Note: Minimum Uncertainty States (MUS)

- ☑ Heisenberg's Uncertainty Relation
- ☑ Minimum Uncertainty States (MUS)
- ☑ Gaussian States
- ☑ Free-particle expansion
- □ Squeezed States

- More on Uncertainty Relations
  - □ Intelligent States
  - □ Robertson–Schrodinger uncertainty relations
  - □ Quantum entropic uncertainty principle
  - □ Quantum Metrology
  - □ Heisenberg limit ➡ Quantum Cramer-Rao bound
  - □ Quantum Non-Demolition (QND) Measurement

Note: Phase Space

- ☑ Quantum Distribution Theory
- ☑ Ordering
- □ Normally Ordering, P-representation (Glauber-Sudarshan)
- □ Anti-Normally Ordering, Q-representation (Husimi)
- ☑ Symmetrically Ordering, W-representation (Wigner-Weyl)
- More on Phase Space
- □ Fokker-Planck equation
- □ Quantum Liouville equation
- □ Moyal function
- □ Homodyne detection
- □ Wigner Flow

Note: Squeezed States

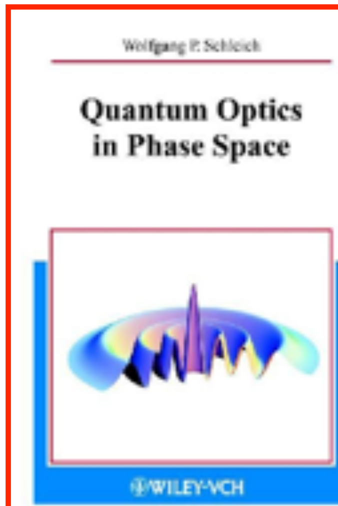
- □ Squeezed Operator
- □ Squeezed Vacuum
- □ Squeezed Coherent v.s. Coherent Squeezed states
- □ Squeezed states as the Minimum Uncertainty states
- □ Generation of Squeezed states by OPO
- □ Homodyne detections
- More on Squeezed States
  - □ OPA
  - □ FWM Squeezing
  - □ Soliton Squeezing
  - □ Squeezing spectrum
  - □ Applications to Gravitational Wave Detectors

Number (Fock) states

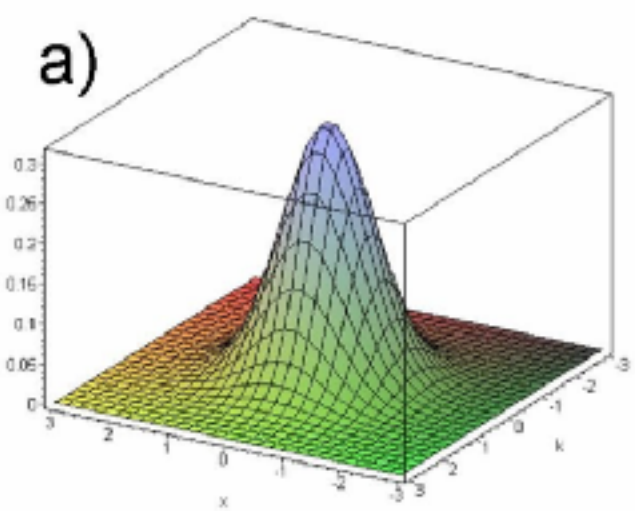
non-classical states

- Wigner quasiprobability distribution

$$W(x,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\xi \psi^*(x - \frac{\xi}{2})\psi(x + \frac{\xi}{2})e^{-ik\xi}$$



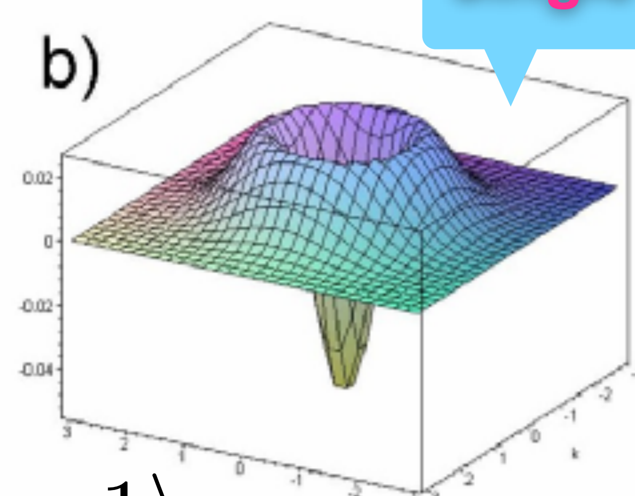
a)



$|0\rangle$

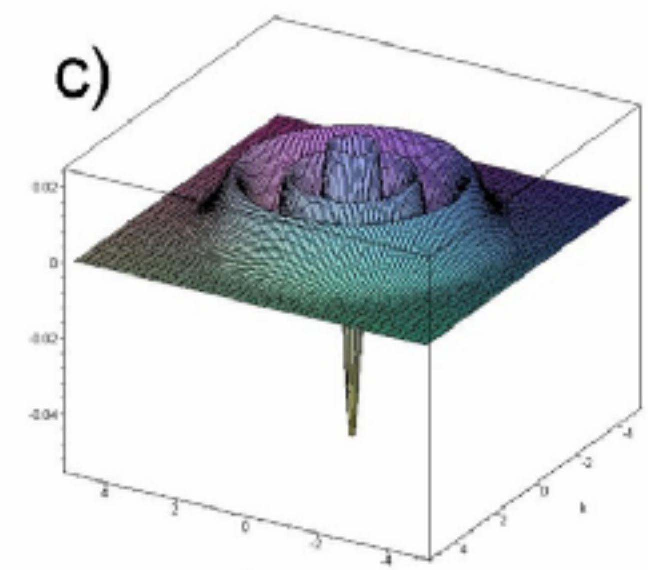
single-photon

b)



$|n = 1\rangle$

c)

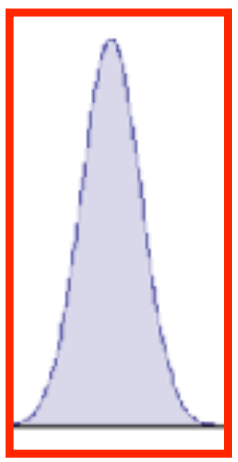


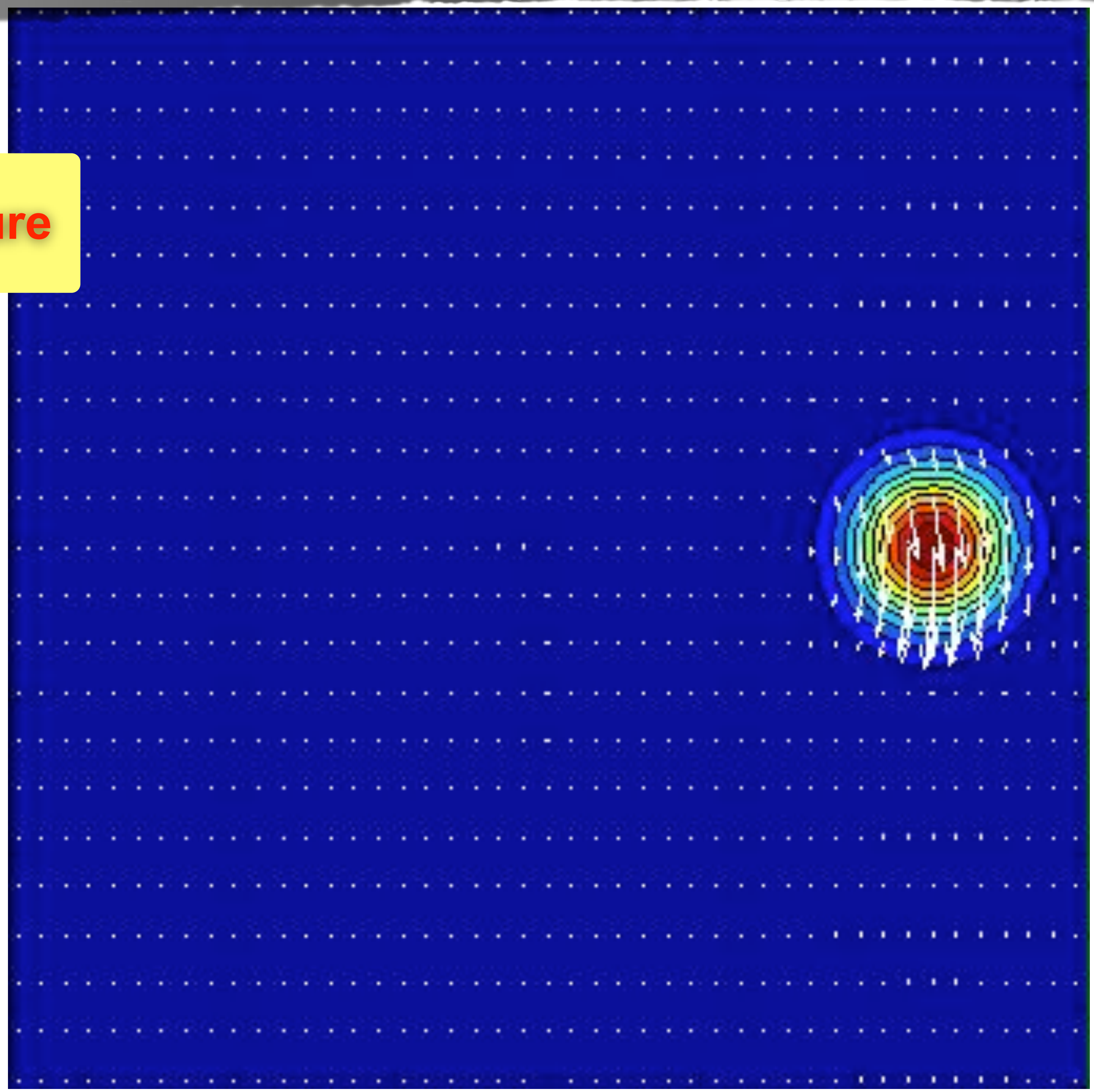
negative probability

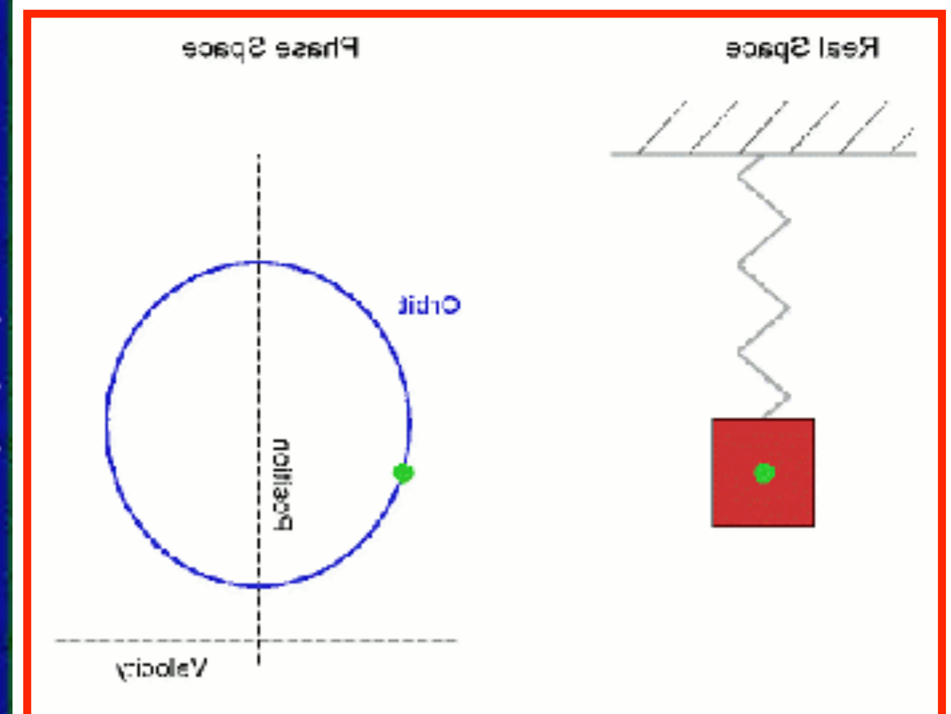
with Ludmila Praxmeyer

Coherent states

wave-nature



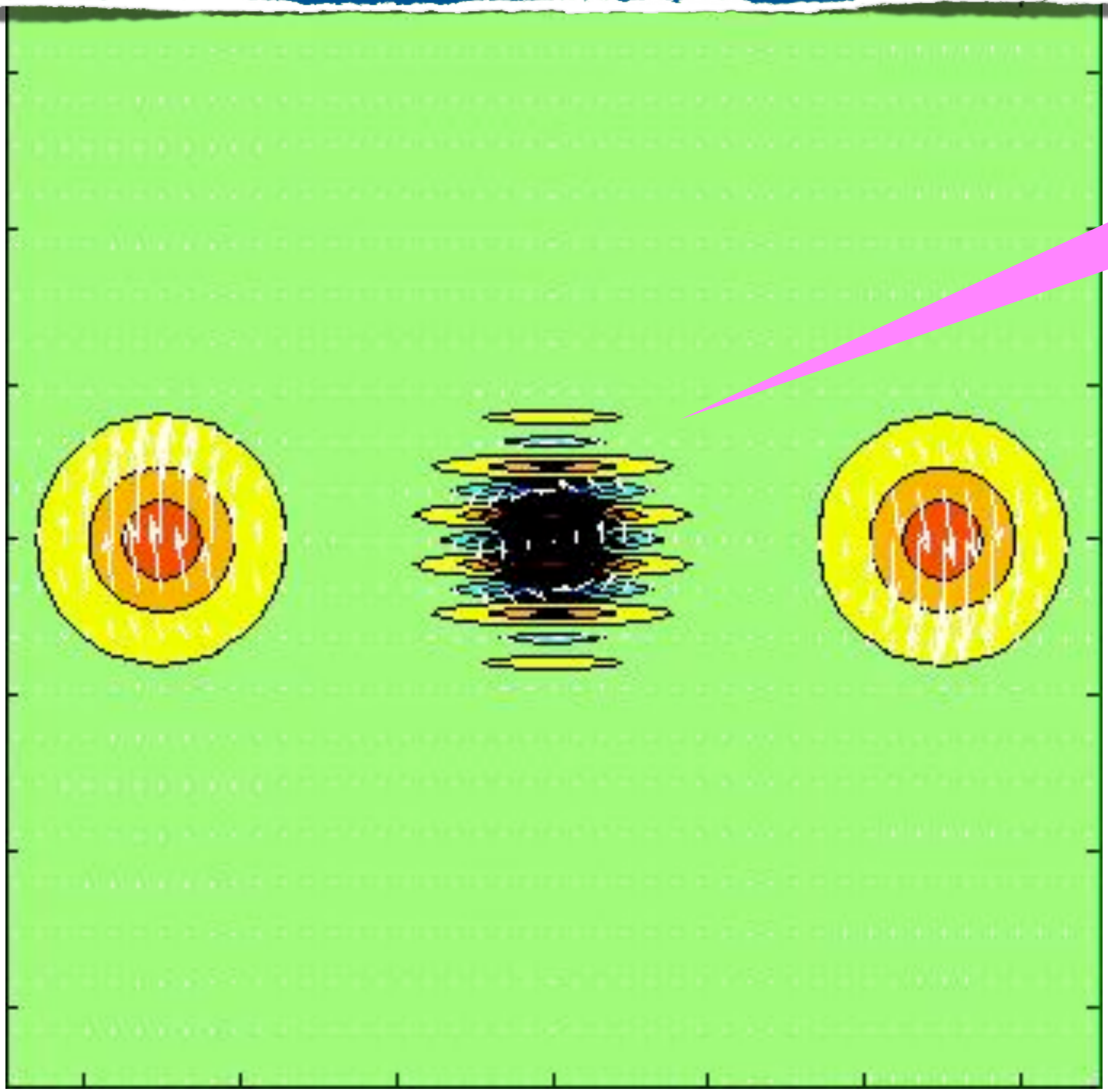




with Popo Yang

Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs, Physica Scripta for the New Focus issue: Quantum Optics and Beyond - in honour of Wolfgang Schleich.

Cat States in Phase Space



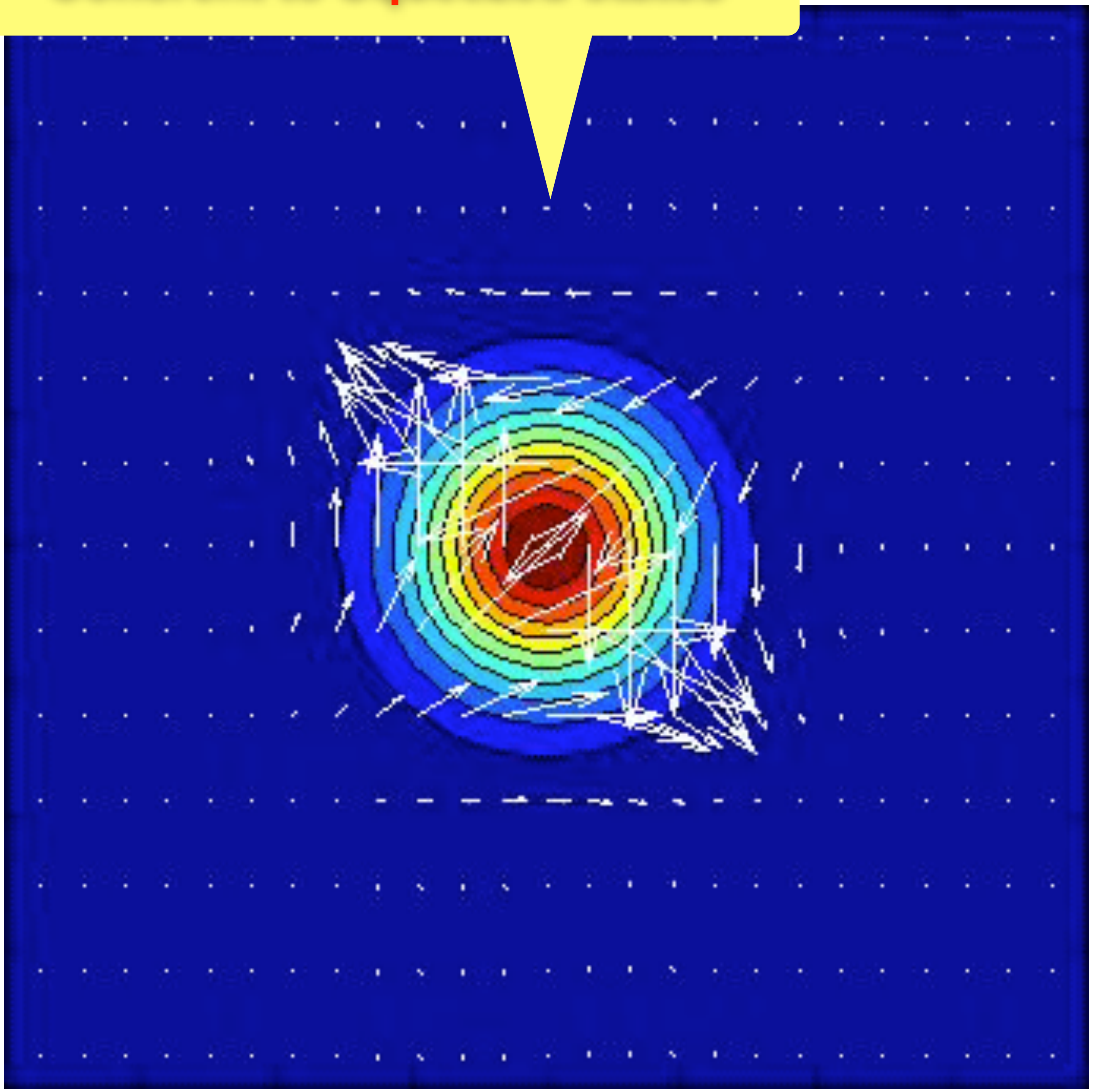
Non-classicality

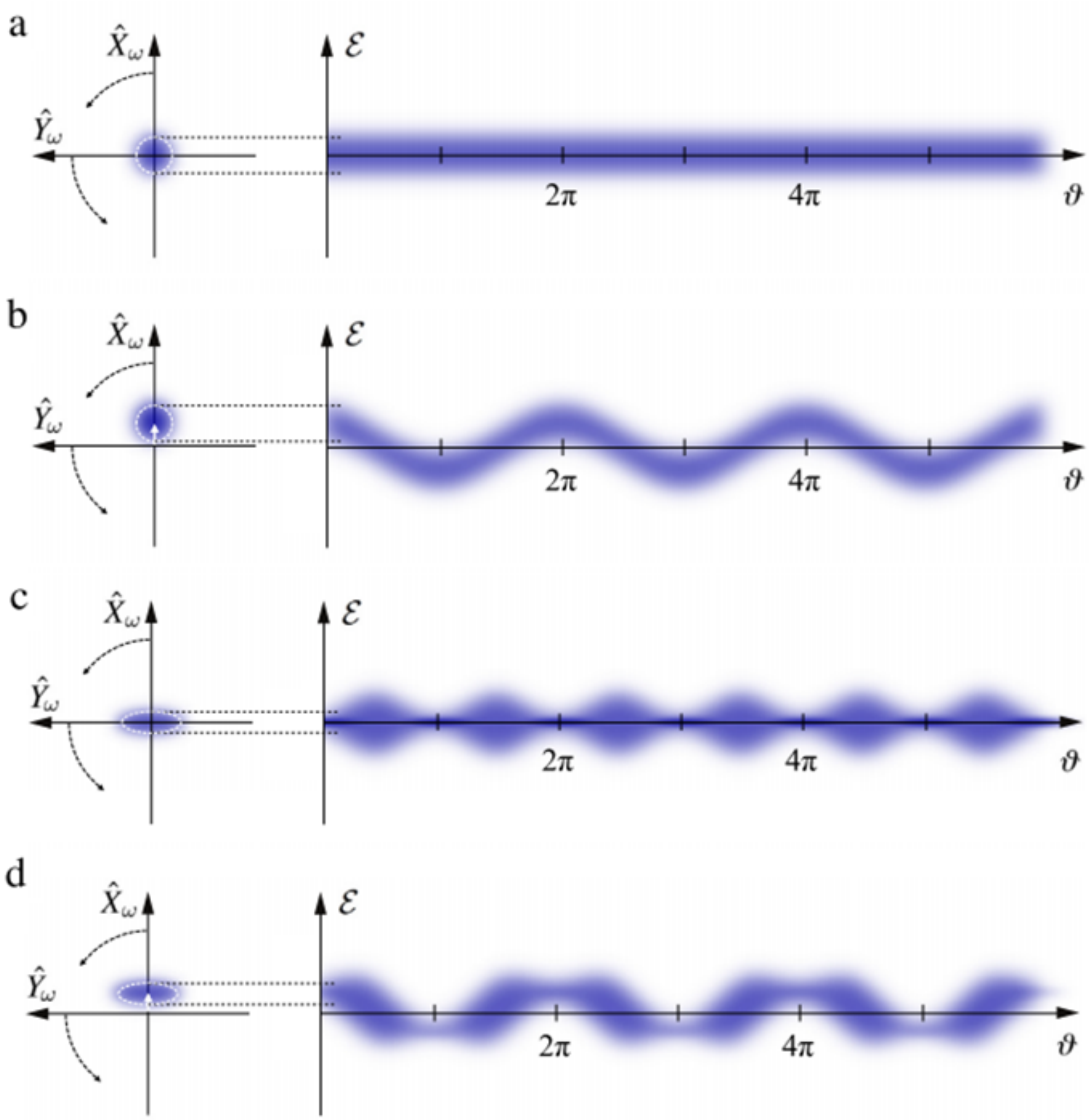
with Popo Yang

Popo Yang, Ivan F. Valtierra, Andrei B. Klimov, Shin-Tza Wu, RKL, Luis L. Sanchez-Soto, and Gerd Leuchs, Physica Scripta for the New Focus issue: Quantum Optics and Beyond - in honour of Wolfgang Schleich.

Squeezed States

Coherent to Squeezed states



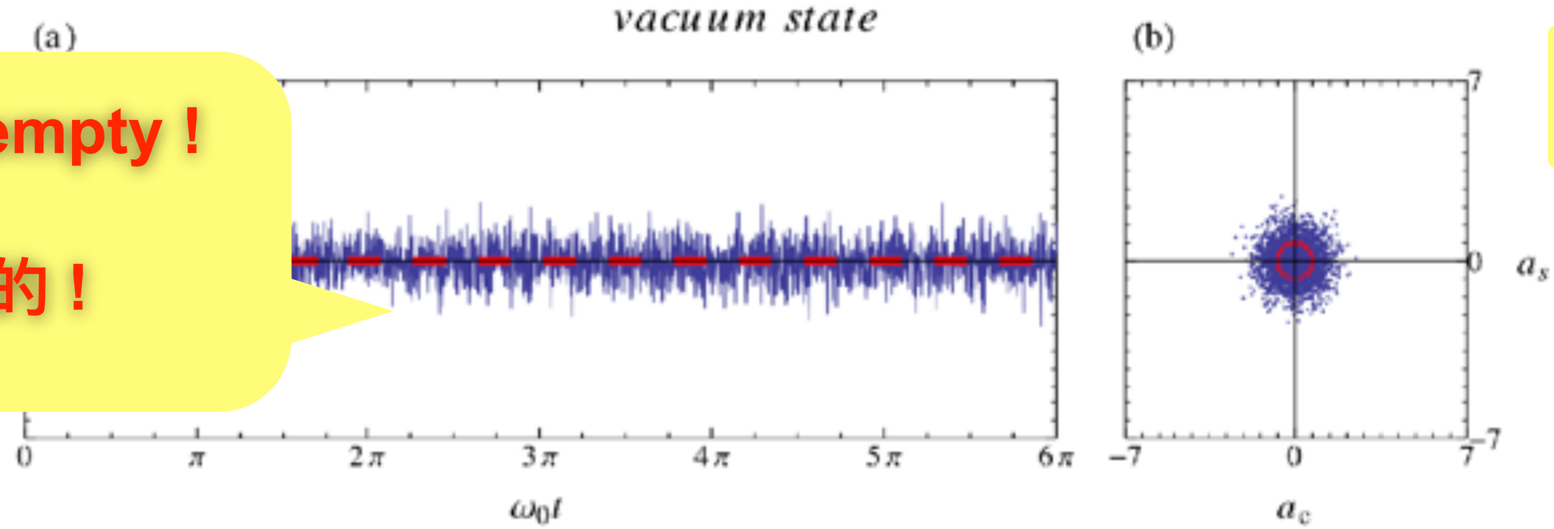


Courtesy:
Roman Schnabel (2017).

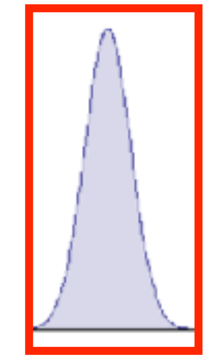
by Popo Yang

Vacuum is NOT empty！

真空並不是空的！



wave-nature



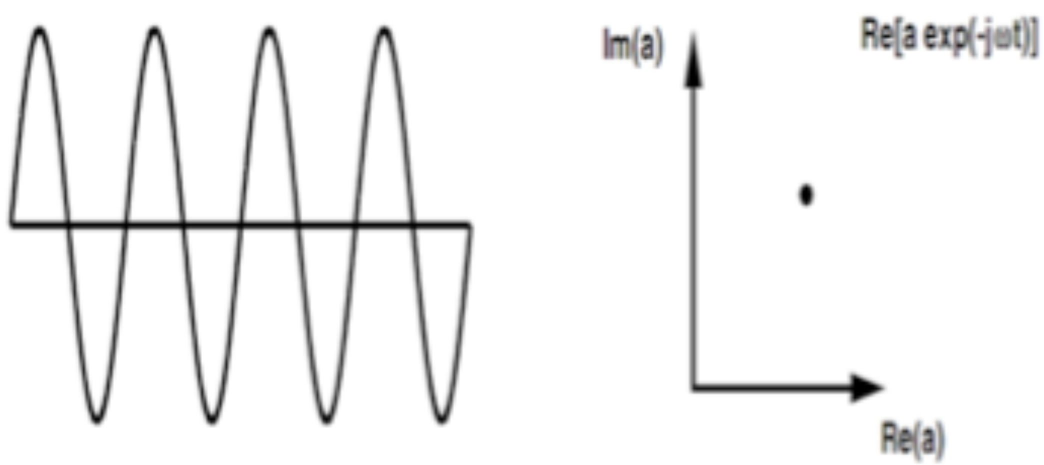
● coherent state

$$(\Delta\hat{X}_1)^2 = \frac{1}{4}, (\Delta\hat{X}_2)^2 = \frac{1}{4}, \quad \Delta\hat{X}_1\Delta\hat{X}_2 = \frac{1}{4}$$

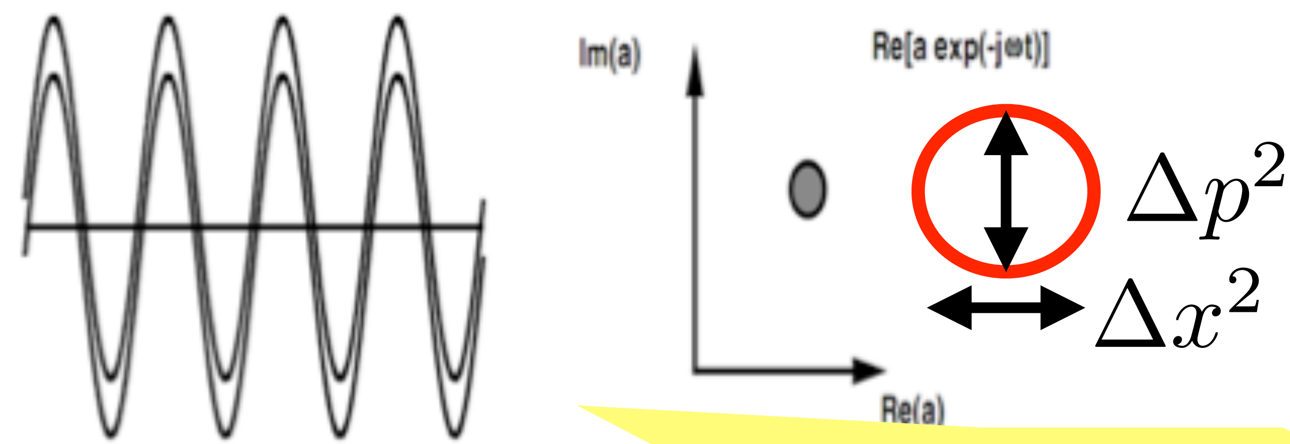
● squeezed state

$$(\Delta\hat{Y}_1)^2 = \frac{1}{4}e^{-2r}, (\Delta\hat{Y}_2)^2 = \frac{1}{4}e^{2r}, \ \Delta\hat{Y}_1\Delta\hat{Y}_2 = \frac{1}{4}$$

● classical field

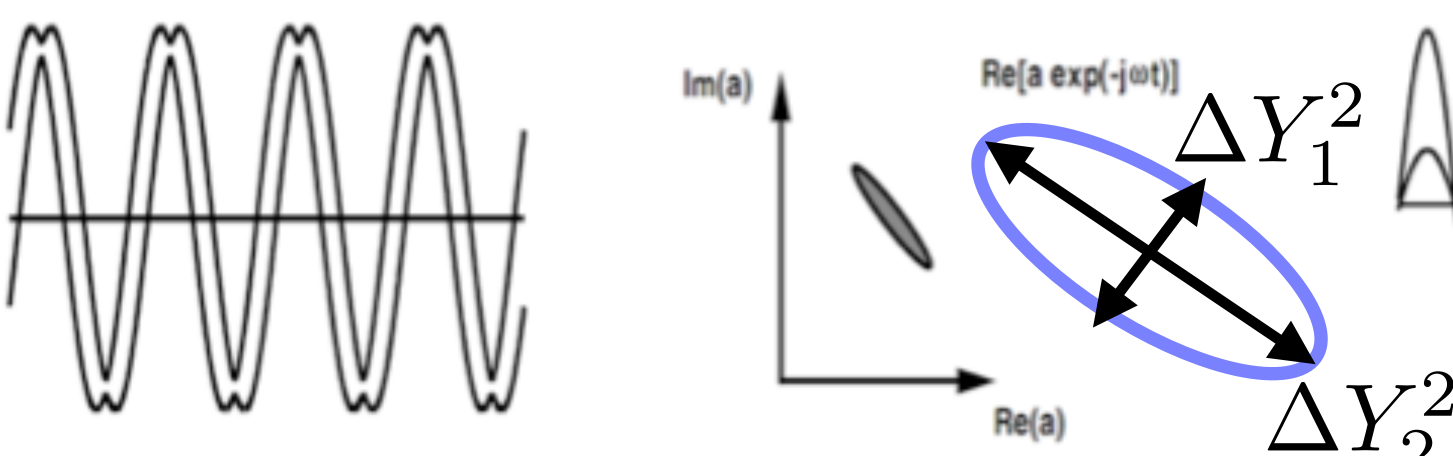


● coherent field

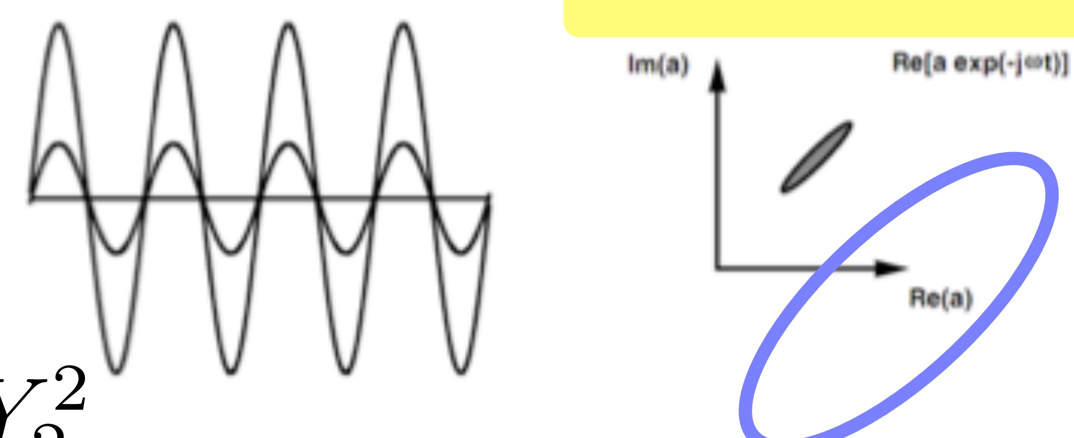


Non-classical states

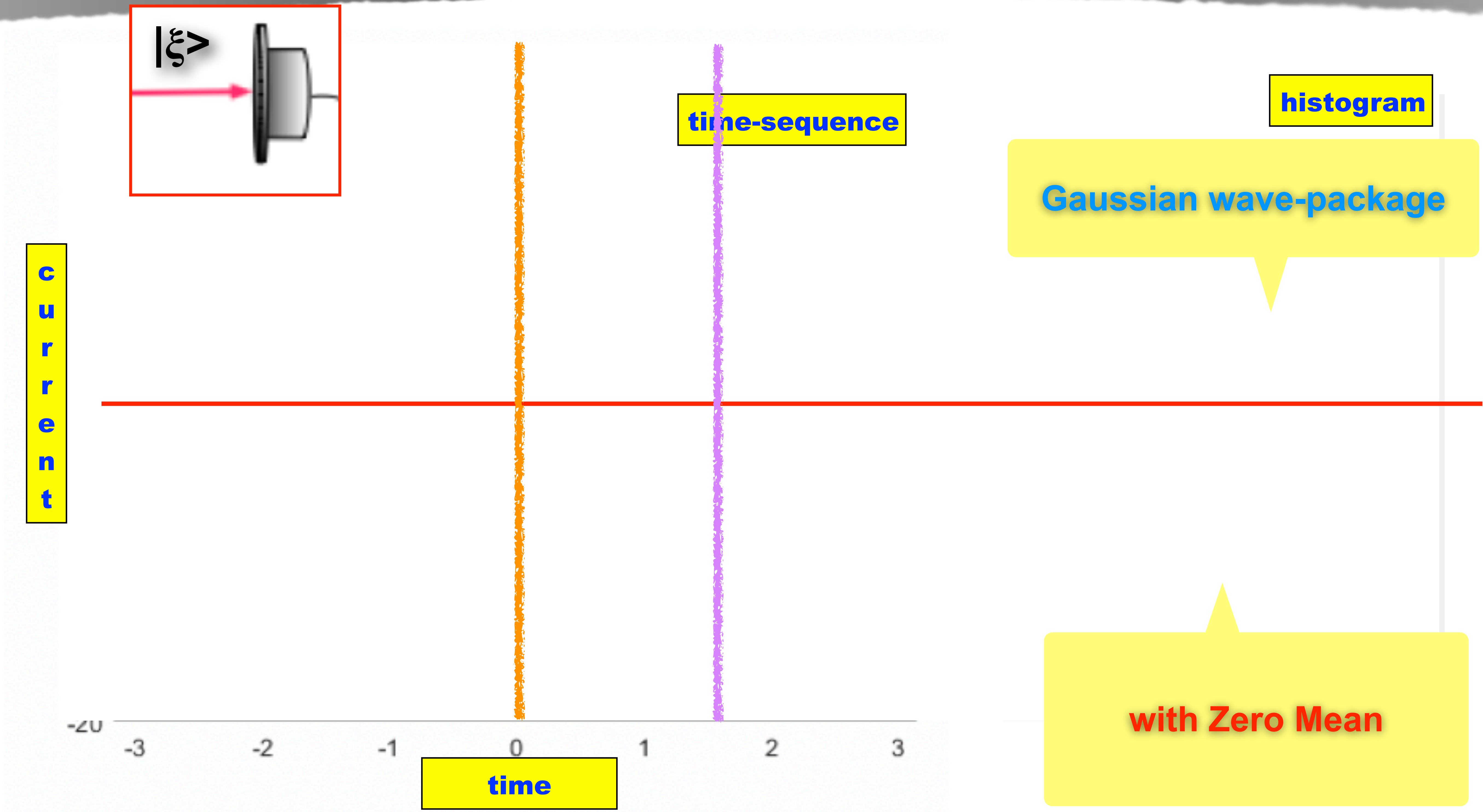
● amplitude-squeezed field



● phase-squeezed field

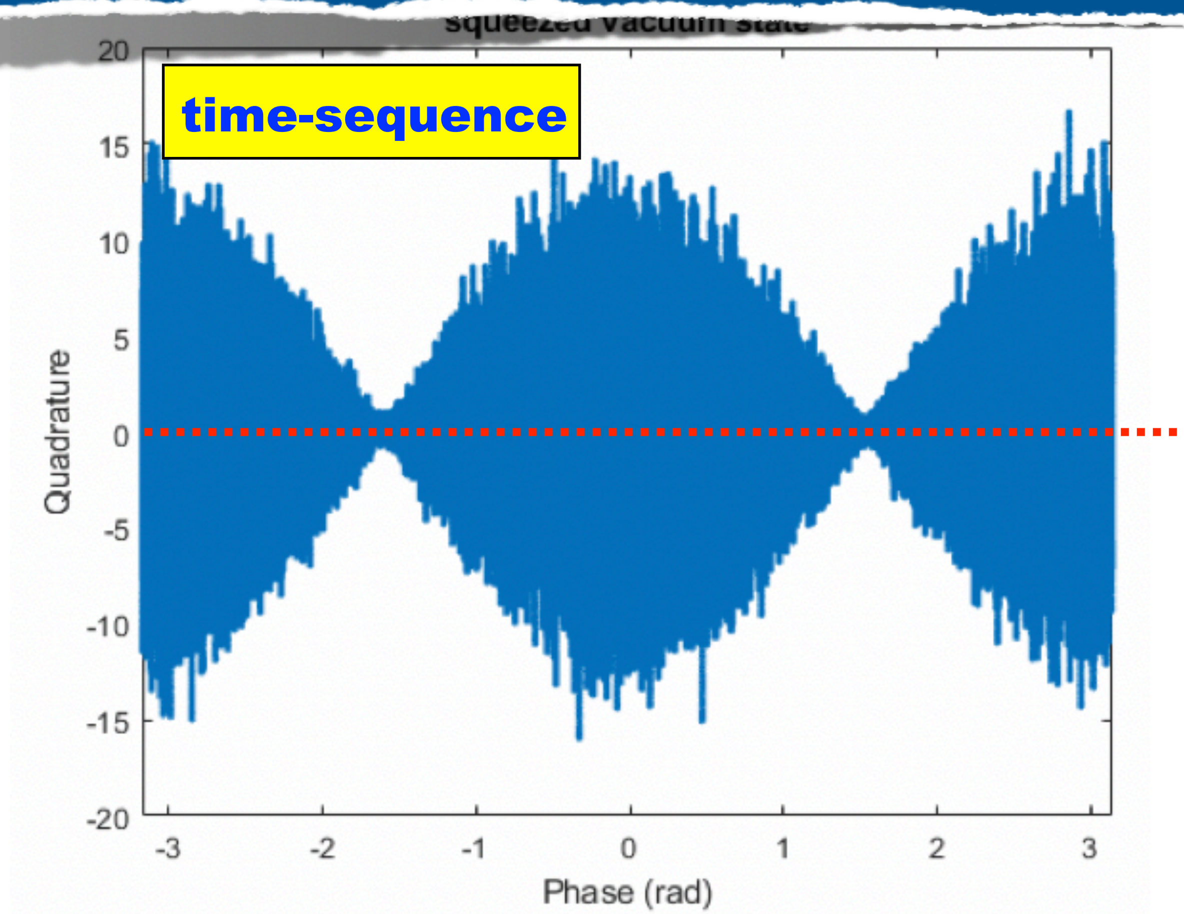


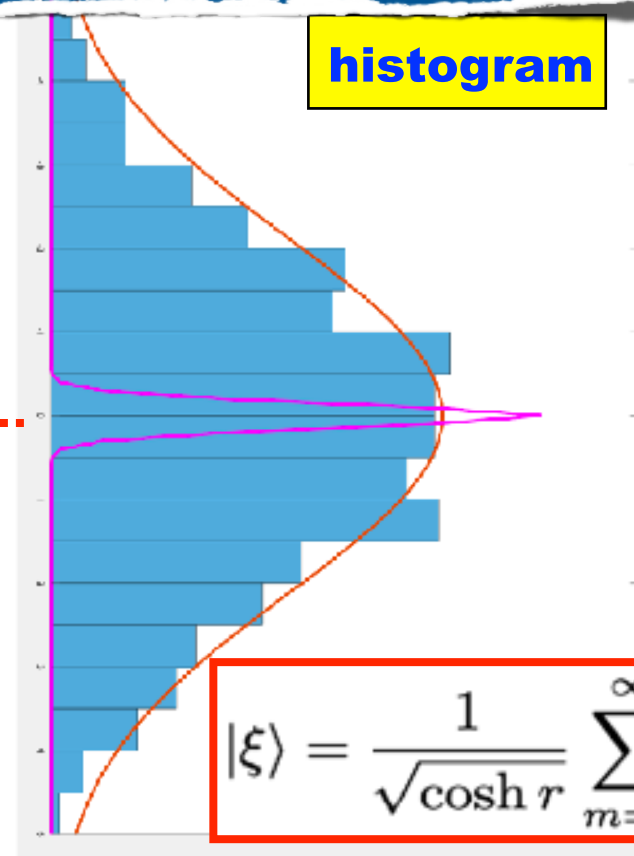
Squeezed Vacuum State: |ξ>

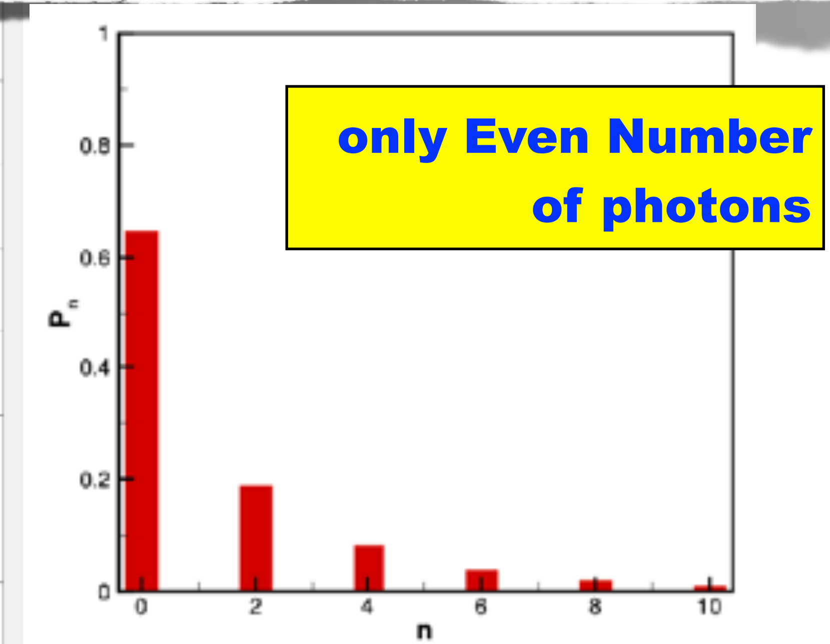


with Yi-Ru Chen, Chien-Ming Wu

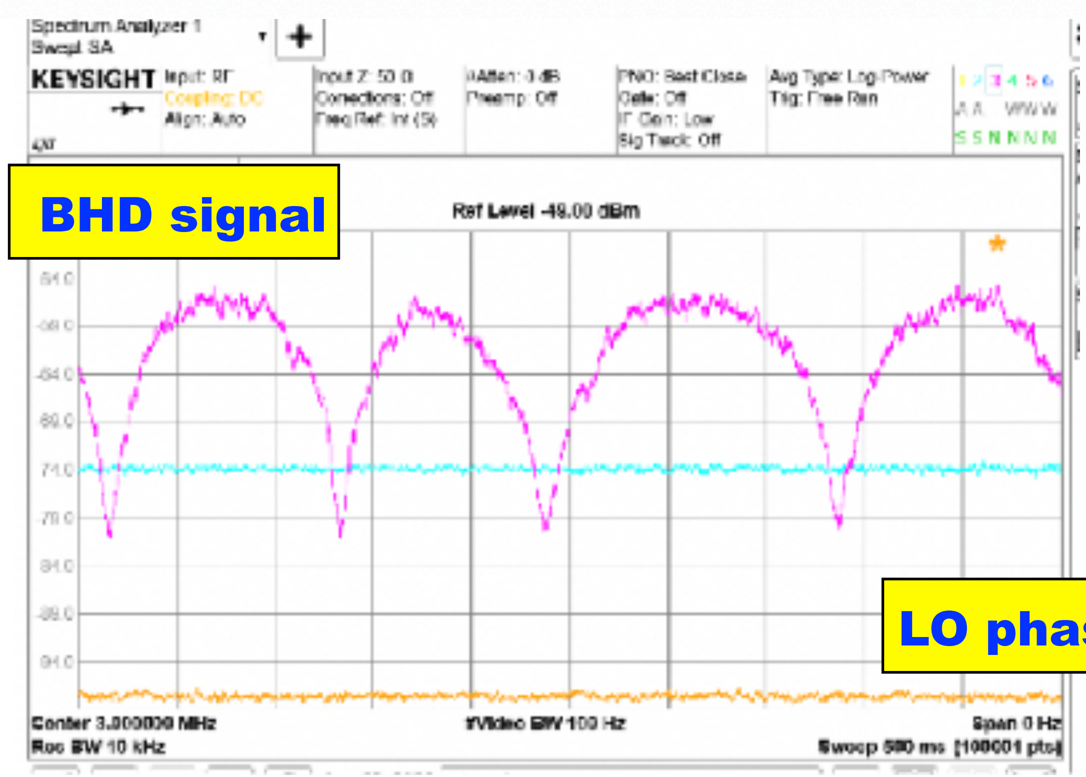
Squeezed States

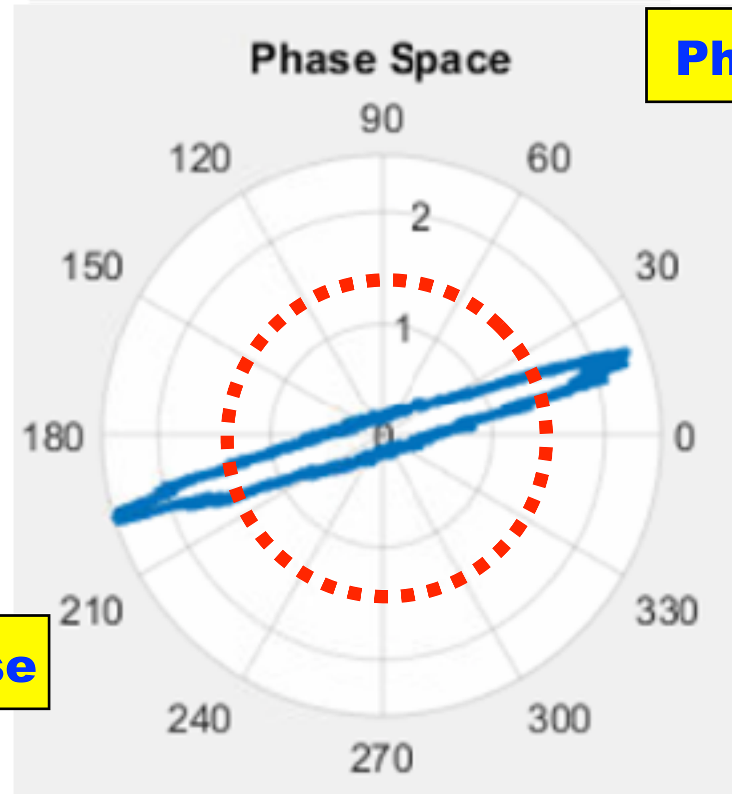


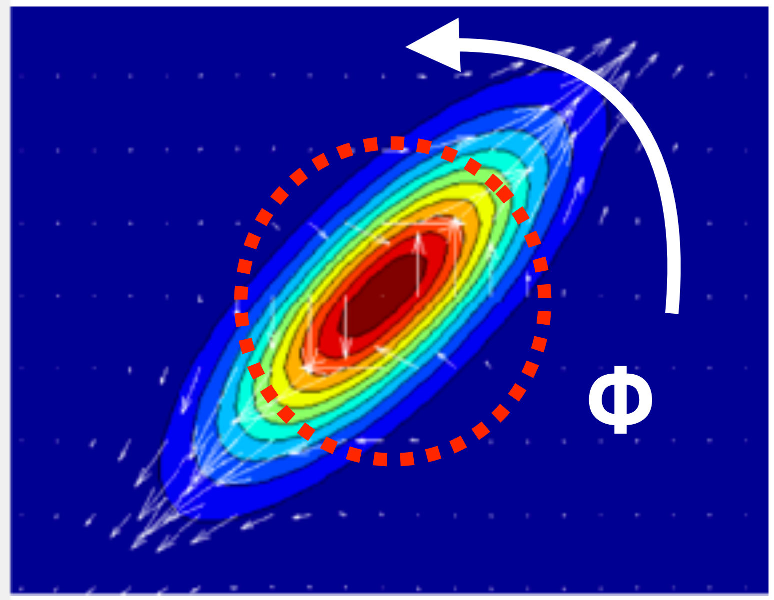




$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle$$







Squeezed States and SHO:

Optical Parametric Oscillator, OPO

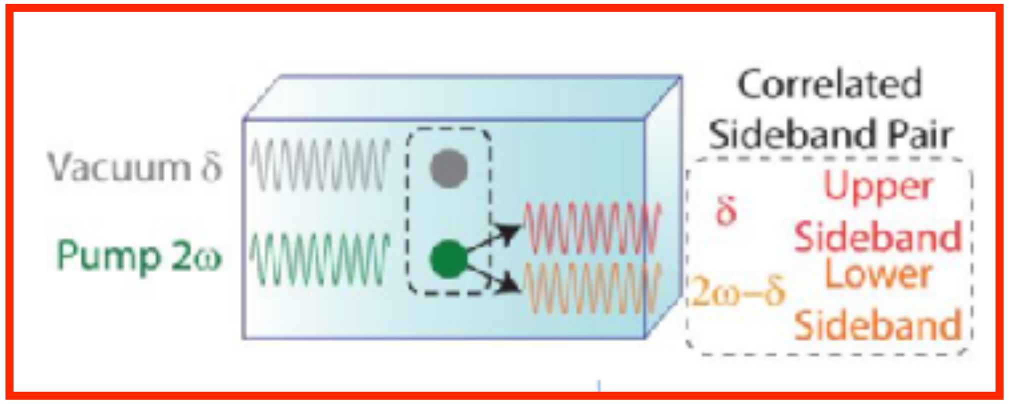
For the degenerate parametric process, *i.e.*, two-photon process, its Hamiltonian is

$$\hat{H} = i\hbar(g\hat{a}^{\dagger 2} - g^*\hat{a}^2),$$

Quantum Correlation

where g is a coupling constant. The state of fields generated by this Hamiltonian is

$$|\Psi(t)\rangle = \exp[(g\hat{a}^{\dagger 2} - g^*\hat{a}^2)t]|0\rangle.$$



Then, one can define the unitary squeeze operator

$$\hat{S}(\xi) = \exp[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}],$$

where $\xi = r\exp(i\theta)$ is an arbitrary complex number.

Squeezed Operator

$$\boxed{\hat{S}^{\dagger}(\xi) = \hat{S}^{-1}(\xi) = \hat{S}(-\xi),}$$

Squeezed Operator

- If $|\Psi\rangle$ is the vacuum state $|0\rangle$, then $|\Psi_s\rangle$ state is the *squeezed vacuum*,

$$|\xi\rangle = \hat{S}(\xi)|0\rangle.$$

- The variances for squeezed vacuum are

$$\begin{aligned}
\Delta\hat{a}_1^2 &= \frac{1}{4}[\cosh^2 r + \sinh^2 r - 2\sinh r \cosh r \cos\theta], \\
\Delta\hat{a}_2^2 &= \frac{1}{4}[\cosh^2 r + \sinh^2 r + 2\sinh r \cosh r \cos\theta],
\end{aligned}$$

- For $\theta = 0$, we have

$$\boxed{\Delta\hat{a}_1^2 = \frac{1}{4}e^{-2r}, \quad \text{and} \quad \Delta\hat{a}_2^2 = \frac{1}{4}e^{+2r},}$$

and squeezing exists in the $\hat{a}_1$ quadrature.

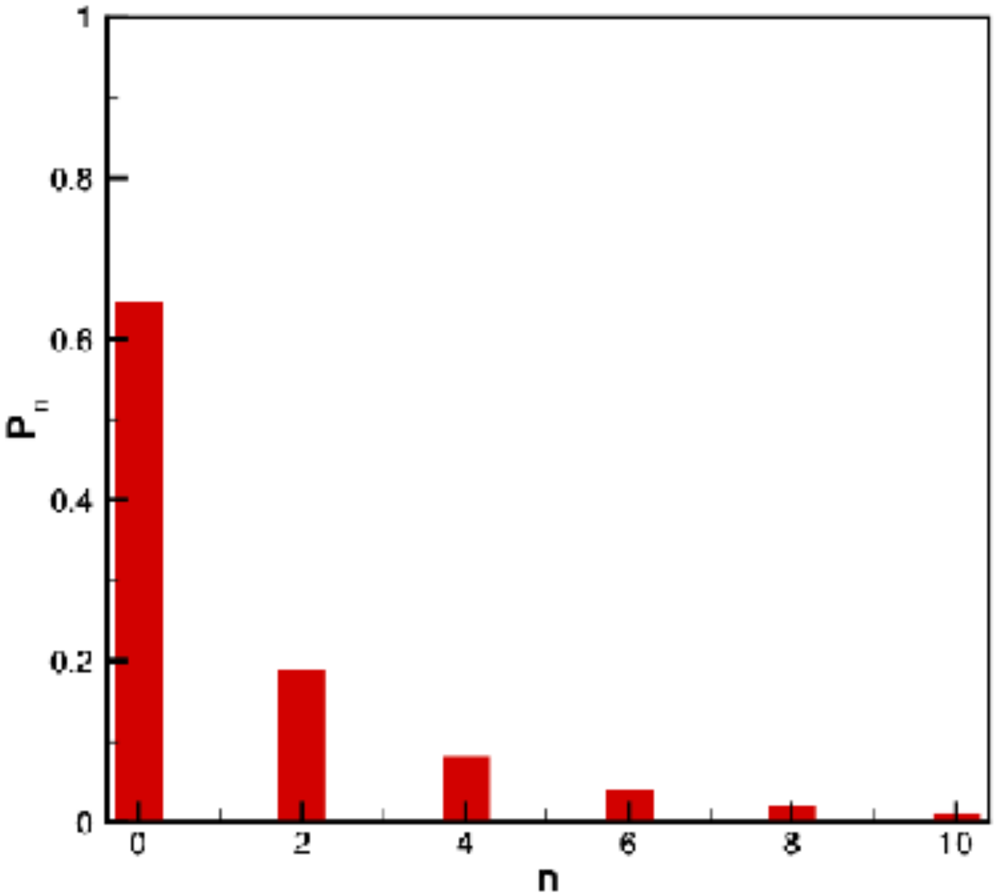
- For $\theta = \pi$, the squeezing will appear in the $\hat{a}_2$ quadrature.

Squeezed State as a Eigen-state of UR

$$\boxed{(\mu\hat{a} + \nu\hat{a}^\dagger)|\xi\rangle = 0,}$$

Squeezed states in the Number state basis

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle$$



Sub-Poisson distribution

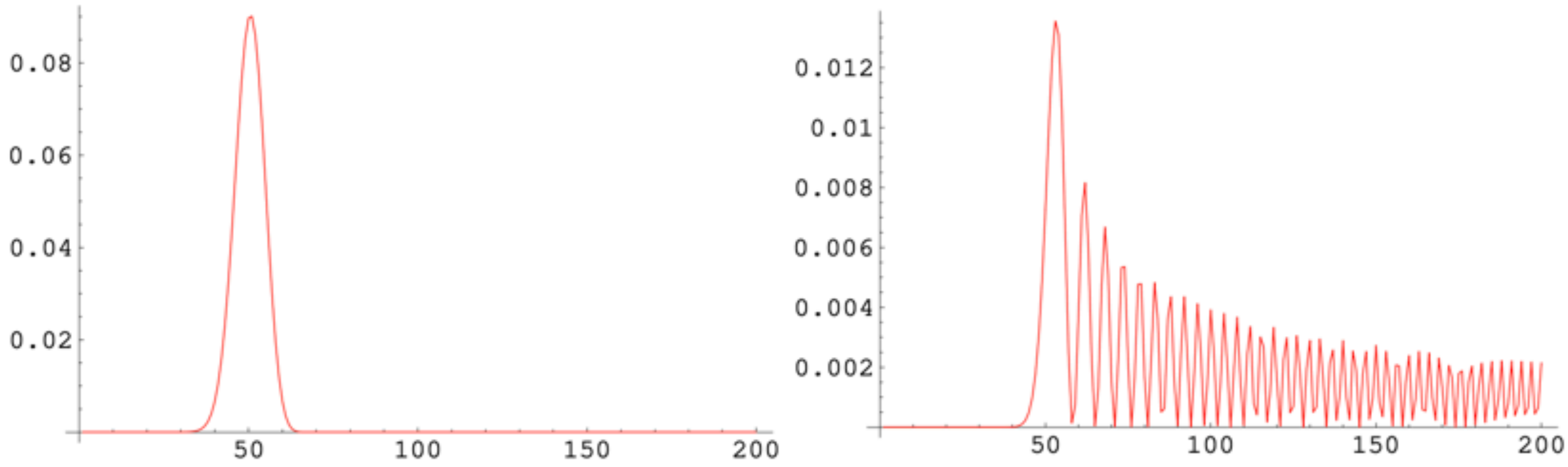
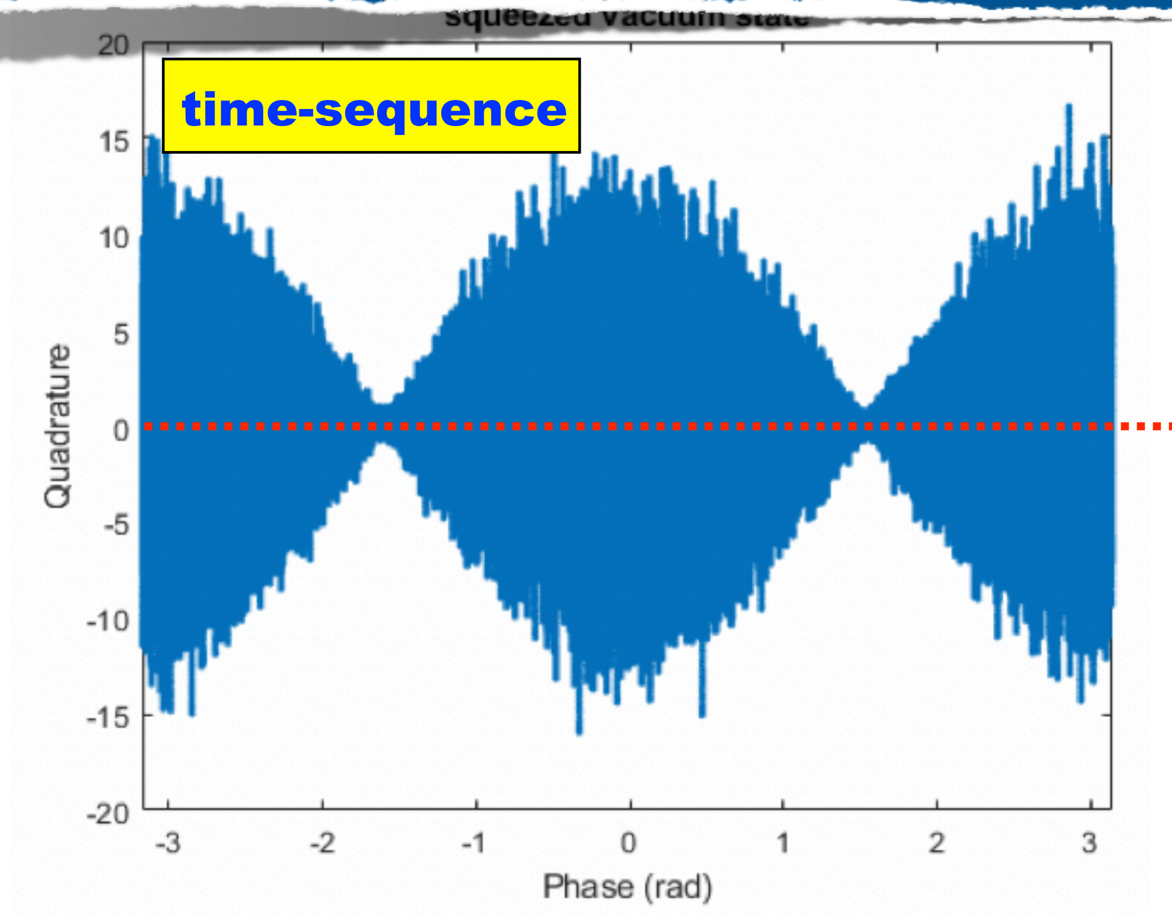


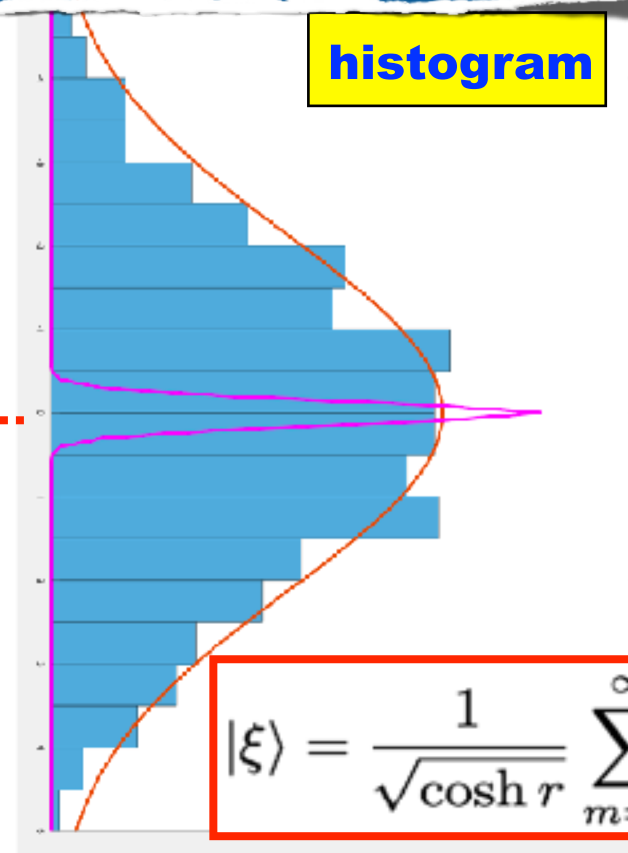
FIG. 2: Photon number distribution for squeezed coherent states. Left: $|\alpha|^2 = 50, \theta = 0, r = 0.5$; Right: $|\alpha|^2 = 50, \theta = 0, r = 4.0$.

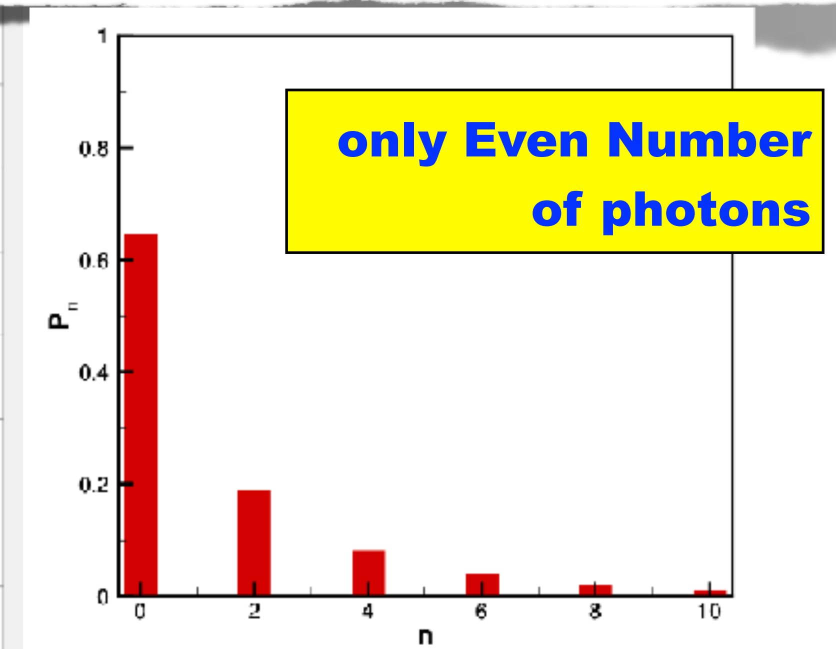
Squeezed State as a Eigen-state of UR

$$\boxed{(\mu\hat{a}+\nu\hat{a}^{\dagger})|\alpha,\xi\rangle=(\alpha\cosh r+\alpha^{*}\sinh r)|\alpha,\xi\rangle\equiv\gamma|\alpha,\xi\rangle,}$$

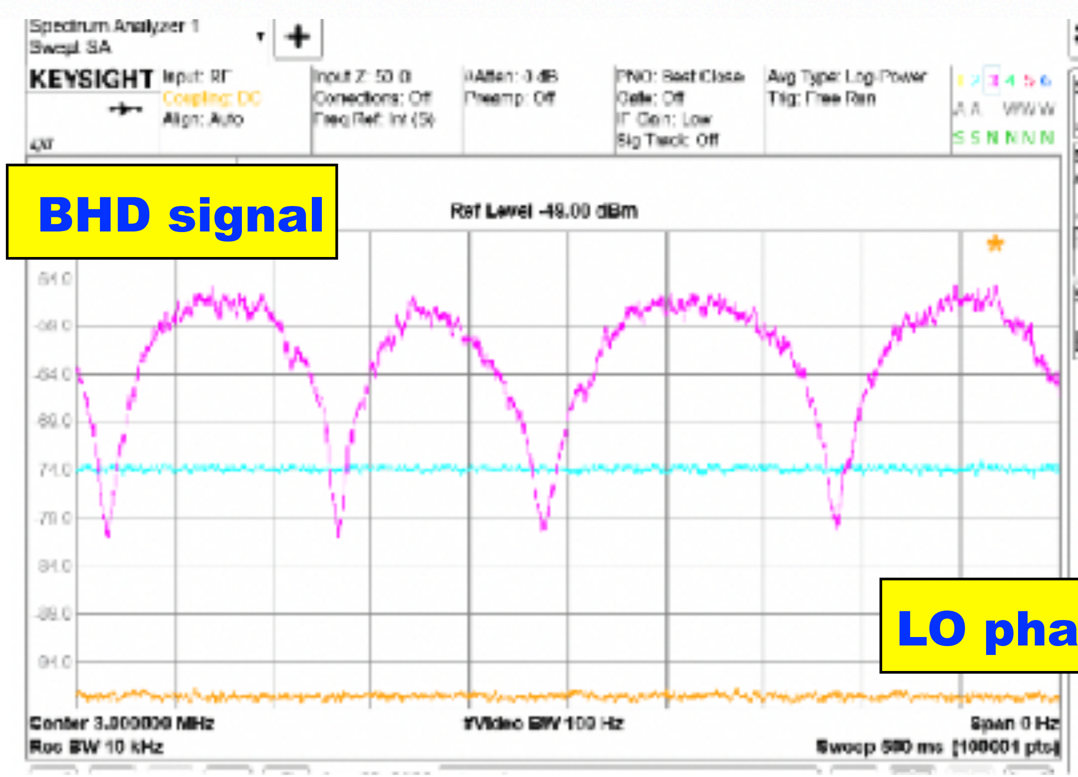
Squeezed States

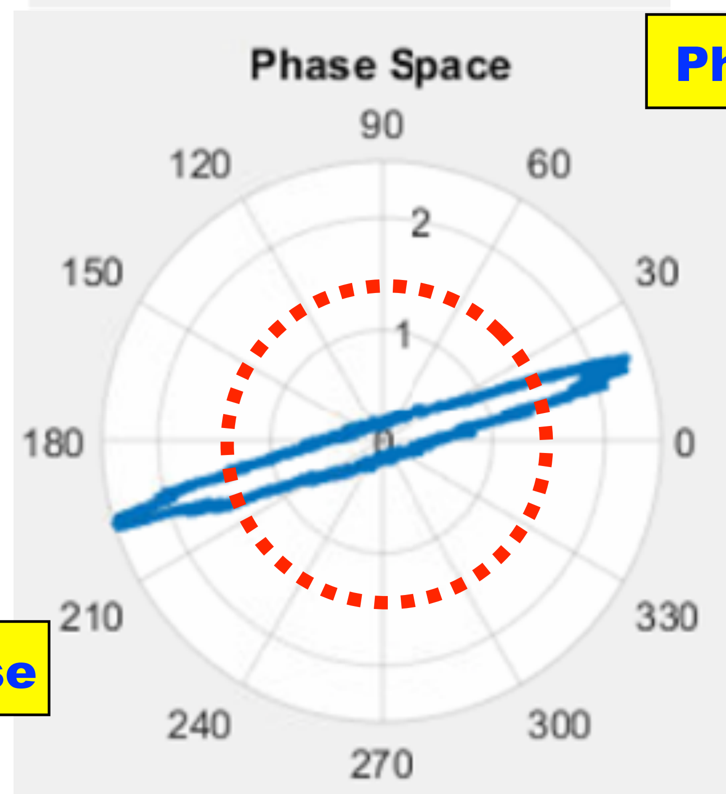


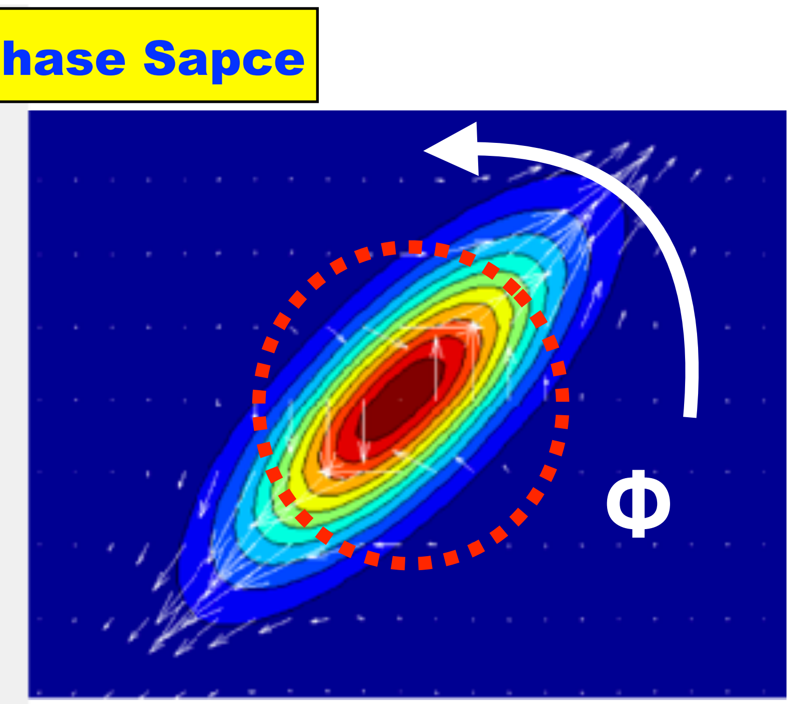




$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle$$

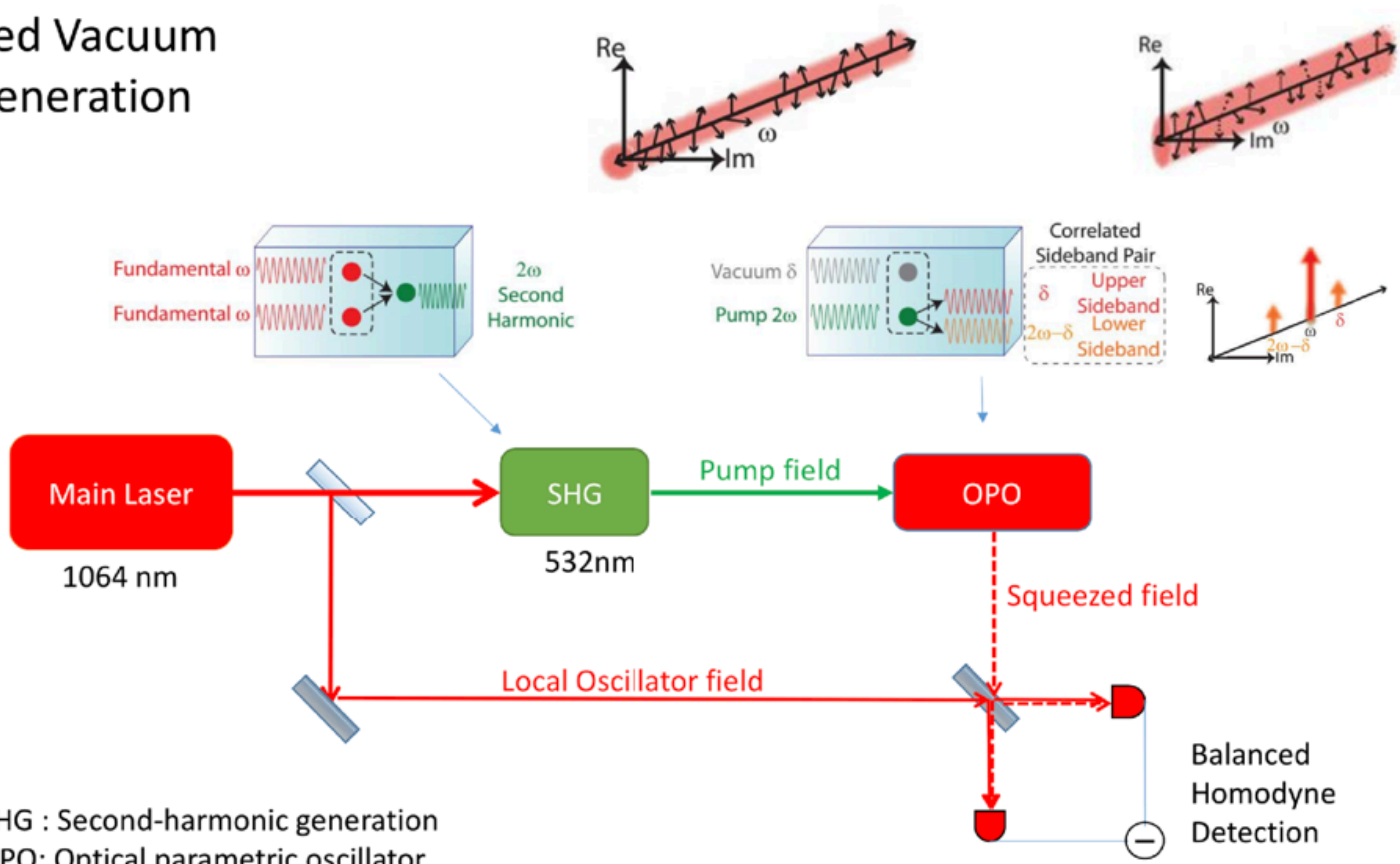






Optical Parametric Oscillator, OPO

Squeezed Vacuum State Generation



VOLUME 57, NUMBER 20 PHYSICAL REVIEW LETT

Generation of Squeezed States by Parametri

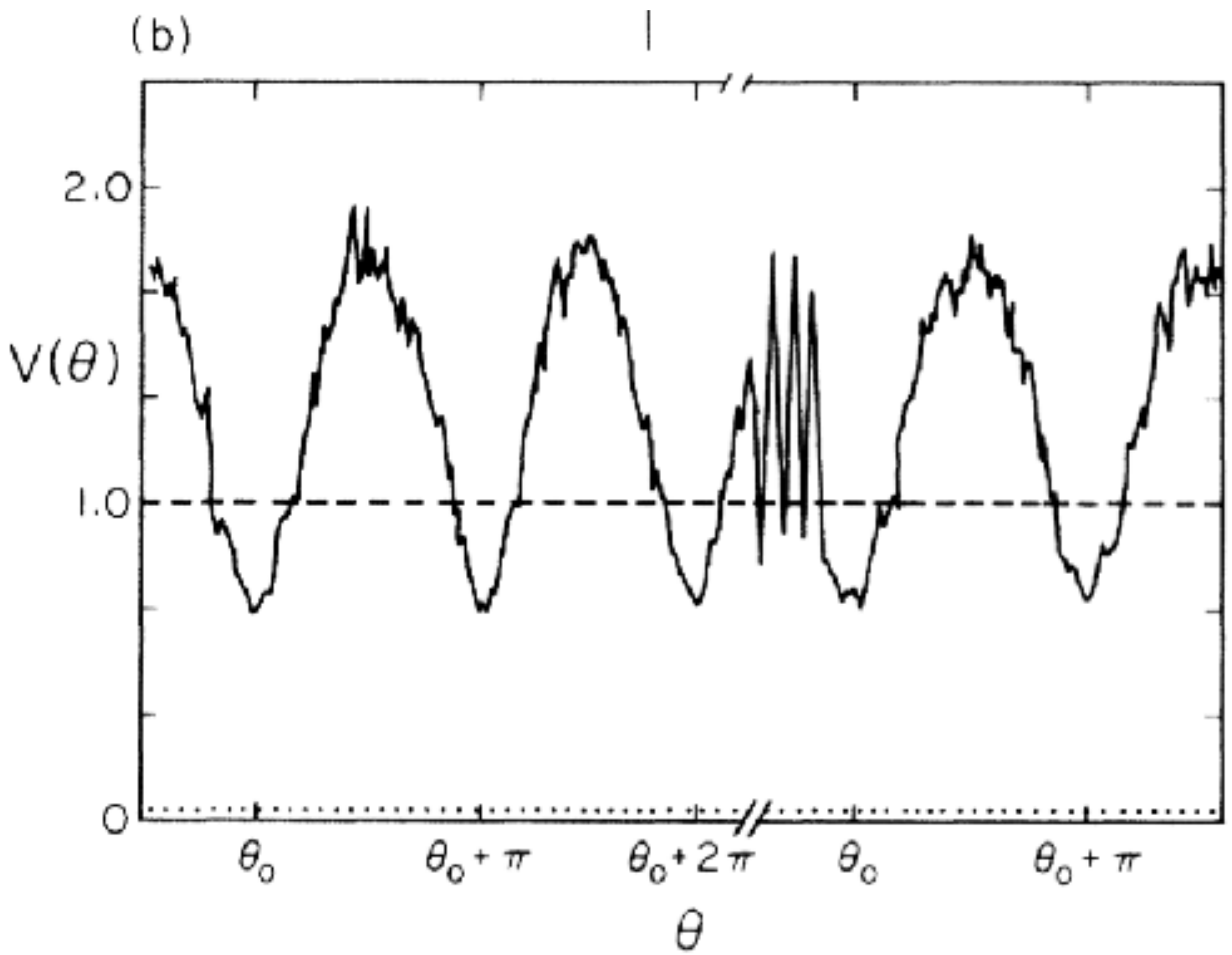
Ling-An Wu, H. J. Kimble, J. L. Hall,[a]

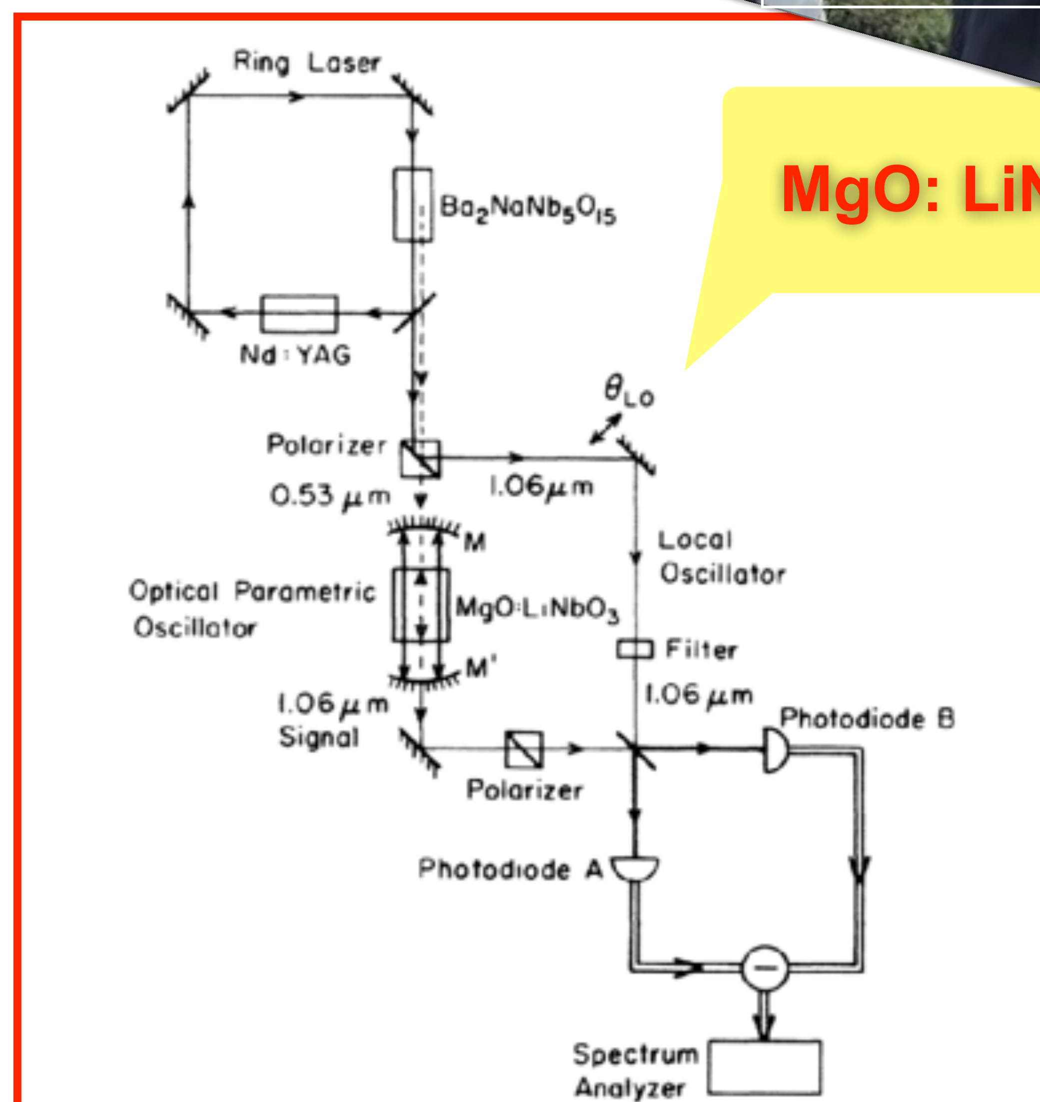
Department of Physics, University of Texas at Aust

(Received 11 September 1986)

Squeezed states of the electromagnetic field are generated b in an optical cavity. Noise reductions greater than 50% relative in a balanced homodyne detector. A quantitative comparison with theo squeezing results from a field that in the absence of linear attenuation would be sq then tenfold.

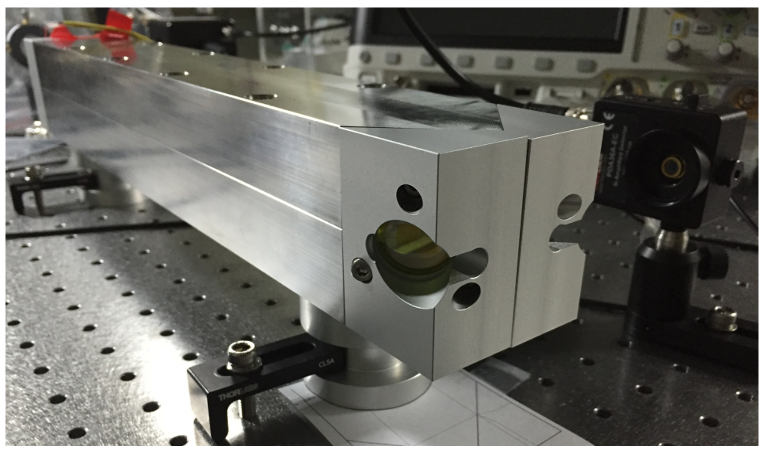
PACS numbers: 42.50.Dv, 03.65−w, 42.65.Ky



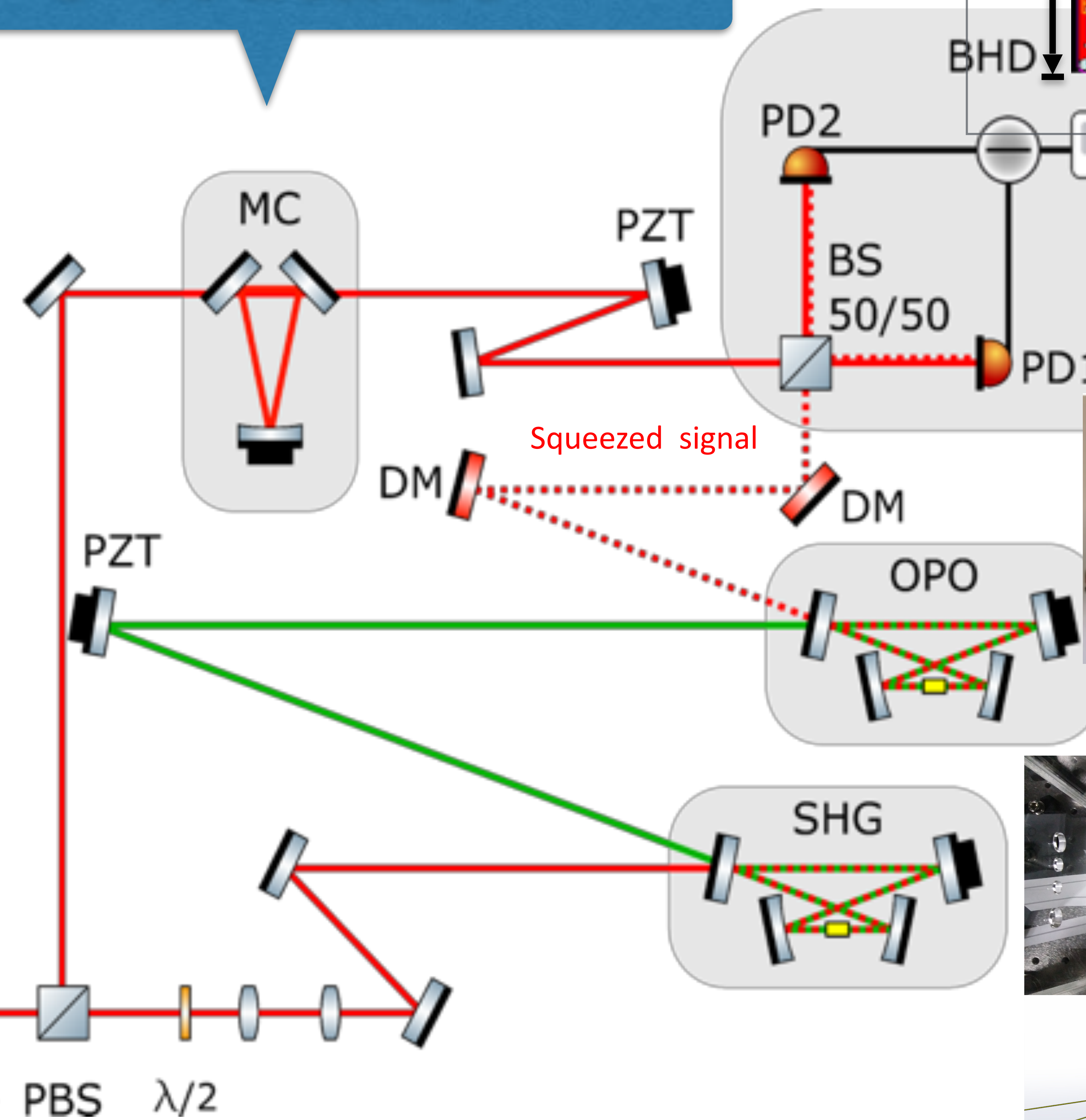


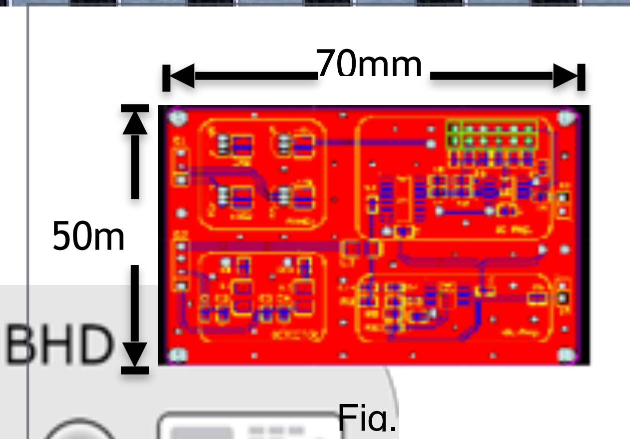
MgO: LiNbO3

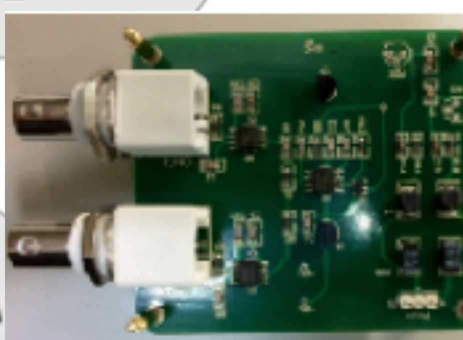
1st Squeezer @ Taiwan

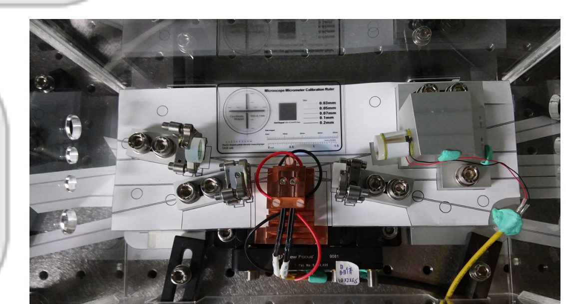


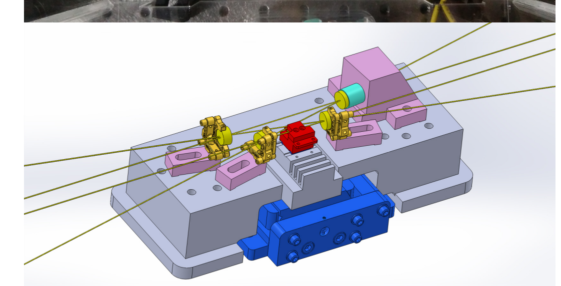
1064 nm, 2W, BW < 1 kHz







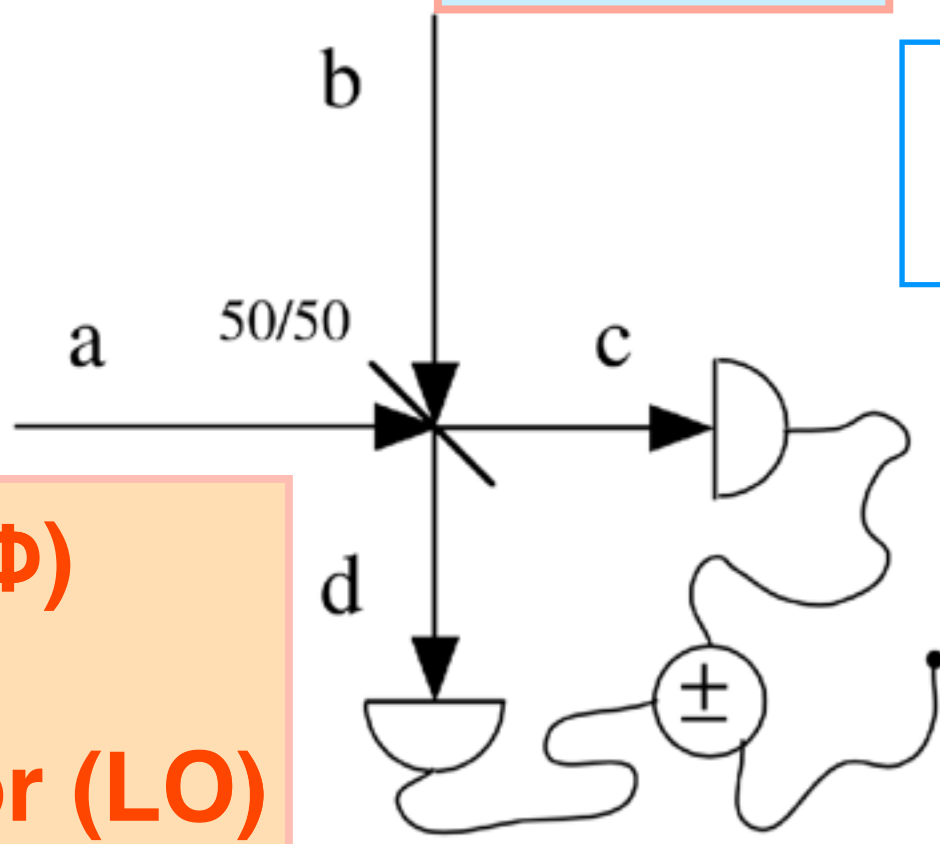




Balanced Homodyne Detector, BHD

|0> or |ξ>

$$\begin{aligned}\tilde{a} &= \alpha + \delta\tilde{a}(\omega)\\ \tilde{b} &= \beta + \delta\tilde{b}(\omega)\end{aligned}$$



$$\begin{aligned}\tilde{c} &= \sqrt{1-\xi}\,\tilde{a} - \sqrt{\xi}\,\tilde{b}\\ \tilde{d} &= \sqrt{\xi}\,\tilde{a} + \sqrt{1-\xi}\,\tilde{b}\end{aligned}$$

|α> *exp(i Φ)

Local Oscillator (LO)

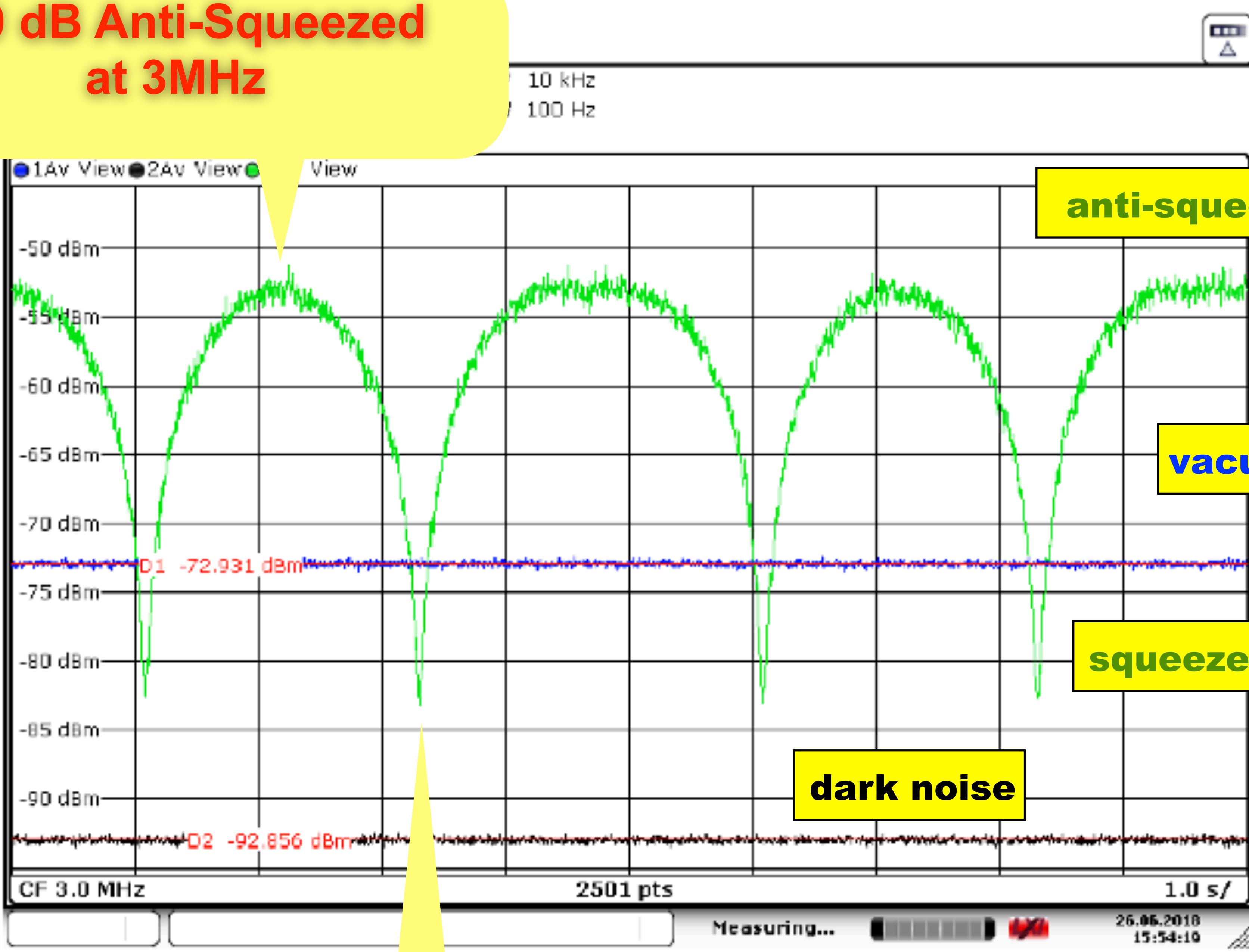
$$\tilde{n}^{-} = 0 + \alpha\,\delta\tilde{X}_1{}^{b}$$

$$\begin{aligned}\tilde{n}^{-} &= \tilde{c}^{\dagger}\tilde{c} - \tilde{d}^{\dagger}\tilde{d}\\ &= 2\alpha\beta + \alpha(\delta\tilde{b} + \delta\tilde{b}^{\dagger}) + \beta(\delta\tilde{a} + \delta\tilde{a}^{\dagger})\\ &= 2\alpha\beta + \alpha\delta\tilde{X}_1{}^{b} + \beta\delta\tilde{X}_1{}^{a}\end{aligned}$$

OPO 532nm incident power: 96mW
MC output(LO beam)= 14.5mW

specially ordered InGaAs Photodiodes
Laser Components GmbH   ϕ=500µm QE≥99%

+20 dB Anti-Squeezed at 3MHz



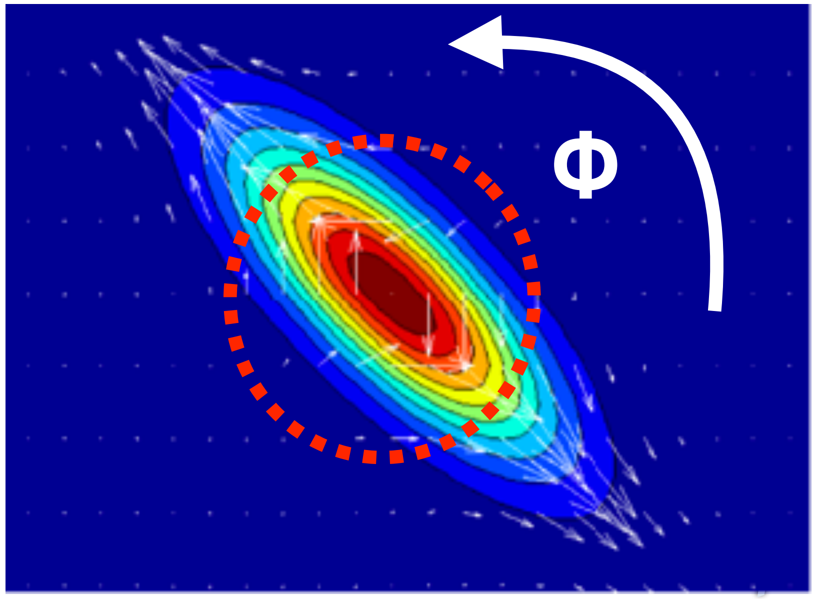
Clearance= 20dB

Zero Span mode at 3MHz
RBW=10kHz
VBW=100Hz

Blue Line: 14.5mW Vacuum noise
Black Line: Dark noise

-10 dB Squeezed vacuum at 3MHz

Squeezing angle, Φ



by Chien-Ming Wu

Date: June 26th, 2018