of $\hat{A}$,

     $$\mathrm{Tr}(\hat{A}) = \sum_i \langle\phi_i|\hat{A}|\phi_i\rangle.$$

     The value of the trace of an operator is independent of the basis.
  10. The eigenvalues of a hermitian operator are real, $\hat{H}|\Psi\rangle = \lambda|\Psi\rangle$, where λ is real.

- Operator <=> Matrix
- Hermitian operator
- unitary operator

Quantum Mechanics: Momentum Operator

For an *infinitesimal* translation, we have

$$\hat{\mathcal{T}}(dx)|x\rangle = |x+dx\rangle,$$

$$\boxed{[\hat{x},\hat{p}] = i\hbar.}$$

We use $|x\rangle$ and $|p\rangle$ for the states representations in the position and momentum spaces, respectively,

$$\begin{aligned}\hat{x}|x\rangle &= x|x\rangle,\\ \hat{p}|p\rangle &= p|p\rangle.\end{aligned}$$

The transformation between these two spaces is

$$\begin{aligned}\hat{p}|x\rangle &= -i\hbar\frac{\partial}{\partial x}|x\rangle,\\ \hat{x}|p\rangle &= i\hbar\frac{\partial}{\partial p}|p\rangle,\\ \langle x|p\rangle &= \frac{1}{\sqrt{2\pi\hbar}}\exp(\frac{i\,p\,x}{\hbar}).\end{aligned}$$

There is a *Fourier transform* between two representations:

$$\begin{aligned}\psi(x) &= \langle x|\phi\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int dp\exp(\frac{i\,p\,x}{\hbar})\Psi(p),\\ \Psi(p) &= \langle p|\phi\rangle = \frac{1}{\sqrt{2\pi\hbar}}\int dx\exp(\frac{-i\,p\,x}{\hbar})\psi(x).\end{aligned}$$

- position-momentum
- time-energy
- spin
- angular momentum
- $[a, a^+]$

- Uncertainty Relation

Quantum Mechanics: Measurement

- Measurement:

  1. Each act of measurement of an observable $\hat{A}$ of a system in state $|\Psi\rangle$ collapses the system to an eigenstate $|\psi_i\rangle$ of $\hat{A}$ with probability $|\langle\phi_i|\Psi\rangle|^2$.
  2. The average or the expectation value of $\hat{A}$ is given by

  $$\langle\hat{A}\rangle = \sum_i \lambda_i |\langle\phi_i|\Psi\rangle|^2 = \langle\Psi|\hat{A}|\Psi\rangle,$$

  where λ_i is the eigenvalue of $\hat{A}$ corresponding to the eigenstate $|\psi_i\rangle$.

- Expectation Value
- Variance
- Co-variance

However, for mixed states, there is no unique way of telling whether statistical fluctuations of observed quantities are caused

- by fluctuations in the state preparation (due to the lack of knowledge), or
- by fluctuations caused by the measurement process (due to the lack of complete control).

Quantum Mechanics: Desity Matrix/Operator

For the quantum mechanical description, if we know that the system is in state $|\psi\rangle$, then an operator $\hat{O}$ has the expectation value,

$$\langle \hat{O} \rangle_{\text{qm}} = \langle \psi | \hat{O} | \psi \rangle.$$

But, typically, we do not know that we are in state $|\psi\rangle$, then an *ensemble average* must be performed,

$$\langle \langle \hat{O} \rangle_{\text{qm}} \rangle_{\text{ensemble}} = \sum_n P_n \langle \psi_n | \hat{O} | \psi_n \rangle,$$

where the P_n is the probability of being in the state $|\psi_n\rangle$ and we introduce a density operator,

$$\hat{\rho} = \sum_n P_n |\psi_n\rangle \langle \psi_n|.$$

- rank-one matrix

The expectation value of any operator $\hat{O}$ is given by,

$$\langle \hat{O} \rangle_{\text{qm}} = \text{Tr}[\hat{\rho}\, \hat{O}],$$

where Tr stands for trace.

Can we see Quantum (state) ?

PHYSICS TODAY / APRIL 1985 PAG. 38-47

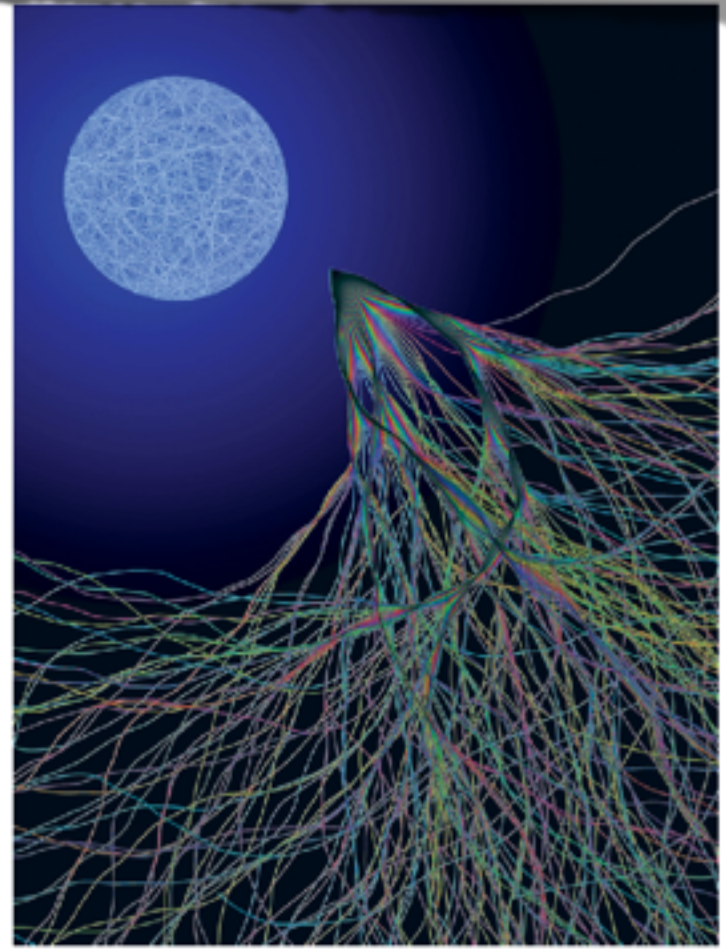
Is the moon there when nobody looks? Reality and the quantum theory

Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments have now shown that what bothered Einstein is not a debatable point but the observed behaviour of the real world.

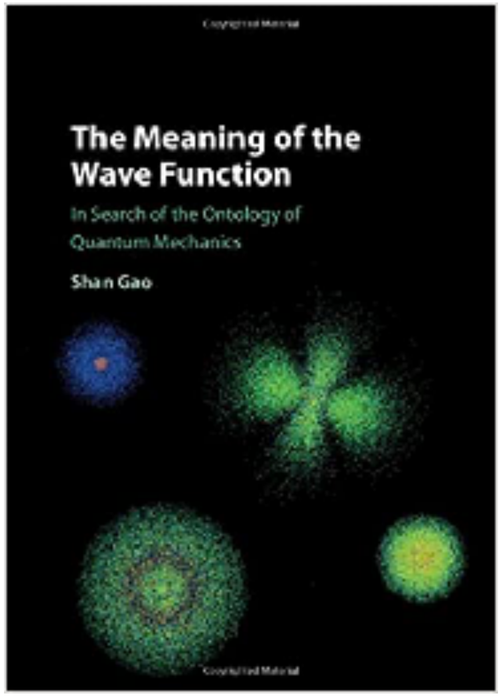
N. David Mermin

[David Mermin is director of the Laboratory of Atomic and Solid State Physics at Cornell University. A solid-state theorist, he has recently come up with some quasithoughts about quasicrystals. He is known to PHYSICS TODAY readers as the person who made "boojum" an internationally accepted scientific term. With N.W.Ashcroft, he is about to start updating the world's funniest solid-state physics text. He says he *is* bothered by Bell's theorem, but may have rocks in his head anyway.]

Quantum mechanics is magic[1]



Eric J. Heller



nature physics

ARTICLES

PUBLISHED ONLINE: 6 MAY 2012 | DOI: 10.1038/NPHYS2309

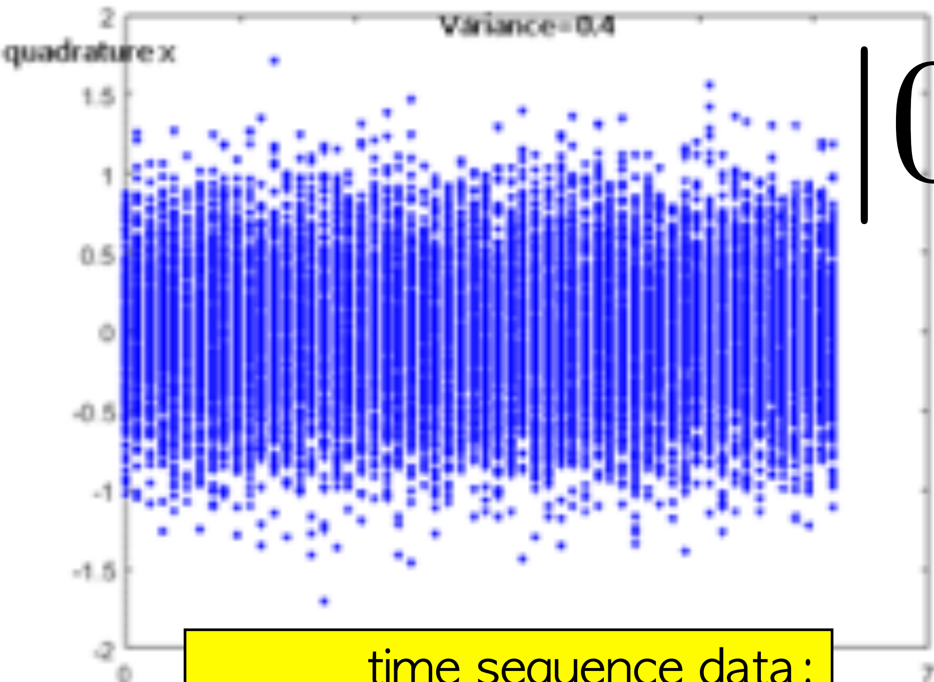
On the reality of the quantum state

Matthew F. Pusey[1]*, Jonathan Barrett[2] and Terry Rudolph[1]

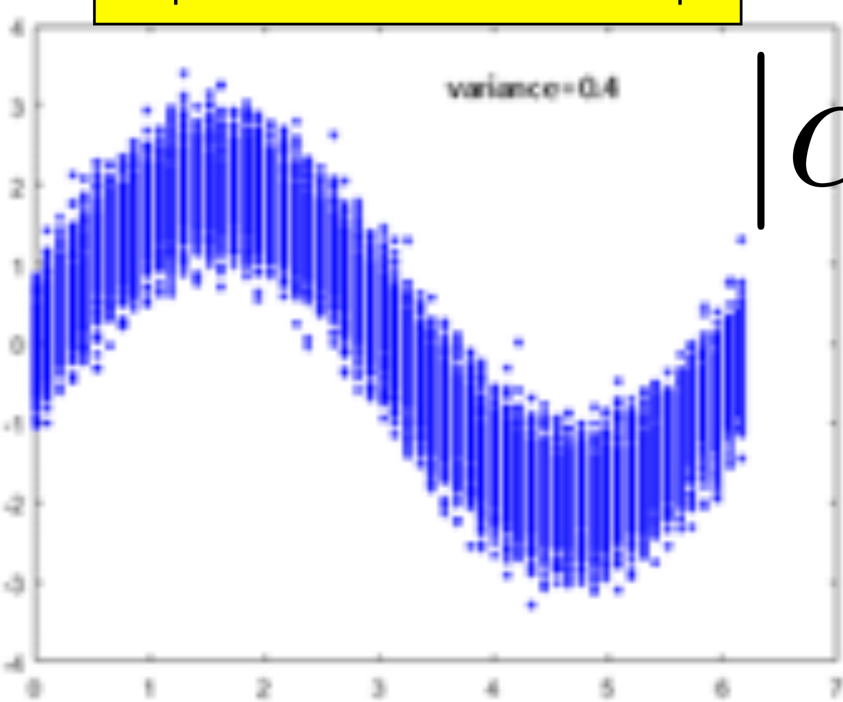
No-go theorem

Quantum states are the key mathematical objects in quantum theory. It is therefore surprising that physicists have been unable to agree on what a quantum state truly represents. One possibility is that a pure quantum state corresponds directly to reality. However, there is a long history of suggestions that a quantum state (even a pure state) represents only knowledge or information about some aspect of reality. Here we show that any model in which a quantum state represents mere information about an underlying physical state of the system, and in which systems that are prepared independently have independent physical states, must make predictions that contradict those of quantum theory.

Quantum State Tomography

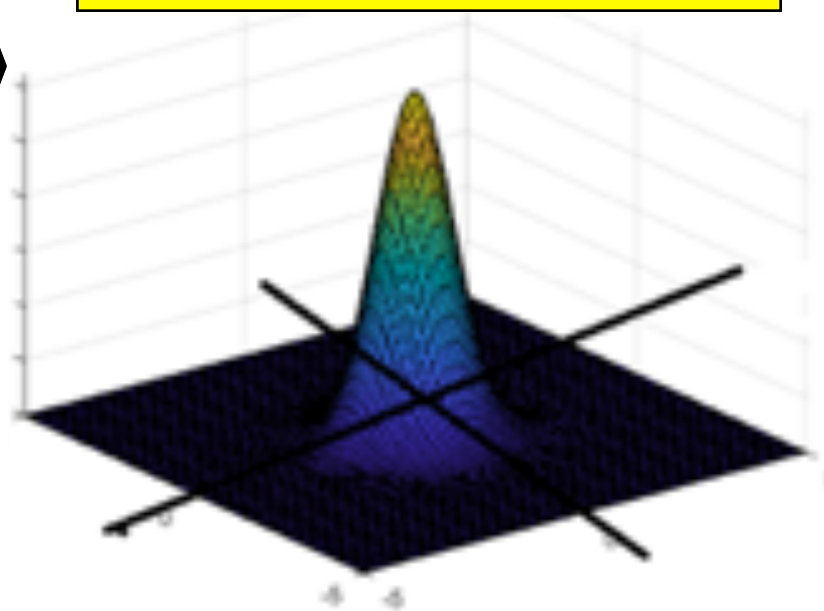


time sequence data:
Exp. data from Oscilloscope

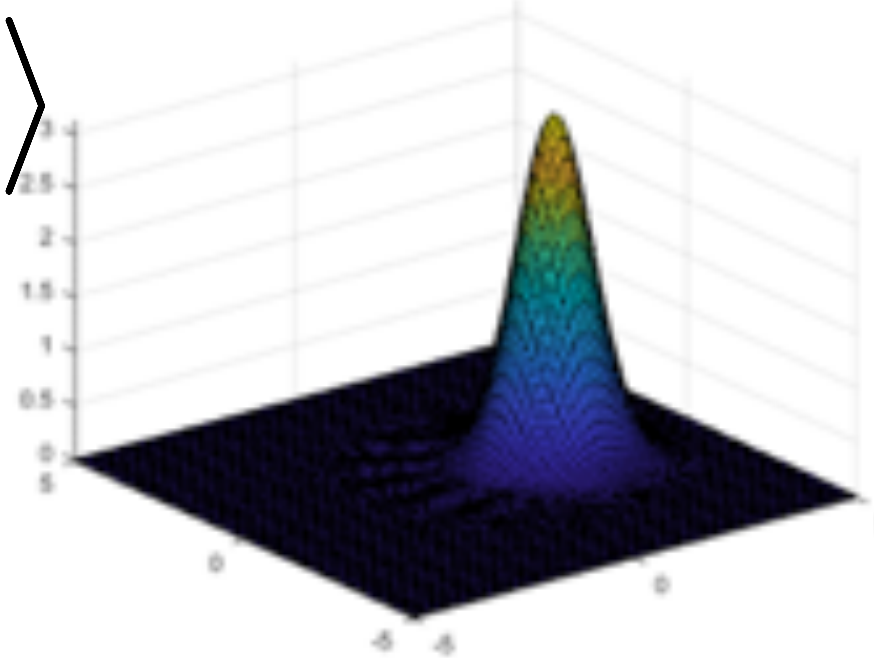


$|0\rangle$

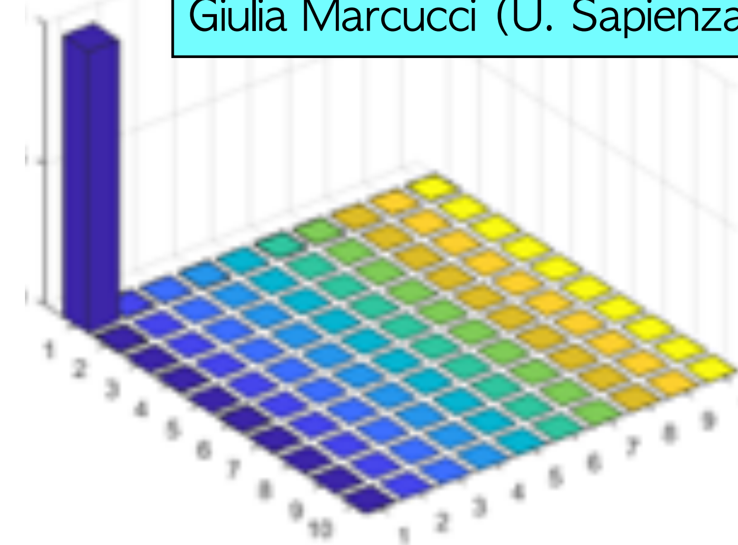
with Max. Likelihood estimation



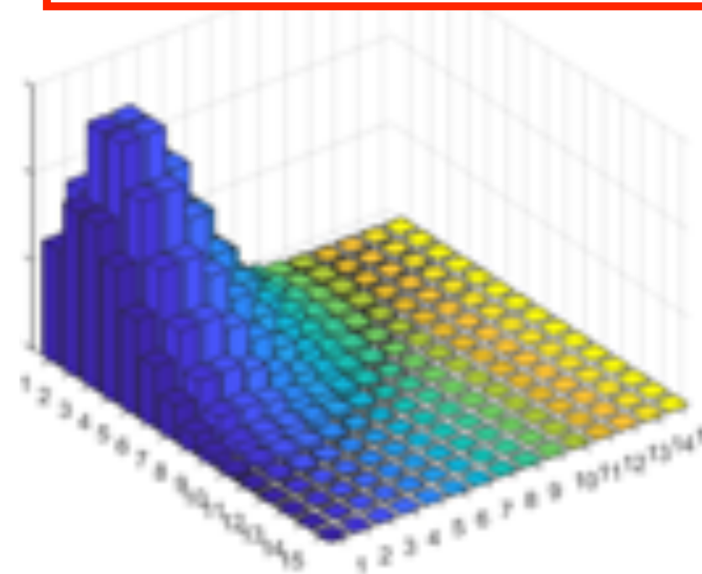
$|\alpha\rangle$



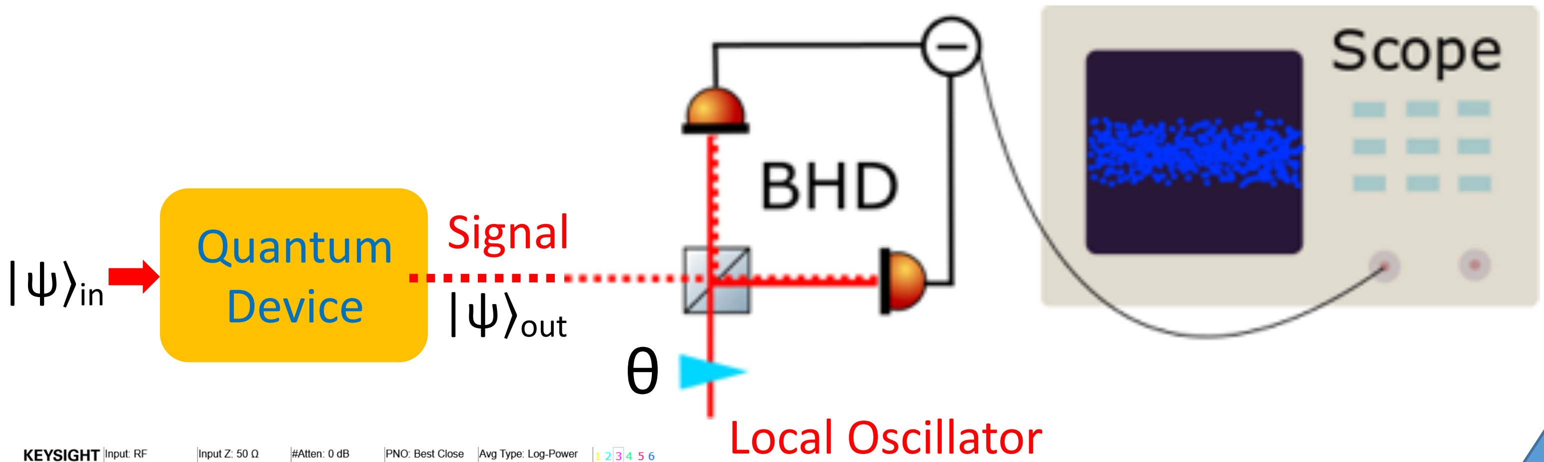
with Hsieh-Yi Hsieh;
Giulia Marcucci (U. Sapienza);



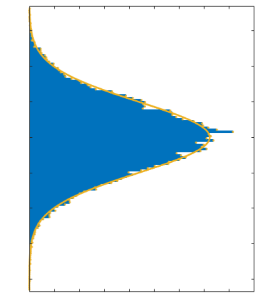
$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$



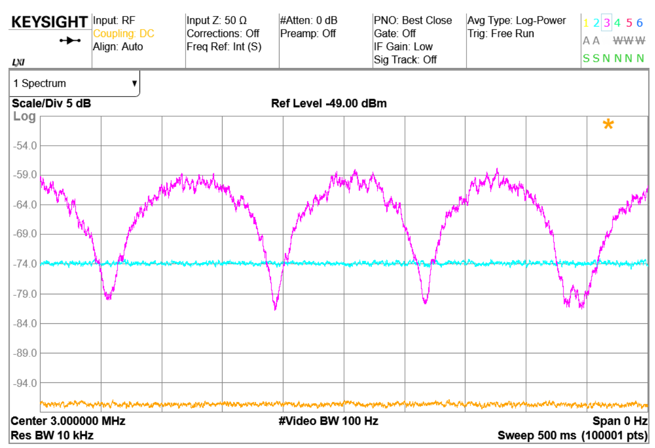
Quantum State Tomography:



Probability distribution
Pr(q, θ)

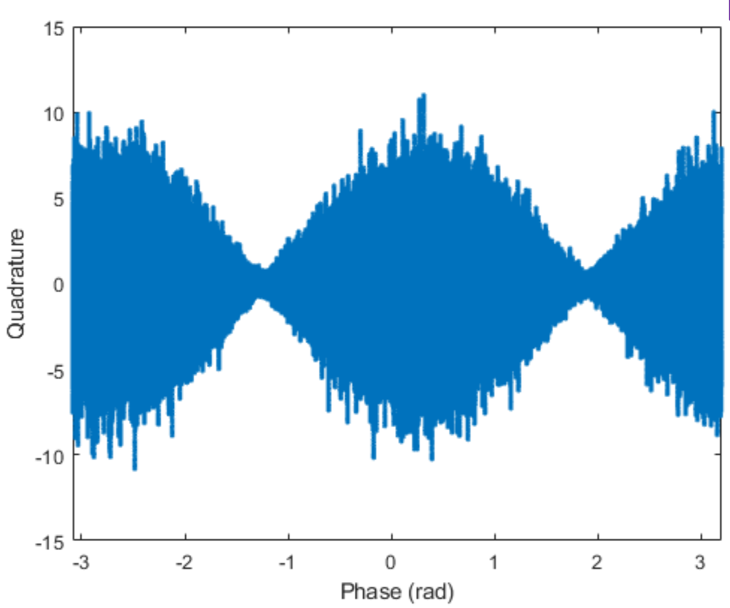


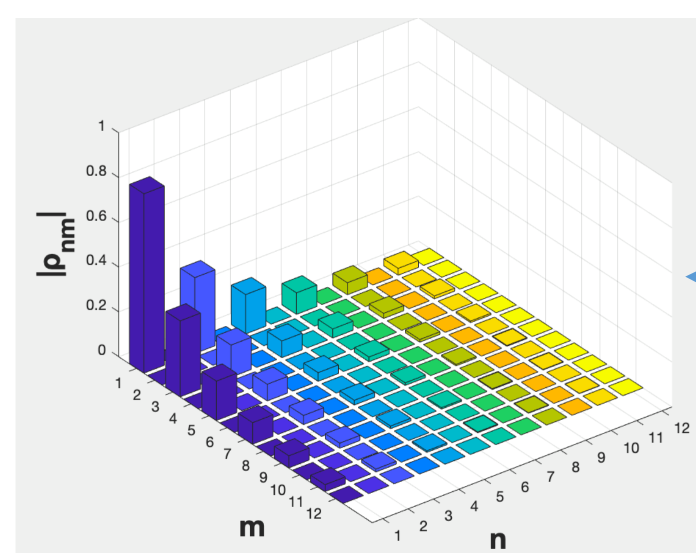
Reconstruction Algorithms

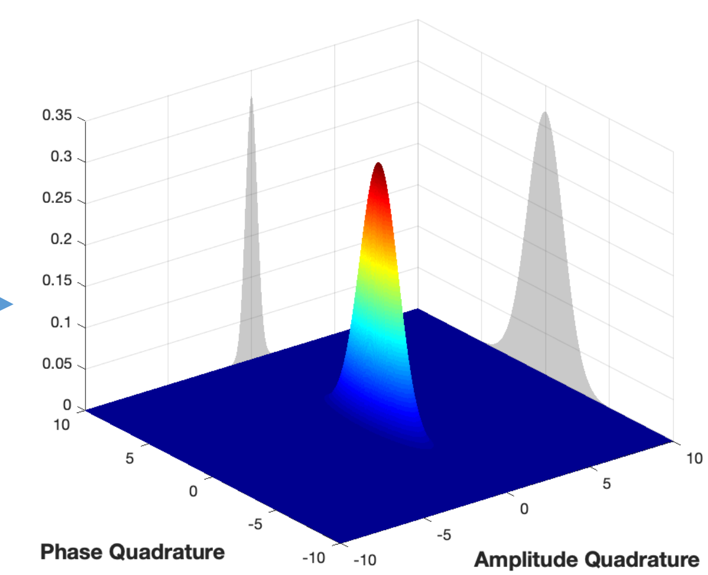


few minutes to reconstruct

Max. Likelihood Estimation, MLE







< 1s to reconstruct

Machine Learning (SQ Learner), CNN

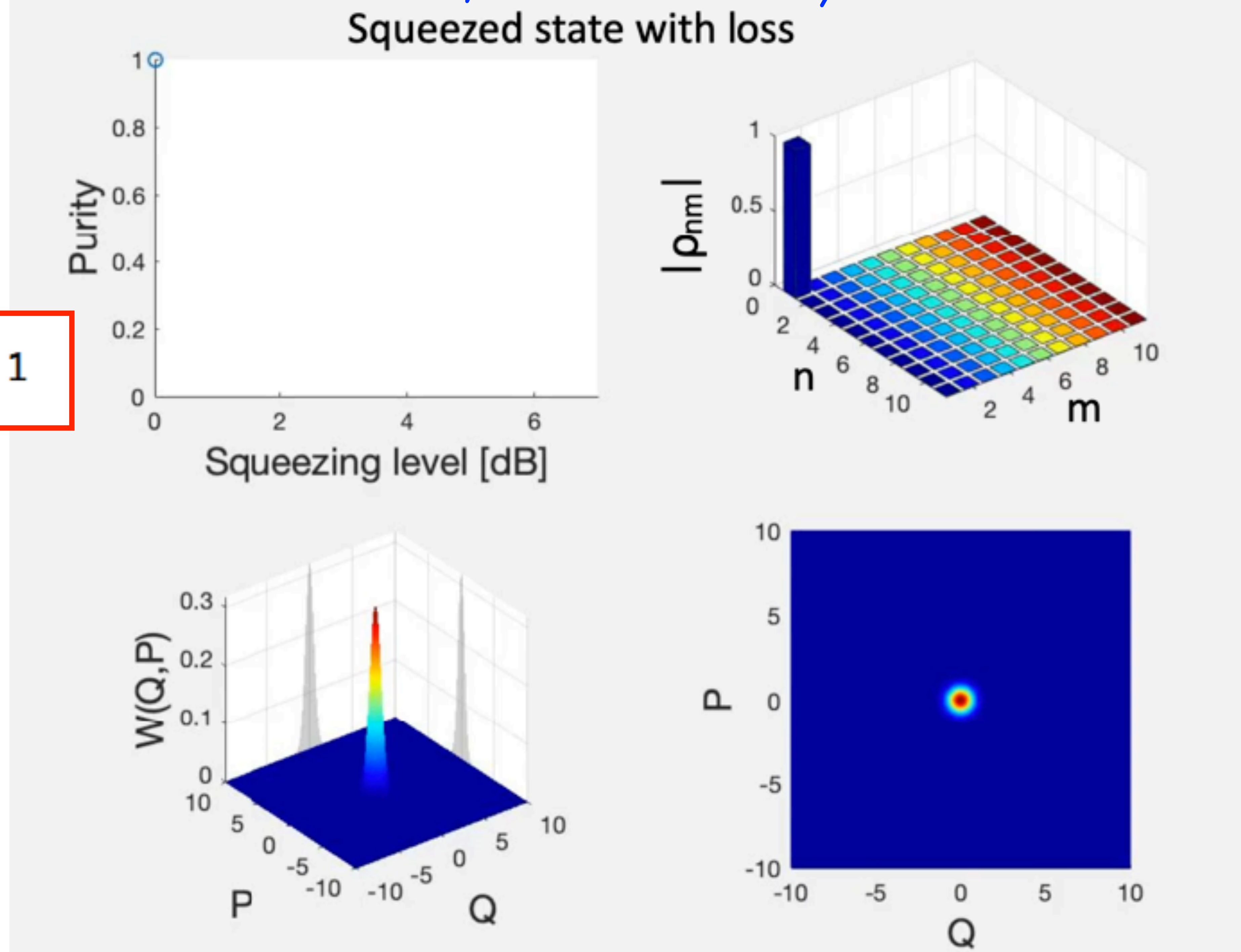
Real-time Q-State Tomography: Dynamics/Decoherence

Monitor the purity of a quantum state in real-time, and reveal the dynamics.

- The purity of a normalized quantum state is a scalar defined as:

$$\boxed{\gamma \equiv \mathrm{tr}(\rho^2) \quad , \quad 0 < \gamma \leq 1}$$

γ =1 for pure squeezed state

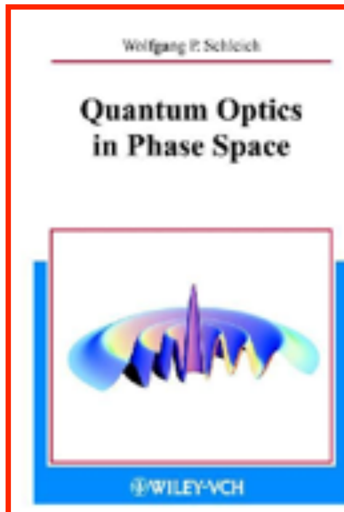


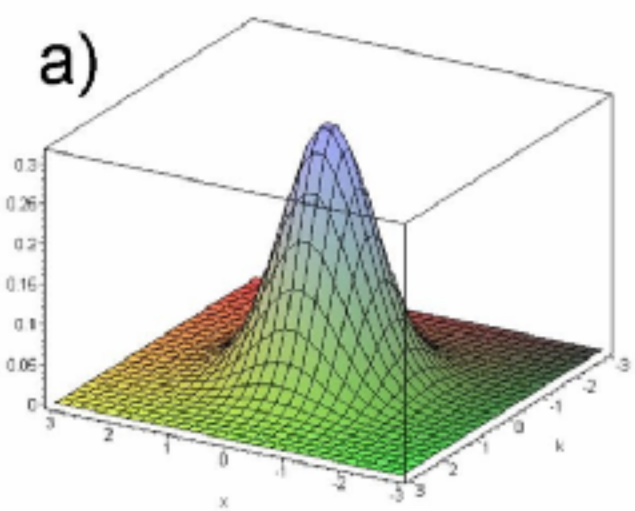
Number (Fock) states

Non-classical states

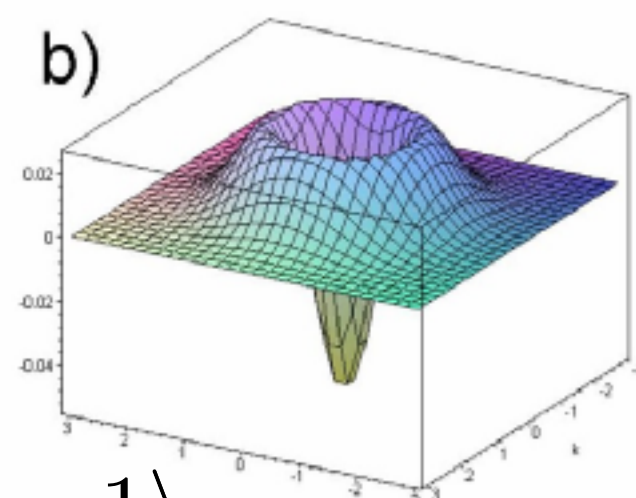
- Wigner quasiprobability distribution

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi \psi^*(x - \frac{\xi}{2})\psi(x + \frac{\xi}{2})e^{-ik\xi}$$

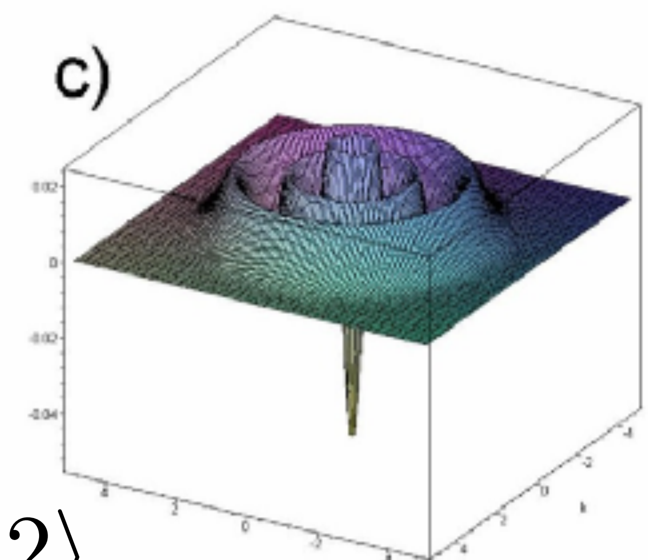




$|0\rangle$



$|n=1\rangle$



$|n=2\rangle$

negative probability

with Ludmila Praxmeyer

Quantum Mechanics: Purity of States

In quantum mechanics, and especially quantum information theory, the purity of a normalized quantum state is a scalar defined as

$$\boxed{\gamma \equiv \mathrm{tr}[\hat{\rho}^2],}$$

where $\hat{\rho}$ is the density matrix of the state. The purity defines a measure on quantum states, giving information on how much a state is mixed.

- The purity of a normalized quantum state satisfies

$$\frac{1}{d} \leq \gamma \leq 1,$$

  where d is the dimension of the Hilbert space upon which the state is defined.

- The upper bound is obtained by $\mathrm{tr}(\rho) = 1$ and

$$\mathrm{tr}(\hat{\rho}^2) \leq tr(\hat{\rho}) = 1.$$

Examples:

- rank-1 matrix

1. $|\Psi\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$, where $\langle\phi_i|\phi_j\rangle = \delta_{ij}$, orthonormal.
2. $\langle\Psi|\Psi\rangle = 1$, normalization condition: $|c_1|^2 + |c_2|^2 = 1$.
3. Pure states: let $|\Psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle$, then we have

$$\hat{\rho}_1 = |\phi_1\rangle\langle\phi_1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1};$$

$$\hat{\rho}_2 = |\phi_2\rangle\langle\phi_2| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1};$$

$$\hat{\rho}_\Psi = |\phi_1\rangle\langle\phi_1| = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^{-1};$$

Quantum Mechanics: Desity Matrix/Operator

The density operator is strictly *non-negative*, that is it has only non-negative eigenvalues, because for all $|\psi\rangle$,

$$\langle\psi|\hat{\rho}|\psi\rangle = \sum_n P_n |\langle\psi_n|\psi_n\rangle|^2 \geq 0.$$

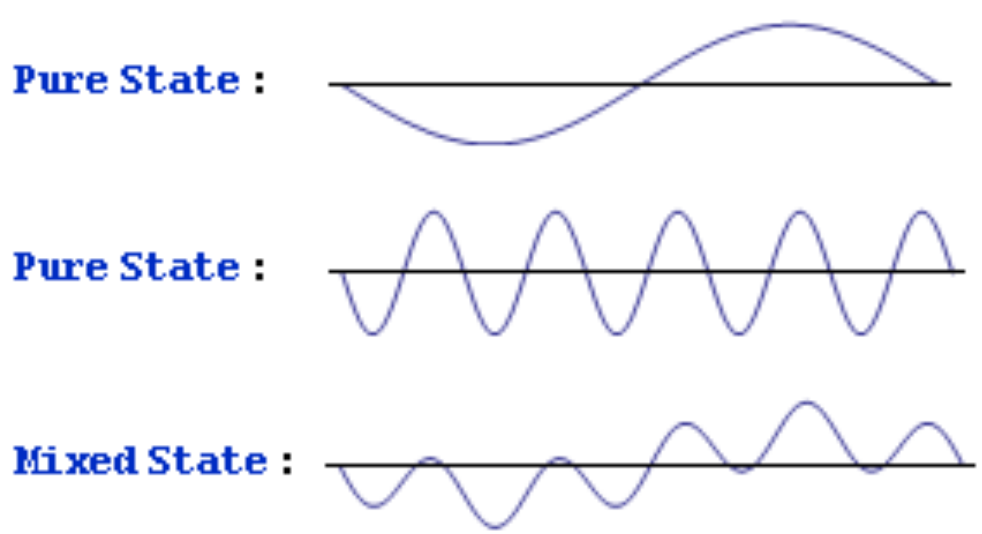
Or equivalently, for A $n \times n$ Hermitian complex matrix $\mathcal{M}$ is said to be *positive-semi-definite* or or *non-negative definite* if

$$\vec{x}^* \mathcal{M} \vec{x} \geq 0, \qquad \text{for all} \quad \vec{x} \in \mathbb{C}^n,$$

where $\vec{x}^*$ is the conjugate transpose of $\vec{x}$.

- Positive semi-definite

- Pure state
- Mixed state



Examples:

$$\boxed{\frac{1}{d} \le \mathrm{tr}[\hat{\rho}^2] \le 1,}$$

4. Mixed states: let $\hat{\rho}_{mix} = \frac{1}{2}\hat{\rho}_1 + \frac{1}{2}\hat{\rho}_2$, then we have

$$\hat{\rho}_{mix} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{d}\bar{\bar{I}}_d; \quad (d=2)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1};$$

- max. mixed state
- rank-2(d) matrix

5. Purity:

$$\mathrm{tr}(\hat{\rho}_1^2) = 1;$$
$$\mathrm{tr}(\hat{\rho}_2^2) = 1;$$
$$\mathrm{tr}(\hat{\rho}_\Psi^2) = 1;$$
$$\mathrm{tr}(\hat{\rho}_{mix}^2) = 1/2;$$

Quantum Mechanics: von Neuman entropy

How can we discriminate pure from mixed states, or more generally, characterize the purity of a state? One option is the *von Neumann entropy*, i.e.,

$$\boxed{S = -k_B \, \mathrm{tr}[\hat{\rho} \ln \hat{\rho}],}$$

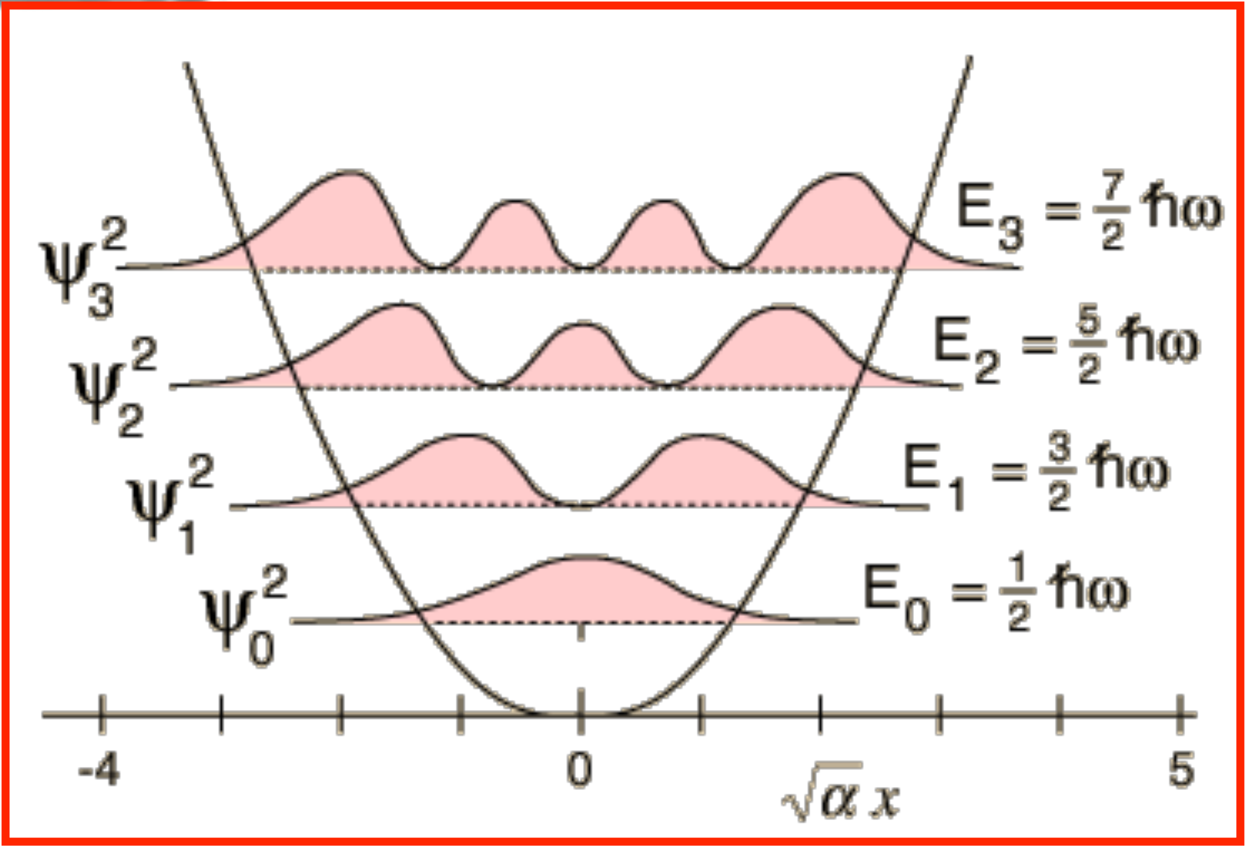
where k_B denotes the Boltzmann constant.

- $S(\rho)$ is zero if and only if ρ represents a pure state.
- $S(\rho)$ is maximal and equal to $\ln N$ for a maximally mixed state, N being the dimension of the Hilbert space.
- $S(\rho)$ is invariant under changes in the basis of ρ, that is, $S(\rho) = S(\hat{U}\rho\hat{U}^\dagger)$, with $\hat{U}$ a unitary transformation.
- $S(\rho)$ is additive for independent systems. Given two density matrices ρ_A, ρ_B describing independent systems A and B, we have

$$S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B).$$

Quantum Fisher Information:

Quantum Simple Harmonic Oscillator (SHO)



- Energy quantization
- Equally spacing in energy difference
- Zero-point energy $\neq 0$

$$\psi(\xi) = H_n(\xi)\exp[-\xi^2/2], \qquad \epsilon = 2n+1, \qquad n = 0,1,2,3\ldots$$

$$E = \frac{\hbar\omega}{2}\epsilon = \hbar\omega(n+\frac{1}{2}), \qquad n = 0,1,2,3,\ldots.$$

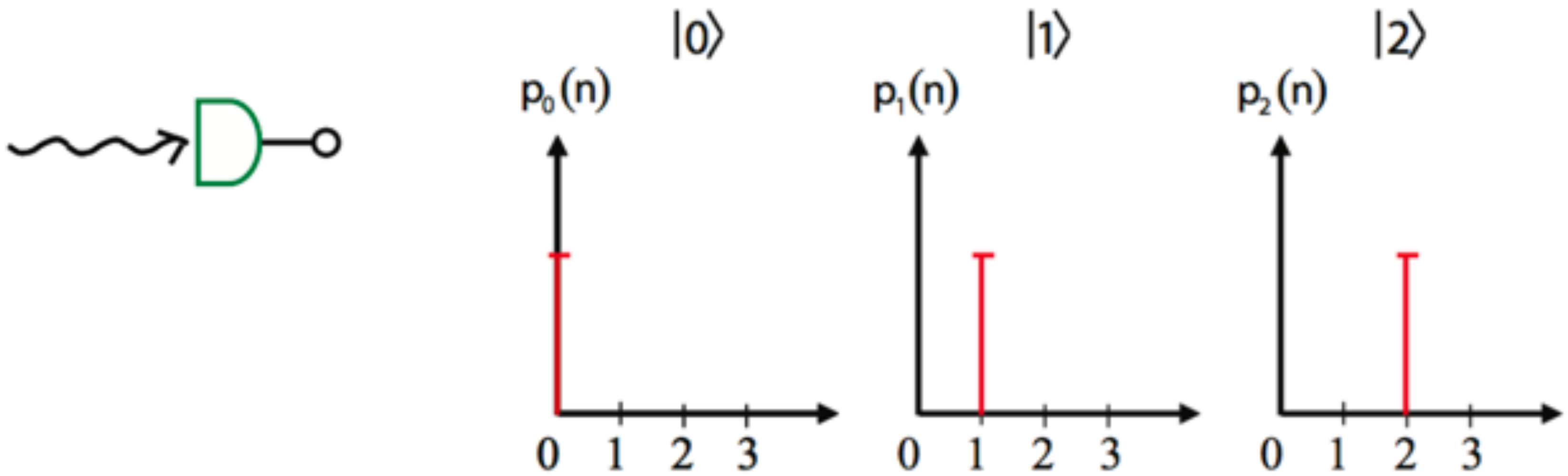
$$\hat{H} = \frac{1}{2}\frac{\hat{p}^2}{m} + \frac{1}{2}k\,\hat{x}^2, \quad [\hat{x},\hat{p}] = i\hbar.$$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}). \quad [\hat{a},\hat{a}^\dagger] = 1,$$

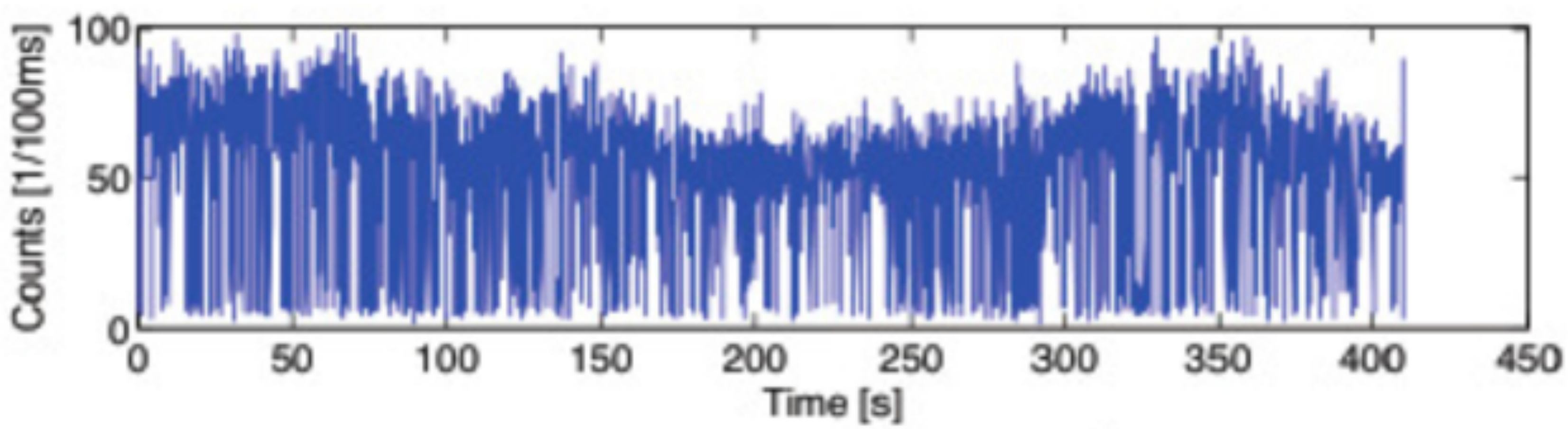
$$\begin{aligned} \hat{N}|n\rangle &= n|n\rangle, \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle, \\ E_n &= \hbar\omega(n+\frac{1}{2}). \end{aligned}$$

Photon Counting:

- Number state

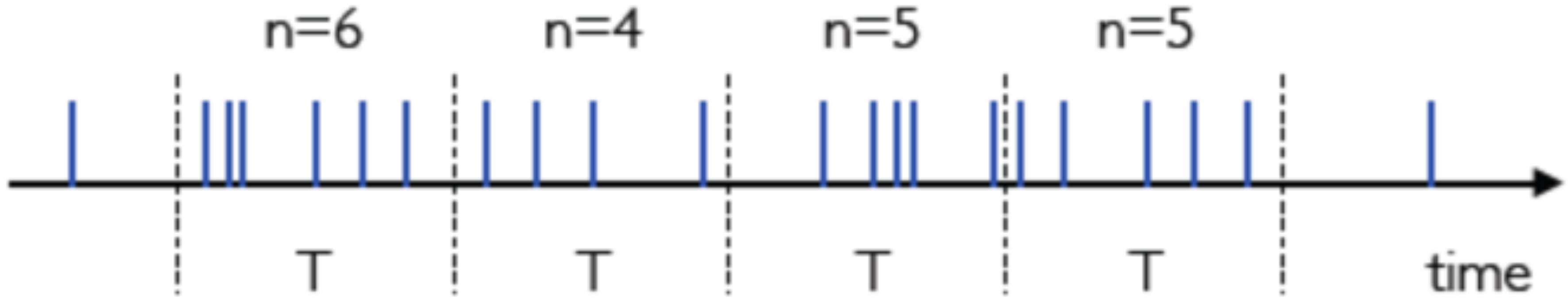


- Laser state



i.i.d. limit:

- idetical
- independent
- distribution
- (large number)



- binomial distribution

$$P(n) = \frac{N!}{n!(N-n)!}p^n(1-p)^{N-n},$$

Poisson Distribution:

$$P(n) = \frac{\bar{n}^n \exp(-\bar{n})}{n!},$$

$$\begin{aligned} \langle \hat{n} \rangle &= \sum_n n P(n) = |\alpha|^2 \equiv \bar{n}, \\ \langle \Delta \hat{n}^2 \rangle &= \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 = |\alpha|^2 = \langle \hat{n} \rangle. \end{aligned}$$

- mean = variance

Bose-Einstein Distribution:

- Boltzmann's law

$$\boxed{P(n) \propto \exp[-E_n/k_B T],}$$

$$\begin{aligned} P(n) &= \frac{\exp[-E_n/k_B T]}{\sum_{n=0}^{\infty} \exp[-E_n/k_B T]}, \\ &= \exp[-E_n/k_B T]\,(1 - \exp[-\hbar\omega/k_B T]); \qquad E_n = n\,\hbar\omega \end{aligned}$$

$$\bar{n} = \sum_{n=0}^{\infty} n\,P(n) = \frac{1}{\exp[\hbar\omega/k_B T] - 1},$$

- average photon number at temperature T

$$\boxed{P(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n,}$$

Bose-Einstein Distribution:

- thermal state

$$\rho_{th} = \sum_n = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n |n\rangle\langle n|.$$

$$\Delta n^2 = \bar{n} + \bar{n}^2,$$

Next and Questions

- ☑ **Axioms**
- ☑ **State**
- ☑ **Operator**
- ☑ **Density Matrix**

“Shut up and Calculate”

Richard Feynman

The real beauty of the Density Matrix Formalism – no thinking…

- **More on States**
  - □ Coherent States
  - □ Squeezed States
  - □ **Uncertainty Relation ➡ Minimum Uncertainty States**
  - □ Entropy
  - □ Purity
  - □ bi-particle States ➡ **Entanglement** (Schmidt decomposition)
  - □ Cat states

Quantum Mechanics: States

1. Non-commuting observable do not admit common eigenvectors.

2. Non-commuting observables can not have definite values simultaneously.

3. Simultaneous measurement of non-commuting observables to an arbitrary degree of accuracy is thus *incompatible*.

4. Variance: one can define

$$\Delta\hat{A}^2 = \langle\Psi|(\hat{A} - \langle\hat{A}\rangle)^2|\Psi\rangle = \langle\Psi|\hat{A}^2|\Psi\rangle - \langle\Psi|\hat{A}|\Psi\rangle^2.$$

5. For any two non-commuting observables,

$$[\hat{A}, \hat{B}] = i\hat{C},$$

we have the *uncertainty relation*:

$$\boxed{\Delta A^2 \Delta B^2 \geq \frac{1}{4}[\langle\hat{F}\rangle^2 + \langle\hat{C}\rangle^2],}$$

where

$$\hat{F} = \hat{A}\hat{B} + \hat{B}\hat{A} - 2\langle\hat{A}\rangle\langle\hat{B}\rangle,$$

where the operator $\hat{F}$ is a measure of correlations between $\hat{A}$ and$\hat{B}$.

'or example, take the operators $\hat{A} = \hat{q}$ (position) and $\hat{B} = \hat{p}$ (momentum) for a free particle, one have

$$[\hat{q}, \hat{p}] = i\hbar \rightarrow \langle\Delta\hat{q}^2\rangle\langle\Delta\hat{p}^2\rangle \geq \frac{\hbar^2}{4}.$$

A 2-qubits pure state $|\psi\rangle_{AB} \in H_A \otimes H_B$ can be written (using Schmidt decomposition) as $|\psi\rangle_{AB} = \sum_j \lambda_j |j\rangle_A |j\rangle_B$, where $\{|j\rangle_A\}, \{|j\rangle_B\}$ are the bases of H_A, H_B respectively, and $\sum_j \lambda_j^2 = 1, \lambda_j \geq 0$. Its density matrix is

- 2-qubit

$$\rho^{AB} = \sum_{i,j} \lambda_i \lambda_j |i\rangle_A \langle j|_A \otimes |i\rangle_B \langle j|_B.$$

- The degree in which it is entangled is related to the purity of the states of its subsystems,

$$\boxed{\rho_A = tr_B(\rho_{AB}) = \sum_j \lambda_j^2 |j\rangle_A \langle j|_A,}$$

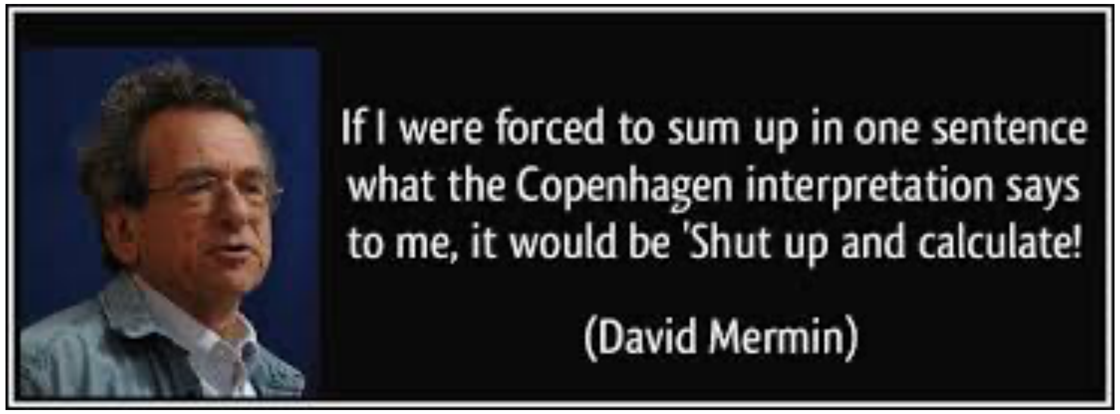
  and similarly for ρ_B.
- If this initial state is separable (i.e. there's only a single $\lambda_j \neq 0$, then ρ_A and ρ_B are both pure.
- Otherwise, this state is entangled and ρ_A, ρ_B are both mixed.
- For 2-qubits (pure or mixed) states, the Schmidt number (number of Schmidt coefficients) is at most 2.
- Using this and Peres–Horodecki criterion (for 2-qubits), a state is entangled if its *partial transpose* has at least one negative eigenvalue.
- The Peres–Horodecki criterion is a necessary condition, for the joint density matrix ρ of two quantum mechanical systems A and B, to be separable.
- It is also called the PPT criterion, for positive partial transpose.
- In the $2x2$ and $2x3$ dimensional cases the condition is also sufficient. It is used to decide the separability of mixed states, where the Schmidt decomposition does not apply.
- In higher dimensions, the test is inconclusive, and one should supplement it with more advanced tests, such as those based on *entanglement witnesses*
- In the context of localization, a quantity closely related to the purity, the so-called inverse participation ratio (IPR) turns out to be useful. It is defined as the inverse of the integral (or sum for finite system size) over the square of the density in some space, e.g., real space, momentum space, or even phase space, where the densities would be the square of the real space wave function $|\psi(x)|^2$, the square of the momentum space wave function

$$|\tilde{\psi}(k)|^2,$$

  or some phase space density like the Husimi distribution, respectively.

More on Strangeness in QM

- □ Geometric (Berry) phase
- □ Schrodinger's Cat paradox
- □ Einstein-Podolosky-Rosen paradox
- □ Local Hidden Variables theory
- □ Bell's inequality
- □ Quantum Zeno effect
- □ Entangled-History theory
- □ The Measurement Problem
- □ The Many Worlds Interpretation
- □ …



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The Trouble with Quantum Mechanics

Steven Weinberg

The Trouble with Quantum Physics, and Why it Matters

"Quantum physics—the physics of atoms and other ultratiny objects, like molecules and subatomic particles—is the most successful theory in all of science. But there's something troubling here. Quantum physics doesn't seem to apply to humans."

By Adam Becker | Mar 30, 2018

The New York Times

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Opinion

Even Physicists Don't Understand Quantum Mechanics

Worse, they don't seem to want to understand it.

By Sean Carroll

Dr. Carroll is a physicist.

Sept. 7, 2019