3, Coherent and Squeezed States

1. Coherent states
2. Squeezed states
3. Field Correlation Functions
4. Hanbury Brown and Twiss experiment
5. Photon Antibunching
6. Quantum Phenomena in Simple Nonlinear Optics

Ref:

Ch. 2, 4, 16 in *"Quantum Optics,"* by M. Scully and M. Zubairy.

Ch. 3, 4 in *"Mesoscopic Quantum Optics,"* by Y. Yamamoto and A. Imamoglu.

Ch. 6 in *"The Quantum Theory of Light,"* by R. Loudon.

Ch. 5, 7 in *"Introductory Quantum Optics,"* by C. Gerry and P. Knight.

Ch. 5, 8 in *"Quantum Optics,"* by D. Wall and G. Milburn.

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Uncertainty relation

- Non-commuting observable do not admit common eigenvectors.
- Non-commuting observables can not have definite values simultaneously.
- Simultaneous measurement of non-commuting observables to an arbitrary degree of accuracy is thus *incompatible*.
- variance: $\Delta\hat{A}^2 = \langle\Psi|(\hat{A} - \langle\hat{A}\rangle)^2|\Psi\rangle = \langle\Psi|\hat{A}^2|\Psi\rangle - \langle\Psi|\hat{A}|\Psi\rangle^2$.

$$\Delta A^2 \Delta B^2 \geq \frac{1}{4}[\langle\hat{F}\rangle^2 + \langle\hat{C}\rangle^2],$$

where

$$[\hat{A}, \hat{B}] = i\hat{C}, \qquad \text{and} \qquad \hat{F} = \hat{A}\hat{B} + \hat{B}\hat{A} - 2\langle\hat{A}\rangle\langle\hat{B}\rangle.$$

- Take the operators $\hat{A} = \hat{q}$ (position) and $\hat{B} = \hat{p}$ (momentum) for a free particle,

$$[\hat{q}, \hat{p}] = i\hbar \rightarrow \langle\Delta\hat{q}^2\rangle\langle\Delta\hat{p}^2\rangle \geq \frac{\hbar^2}{4}.$$

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Uncertainty relation

- if *Re*$(\lambda) = 0$, $\hat{A} + i\lambda\hat{B}$ is a normal operator, which have orthonormal eigenstates.
- the variances,

$$\Delta\hat{A}^2 = -\frac{i\lambda}{2}[\langle\hat{F}\rangle + i\langle\hat{C}\rangle], \qquad \Delta\hat{B}^2 = -\frac{i}{2\lambda}[\langle\hat{F}\rangle - i\langle\hat{C}\rangle],$$

- set $\lambda = \lambda_r + i\lambda_i$,

$$\Delta\hat{A}^2 = \frac{1}{2}[\lambda_i\langle\hat{F}\rangle + \lambda_r\langle\hat{C}\rangle], \qquad \Delta\hat{B}^2 = \frac{1}{|\lambda|^2}\Delta\hat{A}^2, \qquad \lambda_i\langle\hat{C}\rangle - \lambda_r\langle\hat{F}\rangle = 0.$$

- if $|\lambda| = 1$, then $\Delta\hat{A}^2 = \Delta\hat{B}^2$, *equal variance minimum uncertainty states*.
- if $|\lambda| = 1$ along with $\lambda_i = 0$, then $\Delta\hat{A}^2 = \Delta\hat{B}^2$ and $\langle\hat{F}\rangle = 0$, **uncorrelated** *equal variance minimum uncertainty states*.
- if $\lambda_r \neq 0$, then $\langle\hat{F}\rangle = \frac{\lambda_i}{\lambda_r}\langle\hat{C}\rangle$, $\qquad \Delta\hat{A}^2 = \frac{|\lambda|^2}{2\lambda_r}\langle\hat{C}\rangle$, $\qquad \Delta\hat{B}^2 = \frac{1}{2\lambda_r}\langle\hat{C}\rangle$.
  If $\hat{C}$ is a positive operator then the minimum uncertainty states exist only if $\lambda_r > 0$.

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Minimum Uncertainty State

- $(\hat{q} - \langle\hat{q}\rangle)|\psi\rangle = -i\lambda(\hat{p} - \langle\hat{p}\rangle)|\psi\rangle$
- if we define $\lambda = e^{-2r}$, then

$$(e^r\hat{q} + ie^{-r}\hat{p})|\psi\rangle = (e^r\langle\hat{q}\rangle + ie^{-r}\langle\hat{p}\rangle)|\psi\rangle,$$

- the minimum uncertainty state is defined as an *eigenstate* of a non-Hermitian operator $e^r\hat{q} + ie^{-r}\hat{p}$ with a c-number eigenvalue $e^r\langle\hat{q}\rangle + ie^{-r}\langle\hat{p}\rangle$.
- the variances of $\hat{q}$ and $\hat{p}$ are

$$\langle\Delta\hat{q}^2\rangle = \frac{\hbar}{2}e^{-2r}, \qquad \langle\Delta\hat{p}^2\rangle = \frac{\hbar}{2}e^{2r}.$$

- here r is referred as the *squeezing parameter*.

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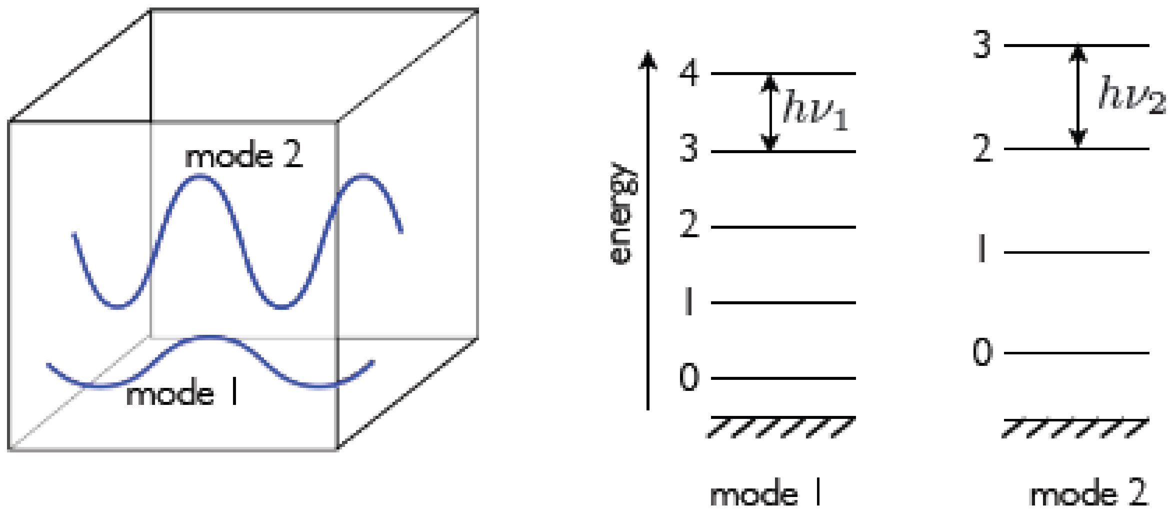
Quantization of EM fields

- the Hamiltonian for EM fields becomes: $\hat{H} = \sum_j \hbar\omega_j (\hat{a}_j^\dagger \hat{a}_j + \frac{1}{2})$,
- the electric and magnetic fields become,

$$\begin{aligned}\hat{E}_x(z,t) &= \sum_j \left(\frac{\hbar\omega_j}{\epsilon_0 V}\right)^{1/2} [\hat{a}_j e^{-i\omega_j t} + \hat{a}_j^\dagger e^{i\omega_j t}] \sin(k_j z), \\ &= \sum_j c_j [\hat{a}_{1j} \cos\omega_j t + \hat{a}_{2j} \sin\omega_j t] u_j(r),\end{aligned}$$



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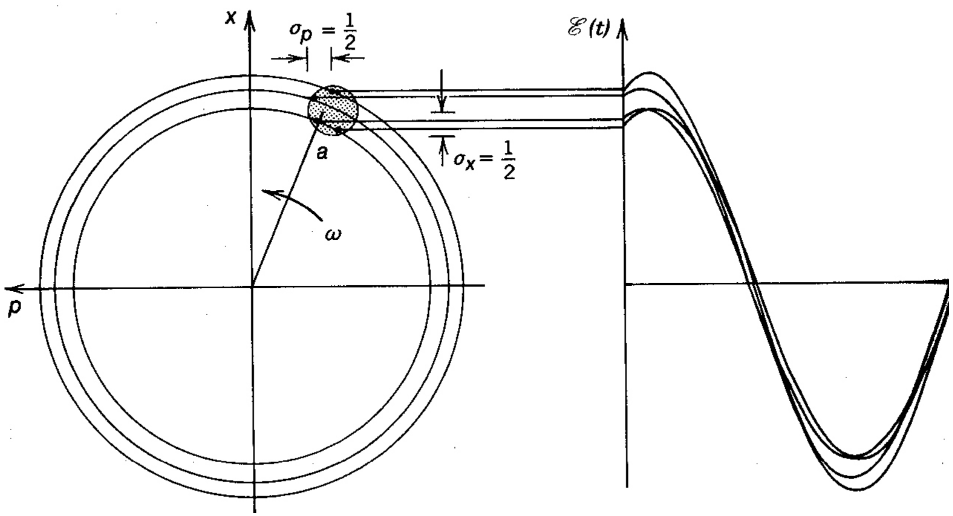
Phase diagram for EM waves

Electromagnetic waves can be represented by

$$\hat{E}(t) = E_0[\hat{X}_1 \sin(\omega t) - \hat{X}_2 \cos(\omega t)]$$

where

$$\hat{X}_1 = \text{amplitude quadrature}$$
$$\hat{X}_2 = \text{phase quadrature}$$



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Quadrature operators

- the electric and magnetic fields become,

$$\hat{E}_x(z,t) = \sum_j \left(\frac{\hbar\omega_j}{\epsilon_0 V}\right)^{1/2}[\hat{a}_j e^{-i\omega_j t} + \hat{a}_j^\dagger e^{i\omega_j t}]\sin(k_j z),$$
$$= \sum_j c_j[\hat{a}_{1j}\cos\omega_j t + \hat{a}_{2j}\sin\omega_j t]u_j(r),$$

- note that $\hat{a}$ and $\hat{a}^\dagger$ are not hermitian operators, but $(\hat{a}^\dagger)^\dagger = \hat{a}$.
- $\hat{a}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$ and $\hat{a}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$ are two Hermitian (quadrature) operators.
- the commutation relation for $\hat{a}$ and $\hat{a}^\dagger$ is $[\hat{a}, \hat{a}^\dagger] = 1$,
- the commutation relation for $\hat{a}$ and $\hat{a}^\dagger$ is $[\hat{a}_1, \hat{a}_2] = \frac{i}{2}$,
- and $\langle\Delta\hat{a}_1^2\rangle\langle\Delta\hat{a}_2^2\rangle \geq \frac{1}{16}$.

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Minimum Uncertainty State

- $(\hat{a}_1 - \langle\hat{a}_1\rangle)|\psi\rangle = -i\lambda(\hat{a}_2 - \langle\hat{a}_2\rangle)|\psi\rangle$
- if we define $\lambda = e^{-2r}$, then $(e^r\hat{a}_1 + ie^{-r}\hat{a}_2)|\psi\rangle = (e^r\langle\hat{a}_1\rangle + ie^{-r}\langle\hat{a}_2\rangle)|\psi\rangle$,
- the minimum uncertainty state is defined as an *eigenstate* of a non-Hermitian operator $e^r\hat{a}_1 + ie^{-r}\hat{a}_2$ with a c-number eigenvalue $e^r\langle\hat{a}_1\rangle + ie^{-r}\langle\hat{a}_2\rangle$.
- the variances of $\hat{a}_1$ and $\hat{a}_2$ are

$$\langle\Delta\hat{a}_1^2\rangle = \frac{1}{4}e^{-2r}, \qquad \langle\Delta\hat{a}_2^2\rangle = \frac{1}{4}e^{2r}.$$

- here r is referred as the *squeezing parameter*.
- when $r = 0$, the two quadrature amplitudes have identical variances,

$$\langle\Delta\hat{a}_1^2\rangle = \langle\Delta\hat{a}_2^2\rangle = \frac{1}{4},$$

- in this case, the non-Hermitian operator, $e^r\hat{a}_1 + ie^{-r}\hat{a}_2 = \hat{a}_1 + i\hat{a}_2 = \hat{a}$, and this minimum uncertainty state is termed a *coherent state* of the electromagnetic field, an eigenstate of the annihilation operator, $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

Coherent States

- in this case, the non-Hermitian operator, $e^r\hat{a}_1 + ie^{-r}\hat{a}_2 = \hat{a}_1 + i\hat{a}_2 = \hat{a}$, and this minimum uncertainty state is termed a *coherent state* of the electromagnetic field, an eigenstate of the annihilation operator,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle.$$

- expand the coherent states in the basis of number states,

$$|\alpha\rangle = \sum_n |n\rangle\langle n|\alpha\rangle = \sum_n |n\rangle\langle 0|\frac{\hat{a}^n}{\sqrt{n!}}|\alpha\rangle = \sum_n \frac{\alpha^n}{\sqrt{n!}}\langle 0|\alpha\rangle|n\rangle,$$

- imposing the normalization condition, $\langle\alpha|\alpha\rangle = 1$, we obtain,

$$1 = \langle\alpha|\alpha\rangle = \sum_n\sum_m \langle m|n\rangle \frac{(\alpha^*)^m\alpha^n}{\sqrt{m!}\sqrt{n!}} = e^{|\alpha|^2}|\langle 0|\alpha\rangle|^2,$$

- we have

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

Properties of Coherent States

- the coherent state can be expressed using the photon number eigenstates,

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,$$

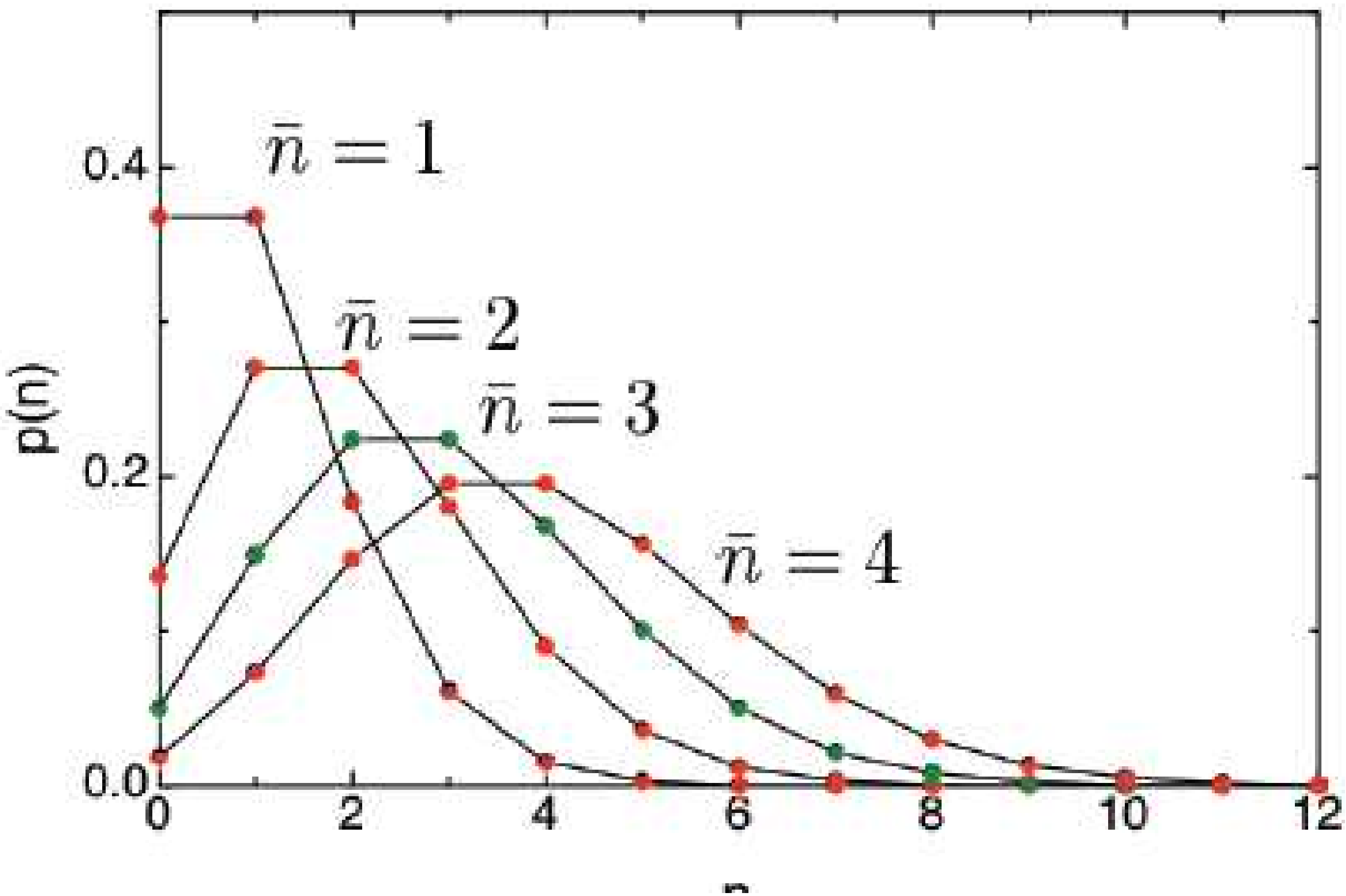
- the probability of finding the photon number n for the coherent state obeys the *Poisson distribution*,

$$P(n) \equiv |\langle n|\alpha\rangle|^2 = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!},$$

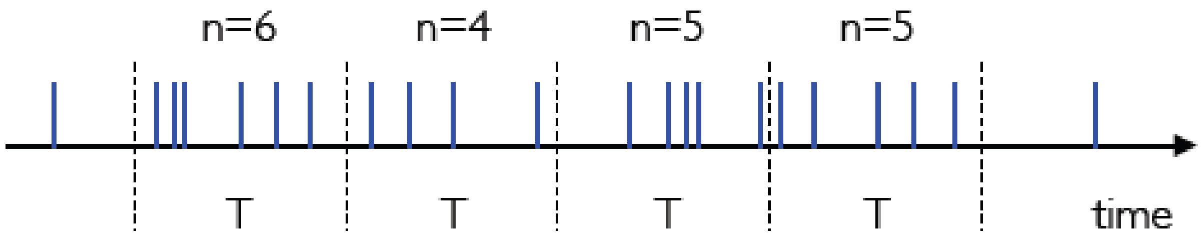
- the mean and variance of the photon number for the coherent state $|\alpha\rangle$ are,

$$\begin{aligned}\langle \hat{n} \rangle &= \sum_n n P(n) = |\alpha|^2, \\ \langle \Delta \hat{n}^2 \rangle &= \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 = |\alpha|^2 = \langle \hat{n} \rangle,\end{aligned}$$

Poisson distribution



Photon number statistics



- For photons are independent of each other, the probability of occurrence of n photons, or photoelectrons in a time interval T is random. Divide T into N intervals, the probability to find one photon per interval is, $p = \bar{n}/N$,
- the probability to find no photon per interval is, $1 - p$,
- the probability to find n photons per interval is,

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n},$$

which is a binomial distribution.

- when $N \to \infty$,

$$P(n) = \frac{\bar{n}^n \text{exp}(-\bar{n})}{n!},$$

this is the *Poisson distribution* and the characteristics of coherent light.

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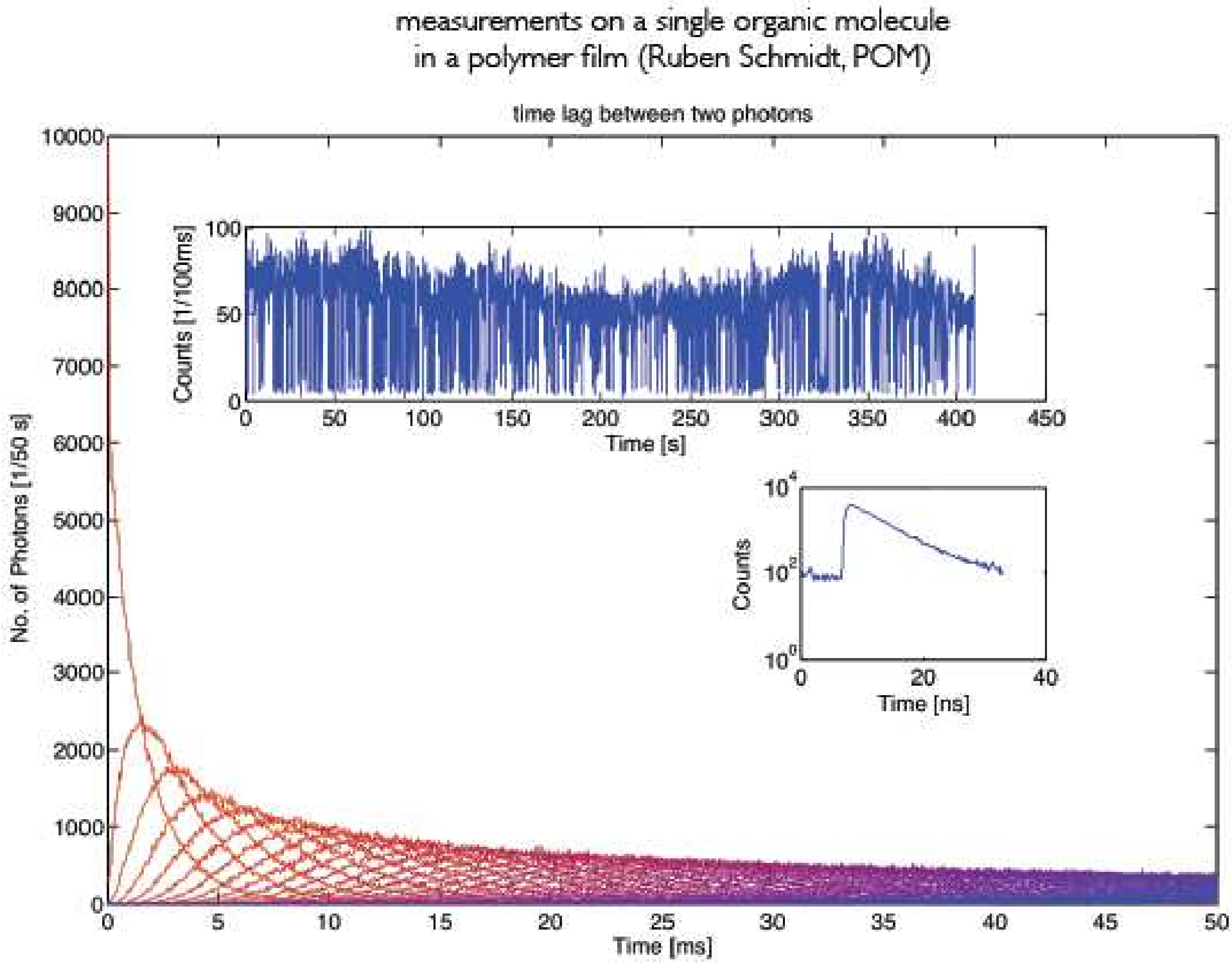
Real life Poisson distribution



Displacement operator

- coherent states are generated by translating the vacuum state $|0\rangle$ to have a finite excitation amplitude α,

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{(\alpha\hat{a}^\dagger)^n}{n!}|0\rangle,$$
$$= e^{-\frac{1}{2}|\alpha|^2}e^{\alpha\hat{a}^\dagger}|0\rangle,$$

- since $\hat{a}|0\rangle = 0$, we have $e^{-\alpha^*\hat{a}}|0\rangle = 0$ and

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}e^{\alpha\hat{a}^\dagger}e^{-\alpha^*\hat{a}}|0\rangle,$$

- any two noncommuting operators $\hat{A}$ and $\hat{B}$ satisfy the Baker-Hausdorff relation, $e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-\frac{1}{2}[\hat{A},\hat{B}]}$, provided $[\hat{A},[\hat{A},\hat{B}]] = 0$,
- using $\hat{A} = \alpha\hat{a}^\dagger$, $\hat{B} = -\alpha^*\hat{a}$, and $[\hat{A},\hat{B}] = |\alpha|^2$, we have,

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = e^{-\alpha\hat{a}^\dagger-\alpha^*\hat{a}}|0\rangle,$$

where $\hat{D}(\alpha)$ is the *displacement operator*, which is physically realized by a classical oscillating current.

Displacement operator

- the coherent state is the displaced form of the harmonic oscillator ground state,

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = e^{-\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle,$$

  where $\hat{D}(\alpha)$ is the *displacement operator*, which is physically realized by a classical oscillating current,

- the displacement operator $\hat{D}(\alpha)$ is a unitary operator, i.e.

$$\hat{D}^\dagger(\alpha) = \hat{D}(-\alpha) = [\hat{D}(\alpha)]^{-1},$$

- $\hat{D}(\alpha)$ acts as a displacement operator upon the amplitudes $\hat{a}$ and $\hat{a}^\dagger$, i.e.

$$\begin{aligned}\hat{D}^{-1}(\alpha)\hat{a}\hat{D}(\alpha) &= \hat{a} + \alpha,\\ \hat{D}^{-1}(\alpha)\hat{a}^\dagger\hat{D}(\alpha) &= \hat{a}^\dagger + \alpha^*,\end{aligned}$$

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Radiation from a classical current

- the Hamiltonian (p · A) that describes the interaction between the field and the current is given by

$$\mathbf{V} = \int \mathbf{J}(r,t) \cdot \hat{A}(r,t) \mathrm{d}^3 r,$$

where $\mathbf{J}(r,t)$ is the classical current and $\hat{A}(r,t)$ is quantized vector potential,

$$\hat{A}(r,t) = -i \sum_k \frac{1}{\omega_k} E_k \hat{a}_k e^{-i\omega_k t + ik\cdot r} + \text{H.c.},$$

- the interaction picture Schrödinger equation obeys,

$$\frac{\mathrm{d}}{\mathrm{d}t} |\Psi(t)\rangle = -\frac{i}{\hbar} \mathbf{V} |\Psi(t)\rangle,$$

- the solution is $|\Psi(t)\rangle = \prod_k \exp[\alpha_k \hat{a}^\dagger - \alpha_k^* \hat{a}_k]|0\rangle_k$, where $\alpha_k = \frac{1}{\hbar\omega_k} E_k \int_0^t \mathrm{d}t' \int \mathrm{d}r \mathbf{J}(r,t) e^{i\omega t' - ik\cdot r}$,
- this state of radiation field is called a coherent state,

$$|\alpha\rangle = (\alpha\hat{a}^\dagger - \alpha^* \hat{a})|0\rangle.$$

Properties of Coherent States

- the probability of finding n photons in $|\alpha\rangle$ is given by a Poisson distribution,
- the coherent state is a minimum-uncertainty states,
- the set of all coherent states $|\alpha\rangle$ is a complete set,

$$\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = \pi \sum_n |n\rangle\langle n|, \quad \text{or} \quad \frac{1}{\pi}\int |\alpha\rangle\langle\alpha| \mathrm{d}^2\alpha = 1,$$

- two coherent states corresponding to different eigenstates α and β are not orthogonal,

$$\langle\alpha|\beta\rangle = \exp(-\frac{1}{2}|\alpha|^2 + \alpha^*\beta - \frac{1}{2}|\beta|^2) = \exp(-\frac{1}{2}|\alpha - \beta|^2),$$

- coherent states are *approximately* orthogonal only in the limit of large separation of the two eigenvalues, $|\alpha - \beta| \to \infty$,

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Properties of Coherent States

- therefore, any coherent state can be expanded using other coherent state,

$$|\alpha\rangle = \frac{1}{\pi}\int \mathrm{d}^2\beta |\beta\rangle\langle\beta|\alpha\rangle = \frac{1}{\pi}\int \mathrm{d}^2\beta e^{-\frac{1}{2}|\beta-\alpha|^2}|\beta\rangle,$$

- this means that a coherent state forms an *overcomplete* set,
- the simultaneous measurement of $\hat{a}_1$ and $\hat{a}_2$, represented by the projection operator $|\alpha\rangle\langle\alpha|$, is not an exact measurement but instead an approximate measurement with a finite measurement error.

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q-representation of the coherent state

- coherent state is defined as the eigenstate of the annihilation operator,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle,$$

where $\hat{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} + i\hat{p})$,

- the q-representation of the coherent state is,

$$(\omega q + \hbar\frac{\partial}{\partial q})\langle q|\alpha\rangle = \sqrt{2\hbar\omega}\alpha\langle q|\alpha\rangle,$$

- with the solution,

$$\langle q|\alpha\rangle = (\frac{\omega}{\pi\hbar})^{1/4}\text{exp}[-\frac{\omega}{2\hbar}(q - \langle q\rangle)^2 + i\frac{\langle p\rangle}{\hbar}q + i\theta],$$

where θ is an arbitrary real phase,

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Expectation value of the electric field

- for a single mode electric field, polarized in the x-direction,

$$\hat{E}_x = E_0[\hat{a}(t) + \hat{a}^\dagger(t)] \sin kz,$$

- the expectation value of the electric field operator,

$$\langle\alpha|\hat{E}(t)|\alpha\rangle = E_0[\alpha e^{-i\omega t} + \alpha^* e^{i\omega t}] \sin kz = 2E_0|\alpha| \cos(\omega t + \phi) \sin kz,$$

- similar,

$$\langle\alpha|\hat{E}(t)^2|\alpha\rangle = E_0^2[4|\alpha|^2 \cos^2(\omega t + \phi) + 1] \sin^2 kz,$$

- the root-mean-square deviation int the electric field is,

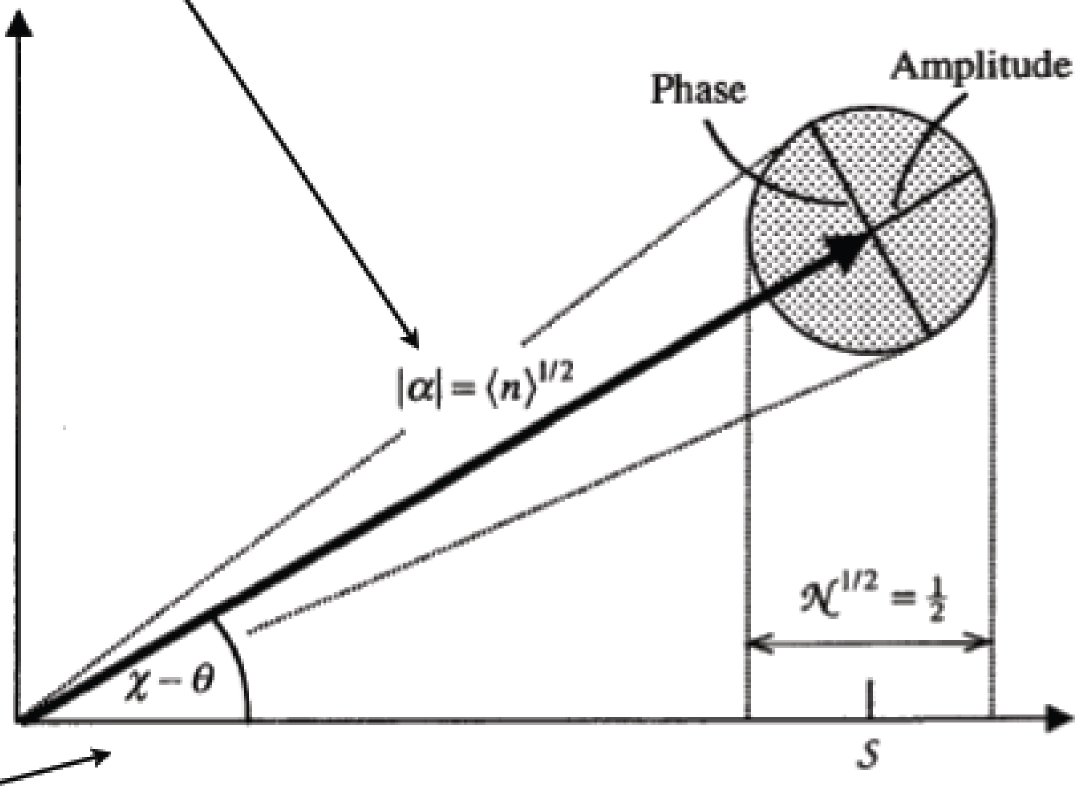
$$\langle\Delta\hat{E}(t)^2\rangle^{1/2} = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} |\sin kz|,$$

- $\langle\Delta\hat{E}(t)^2\rangle^{1/2}$ is independent of the field strength $|\alpha|$,
- quantum noise becomes less important as $|\alpha|^2$ increases, or why a highly excited coherent state $|\alpha| \gg 1$ can be treated as a *classical* EM field.

Phase diagram for coherent states

mean number of photons

$$< \hat{N} >=< \alpha|\hat{N}|\alpha >=< \alpha|\hat{a}^\dagger \hat{a}|\alpha >= |\alpha|^2$$



phase of the field

$$\alpha = |\alpha| \exp(i\theta)$$

Generation of Coherent States

- In classical mechanics we can excite a SHO into motion by, e.g. stretching the spring to a new equilibrium position,

$$
\begin{aligned}
\hat{H} &= \frac{p^2}{2m} + \frac{1}{2}kx^2 - eE_0x, \\
&= \frac{p^2}{2m} + \frac{1}{2}k(x - \frac{eE_0}{k})^2 - \frac{1}{2}(\frac{eE_0}{k})^2,
\end{aligned}
$$

- upon turning off the dc field, i.e. $E_0 = 0$, we will have a coherent state $|\alpha\rangle$ which oscillates without changing its shape,
- applying the dc field to the SHO is mathematically equivalent to applying the displacement operator to the state $|0\rangle$.

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Generation of Coherent States

- a classical external force $f(t)$ couples linearly to the generalized coordinate of the harmonic oscillator,

$$\hat{H} = \hbar\omega(\hat{a}\hat{a}^\dagger + \frac{1}{2}) + \hbar[f(t)\hat{a} + f^*(t)\hat{a}^\dagger],$$

- for the initial state $|\Psi(0)\rangle = |0\rangle$, the solution is

$$|\Psi(t)\rangle = \text{exp}[A(t) + C(t)\hat{a}^\dagger]|0\rangle,$$

where

$$A(t) = -\int_0^t \text{d}t" f(t") \int_0^{t"} \text{d}t' e^{i\omega(t'-t")} f(t'), \qquad C(t) = -i\int_0^t \text{d}t' e^{i\omega(t'-t)} f^*(t'),$$

- When the classical driving force $f(t)$ is resonant with the harmonic oscillator, $f(t) = f_0 e^{i\omega t}$, we have

$$C(t) = -ie^{-i\omega t} f_0 t \equiv \alpha, \quad A(t) = -\frac{1}{2}(f_0 t)^2 = -\frac{|\alpha|^2}{2}, \quad \text{and} \quad |\Psi(t)\rangle = |\alpha\rangle.$$

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Attenuation of Coherent States

- Glauber showed that a classical oscillating current in free space produces a multimode coherent state of light.
- The quantum noise of a laser operating at far above threshold is close to that of a coherent state.
- A coherent state does not change its quantum noise properties if it is attenuated,
- a beam splitter with inputs combined by a coherent state and a vacuum state $|0\rangle$,

$$\hat{H}_I = \hbar\kappa(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger), \qquad \text{interaction Hamiltonian}$$

  where κ is a coupling constant between two modes,
- the output state is, with $\beta = \sqrt{T}\alpha$ and $\gamma = \sqrt{1-T}\alpha$,

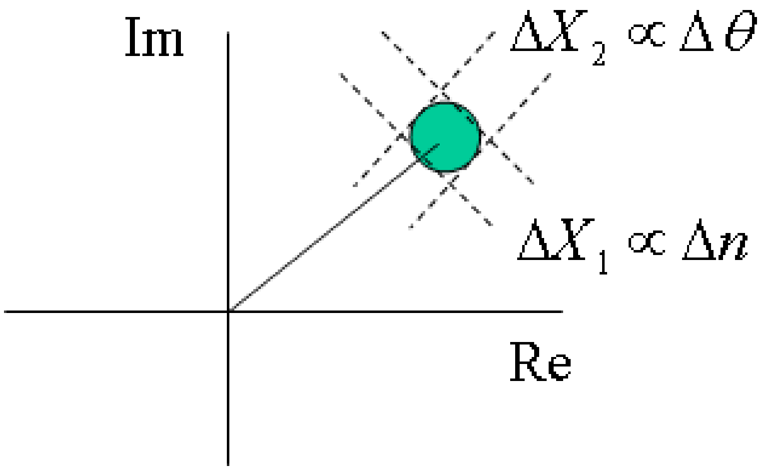
$$|\Psi\rangle_{\text{out}} = \hat{U}|\alpha\rangle_a|0\rangle_b = |\beta\rangle_a|\gamma\rangle_b, \quad \text{with} \quad \hat{U} = \exp[i\kappa(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)t],$$

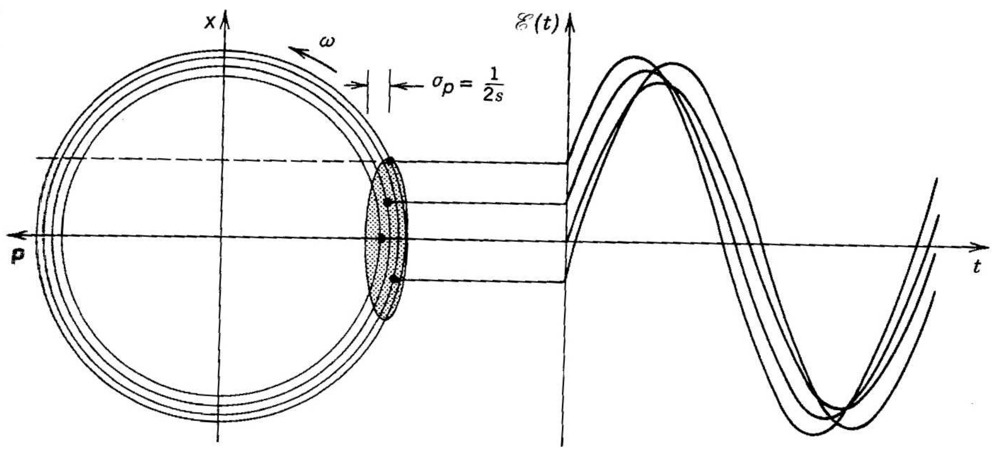
- The reservoirs consisting of ground state harmonic oscillators inject the vacuum fluctuation and partially replace the original quantum noise of the coherent state.
- Since the vacuum state is also a coherent state, the overall noise is unchanged.

Coherent and Squeezed States

Uncertainty Principle: $\Delta\hat{X}_1\Delta\hat{X}_2 \geq 1$.

1. Coherent states: $\Delta\hat{X}_1 = \Delta\hat{X}_2 = 1$,
2. Amplitude squeezed states: $\Delta\hat{X}_1 < 1$,
3. Phase squeezed states: $\Delta\hat{X}_2 < 1$,
4. Quadrature squeezed states.





Squeezed States and SHO

- Suppose we again apply a dc field to SHO but with a *wall* which limits the SHO to a finite region,
- in such a case, it would be expected that the wave packet would be deformed or *'squeezed'* when it is pushed against the barrier.
- Similarly the quadratic displacement potential would be expected to produce a squeezed wave packet,

$$\hat{H} = \frac{p^2}{2m} + \frac{1}{2}kx^2 - eE_0(ax - bx^2),$$

where the ax term will displace the oscillator and the bx^2 is added in order to give us a barrier,

$$\hat{H} = \frac{p^2}{2m} + \frac{1}{2}(k + 2ebE_0)x^2 - eaE_0x,$$

- We again have a displaced ground state, but with the larger effective spring constant $k' = k + 2ebE_0$.

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Squeezed Operator

- To generate squeezed state, we need quadratic terms in x, i.e. terms of the form $(\hat{a} + \hat{a}^\dagger)^2$,
- for the degenerate parametric process, i.e. two-photon, its Hamiltonian is

$$\hat{H} = i\hbar(g\hat{a}^{\dagger 2} - g^*\hat{a}^2),$$

  where g is a coupling constant.
- the state of the field generated by this Hamiltonian is

$$|\Psi(t)\rangle = \text{exp}[(g\hat{a}^{\dagger 2} - g^*\hat{a}^2)t]|0\rangle,$$

- define the unitary squeeze operator

$$\hat{S}(\xi) = \text{exp}[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}]$$

  where $\xi = r\text{exp}(i\theta)$ is an arbitrary complex number.

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Properties of Squeezed Operator

- define the unitary squeeze operator

$$\hat{S}(\xi) = \text{exp}[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}]$$

where $\xi = r\text{exp}(i\theta)$ is an arbitrary complex number.

- squeeze operator is unitary, $\hat{S}^\dagger(\xi) = \hat{S}^{-1}(\xi) = \hat{S}(-\xi)$,and the unitary transformation of the squeeze operator,

$$\begin{aligned} \hat{S}^\dagger(\xi)\hat{a}\hat{S}(\xi) &= \hat{a}\cosh r - \hat{a}^\dagger e^{i\theta}\sinh r, \\ \hat{S}^\dagger(\xi)\hat{a}^\dagger\hat{S}(\xi) &= \hat{a}^\dagger\cosh r - \hat{a}e^{-i\theta}\sinh r, \end{aligned}$$

with the formula $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + [\hat{A},\hat{B}] + \frac{1}{2!}[\hat{A},[\hat{A},\hat{B}]],\ldots$

- A squeezed coherent state $|\alpha,\xi\rangle$ is obtained by first acting with the displacement operator $\hat{D}(\alpha)$ on the vacuum followed by the squeezed operator $\hat{S}(\xi)$, i.e.

$$|\alpha,\xi\rangle = \hat{S}(\xi)\hat{D}(\alpha)|0\rangle,$$

with $\alpha = |\alpha|\text{exp}(i\psi)$.

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Uncertainty relation

- if *Re*$(\lambda) = 0$, $\hat{A} + i\lambda\hat{B}$ is a normal operator, which have orthonormal eigenstates.
- the variances,

$$\Delta\hat{A}^2 = -\frac{i\lambda}{2}[\langle\hat{F}\rangle + i\langle\hat{C}\rangle], \qquad \Delta\hat{B}^2 = -\frac{i}{2\lambda}[\langle\hat{F}\rangle - i\langle\hat{C}\rangle],$$

- set $\lambda = \lambda_r + i\lambda_i$,

$$\Delta\hat{A}^2 = \frac{1}{2}[\lambda_i\langle\hat{F}\rangle + \lambda_r\langle\hat{C}\rangle], \qquad \Delta\hat{B}^2 = \frac{1}{|\lambda|^2}\Delta\hat{A}^2, \qquad \lambda_i\langle\hat{C}\rangle - \lambda_r\langle\hat{F}\rangle = 0.$$

- if $|\lambda| = 1$, then $\Delta\hat{A}^2 = \Delta\hat{B}^2$, *equal variance minimum uncertainty states*.
- if $|\lambda| = 1$ along with $\lambda_i = 0$, then $\Delta\hat{A}^2 = \Delta\hat{B}^2$ and $\langle\hat{F}\rangle = 0$, **uncorrelated** *equal variance minimum uncertainty states*.
- if $\lambda_r \neq 0$, then $\langle\hat{F}\rangle = \frac{\lambda_i}{\lambda_r}\langle\hat{C}\rangle$, $\qquad \Delta\hat{A}^2 = \frac{|\lambda|^2}{2\lambda_r}\langle\hat{C}\rangle$, $\qquad \Delta\hat{B}^2 = \frac{1}{2\lambda_r}\langle\hat{C}\rangle$.
  If $\hat{C}$ is a positive operator then the minimum uncertainty states exist only if $\lambda_r > 0$.

Minimum Uncertainty State

- $(\hat{a}_1 - \langle \hat{a}_1 \rangle)|\psi\rangle = -i\lambda(\hat{a}_2 - \langle \hat{a}_2 \rangle)|\psi\rangle$
- if we define $\lambda = e^{-2r}$, then

$$(e^r \hat{a}_1 + ie^{-r} \hat{a}_2)|\psi\rangle = (e^r \langle \hat{a}_1 \rangle + ie^{-r} \langle \hat{a}_2 \rangle)|\psi\rangle,$$

- the minimum uncertainty state is defined as an *eigenstate* of a non-Hermitian operator $e^r \hat{a}_1 + ie^{-r} \hat{a}_2$ with a c-number eigenvalue $e^r \langle \hat{a}_1 \rangle + ie^{-r} \langle \hat{a}_2 \rangle$.
- the variances of $\hat{a}_1$ and $\hat{a}_2$ are

$$\langle \Delta \hat{a}_1^2 \rangle = \frac{1}{4} e^{-2r}, \qquad \langle \Delta \hat{a}_2^2 \rangle = \frac{1}{4} e^{2r}.$$

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Squeezed State

- define the squeezed state as

$$|\Psi_s\rangle = \hat{S}(\xi)|\Psi\rangle,$$

- where the unitary squeeze operator

$$\hat{S}(\xi) = \text{exp}[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}]$$

where $\xi = r\text{exp}(i\theta)$ is an arbitrary complex number.

- squeeze operator is unitary, $\hat{S}^\dagger(\xi) = \hat{S}^{-1}(\xi) = \hat{S}(-\xi)$,and the unitary transformation of the squeeze operator,

$$\begin{aligned}\hat{S}^\dagger(\xi)\hat{a}\hat{S}(\xi) &= \hat{a}\cosh r - \hat{a}^\dagger e^{i\theta}\sinh r,\\ \hat{S}^\dagger(\xi)\hat{a}^\dagger\hat{S}(\xi) &= \hat{a}^\dagger\cosh r - \hat{a}e^{-i\theta}\sinh r,\end{aligned}$$

- for $|\Psi\rangle$ is the vacuum state $|0\rangle$, the $|\Psi_s\rangle$ state is the *squeezed vacuum*,

$$|\xi\rangle = \hat{S}(\xi)|0\rangle,$$

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Squeezed Vacuum State

- for $|\Psi\rangle$ is the vacuum state $|0\rangle$, the $|\Psi_s\rangle$ state is the *squeezed vacuum*,

$$|\xi\rangle = \hat{S}(\xi)|0\rangle,$$

- the variances for squeezed vacuum are

$$\begin{aligned} \Delta\hat{a}_1^2 &= \frac{1}{4}[\cosh^2 r + \sinh^2 r - 2\sinh r\cosh r\cos\theta], \\ \Delta\hat{a}_2^2 &= \frac{1}{4}[\cosh^2 r + \sinh^2 r + 2\sinh r\cosh r\cos\theta], \end{aligned}$$

- for $\theta = 0$, we have

$$\Delta\hat{a}_1^2 = \frac{1}{4}e^{-2r}, \quad \text{and} \quad \Delta\hat{a}_2^2 = \frac{1}{4}e^{+2r},$$

and squeezing exists in the $\hat{a}_1$ quadrature.

- for $\theta = \pi$, the squeezing will appear in the $\hat{a}_2$ quadrature.

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Quadrature Operators

- define a rotated complex amplitude at an angle $\theta/2$

$$\hat{Y}_1 + i\hat{Y}_2 = (\hat{a}_1 + i\hat{a}_2)e^{-i\theta/2} = \hat{a}e^{-i\theta/2},$$

where

$$\begin{pmatrix} \hat{Y}_1 \\ \hat{Y}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ -\sin\theta/2 & \cos\theta/2 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

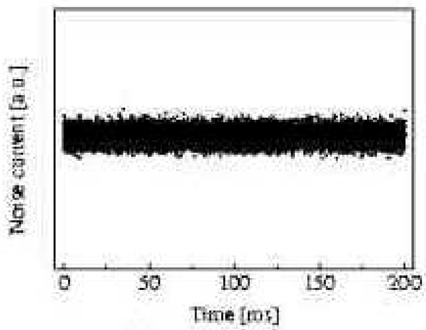
- then $\hat{S}^\dagger(\xi)(\hat{Y}_1 + i\hat{Y}_2)\hat{S}(\xi) = \hat{Y}_1 e^{-r} + i\hat{Y}_2 e^{r}$,
- the quadrature variance

$$\Delta\hat{Y}_1^2 = \frac{1}{4}e^{-2r}, \quad \Delta\hat{Y}_2^2 = \frac{1}{4}e^{+2r}, \quad \text{and} \quad \Delta\hat{Y}_1\Delta\hat{Y}_2 = \frac{1}{4},$$

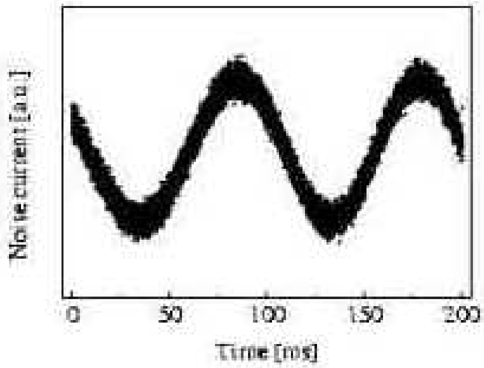
- in the complex amplitude plane the coherent state error circle is squeezed into an *error ellipse* of the same area,
- the degree of squeezing is determined by $r = |\xi|$ which is called the squeezed parameter.

Vacuum, Coherent, and Squeezed states

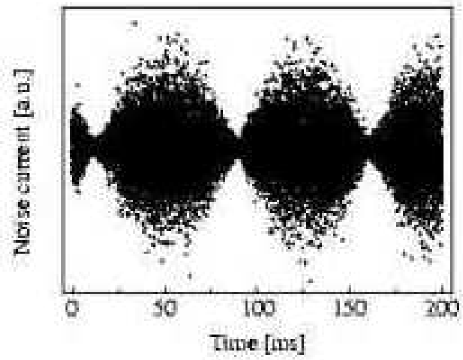
vacuum

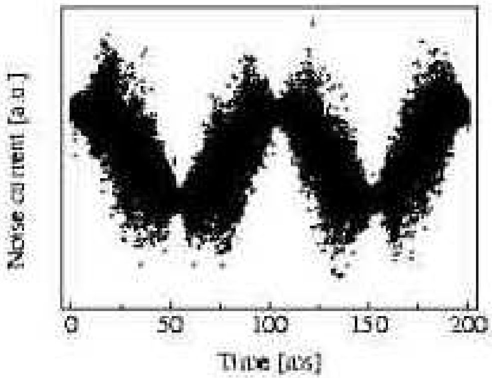


coherent

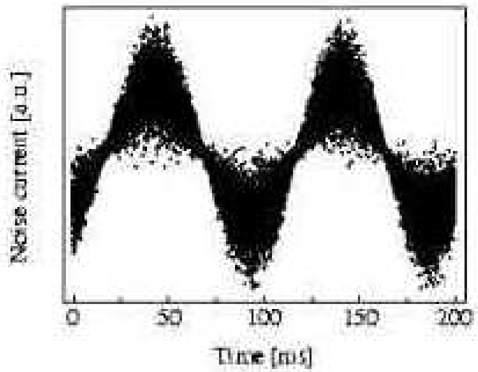


squeezed-vacuum

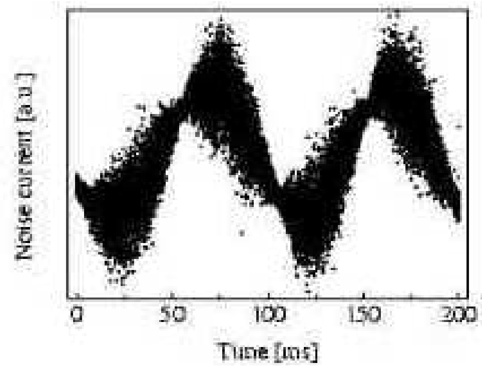




amp-squeezed



phase-squeezed



quad-squeezed

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Squeezed Coherent State

- A squeezed coherent state $|\alpha, \xi\rangle$ is obtained by first acting with the displacement operator $\hat{D}(\alpha)$ on the vacuum followed by the squeezed operator $\hat{S}(\xi)$, i.e.

$$|\alpha, \xi\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle,$$

where $\hat{S}(\xi) = \exp[\frac{1}{2}\xi^*\hat{a}^2 - \frac{1}{2}\xi\hat{a}^{\dagger 2}]$,

- for $\xi = 0$, we obtain just a coherent state.
- the expectation values,

$$\langle\alpha, \xi|\hat{a}|\alpha, \xi\rangle = \alpha, \quad \langle\hat{a}^2\rangle = \alpha^2 - e^{i\theta}\sinh r\cosh r, \quad \text{and} \quad \langle\hat{a}^\dagger\hat{a}\rangle = |\alpha|^2 + \sinh^2 r,$$

with helps of $\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha$ and $\hat{D}^\dagger(\alpha)\hat{a}^\dagger\hat{D}(\alpha) = \hat{a}^\dagger + \alpha^*$,

- for $r \to 0$ we have coherent state, and $\alpha \to 0$ we have squeezed vacuum.
- furthermore

$$\langle\alpha, \xi|\hat{Y}_1 + i\hat{Y}_2|\alpha, \xi\rangle = \alpha e^{-i\theta/2}, \quad \langle\Delta\hat{Y}_1^2\rangle = \frac{1}{4}e^{-2r}, \quad \text{and} \quad \langle\Delta\hat{Y}_2^2\rangle = \frac{1}{4}e^{+2r},$$

Squeezed State

- from the vacuum state $\hat{a}|0\rangle = 0$, we have

$$\hat{S}(\xi)\hat{a}\hat{S}^\dagger(\xi)\hat{S}(\xi)|0\rangle = 0, \quad \text{or} \quad \hat{S}(\xi)\hat{a}\hat{S}^\dagger(\xi)|\xi\rangle = 0,$$

- since $\hat{S}(\xi)\hat{a}\hat{S}^\dagger(\xi) = \hat{a}\cosh r + \hat{a}^\dagger e^{i\theta}\sinh r \equiv \mu\hat{a} + \nu\hat{a}^\dagger$, we have,

$$(\mu\hat{a} + \nu\hat{a}^\dagger)|\xi\rangle = 0,$$

  the squeezed vacuum state is an eigenstate of the operator $\mu\hat{a} + \nu\hat{a}^\dagger$ with eigenvalue zero.

- similarly,

$$\hat{D}(\alpha)\hat{S}(\xi)\hat{a}\hat{S}^\dagger(\xi)\hat{D}^\dagger(\alpha)\hat{D}(\alpha)|\xi\rangle = 0,$$

  with the relation $\hat{D}(\alpha)\hat{a}\hat{D}^\dagger(\alpha) = \hat{a} - \alpha$, we have

$$(\mu\hat{a} + \nu\hat{a}^\dagger)|\alpha,\xi\rangle = (\alpha\cosh r + \alpha^*\sinh r)|\alpha,\xi\rangle \equiv \gamma|\alpha,\xi\rangle,$$

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Squeezed State and Minimum Uncertainty State

- write the eigenvalue problem for the squeezed state

$$(\mu\hat{a}+\nu\hat{a}^\dagger)|\alpha,\xi\rangle = (\alpha\cosh r+\alpha^*\sinh r)|\alpha,\xi\rangle \equiv \gamma|\alpha,\xi\rangle,$$

- in terms of in terms of $\hat{a}=(\hat{Y}_1+i\hat{Y}_2)e^{i\theta/2}$ we have

$$(\hat{Y}_1+ie^{-2r}\hat{Y}_2)|\alpha,\xi\rangle = \beta_1|\alpha,\xi\rangle,$$

where

$$\beta_1 = \gamma e^{-r}e^{-i\theta/2} = \langle\hat{Y}_1\rangle + i\langle\hat{Y}_2\rangle e^{-2r},$$

- in terms of $\hat{a}_1$ and $\hat{a_2}$ we have

$$(\hat{a}_1+i\lambda\hat{a}_2^\dagger)|\alpha,\xi\rangle = \beta_2|\alpha,\xi\rangle,$$

where

$$\lambda = \frac{\mu-\nu}{\mu+\nu}, \quad \text{and} \quad \beta_2 = \frac{\gamma}{\mu+\nu},$$

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Squeezed State in the basis of Number states

- consider squeezed vacuum state first,

$$|\xi\rangle = \sum_{n=0}^{\infty} C_n |n\rangle,$$

with the operator of $(\mu\hat{a} + \nu\hat{a}^{\dagger})|\xi\rangle = 0$, we have

$$C_{n+1} = -\frac{\nu}{\mu}(\frac{n}{n+1})^{1/2} C_{n-1},$$

- only the even photon states have the solutions,

$$C_{2m} = (-1)^m (e^{i\theta}\tanh r)^m [\frac{(2m-1)!!}{(2m)!!}]^{1/2} C_0,$$

where C_0 can be determined from the normalization, i.e. $C_0 = \sqrt{\cosh r}$,

- the squeezed vacuum state is

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle,$$

Squeezed State in the basis of Number states

- the squeezed vacuum state is

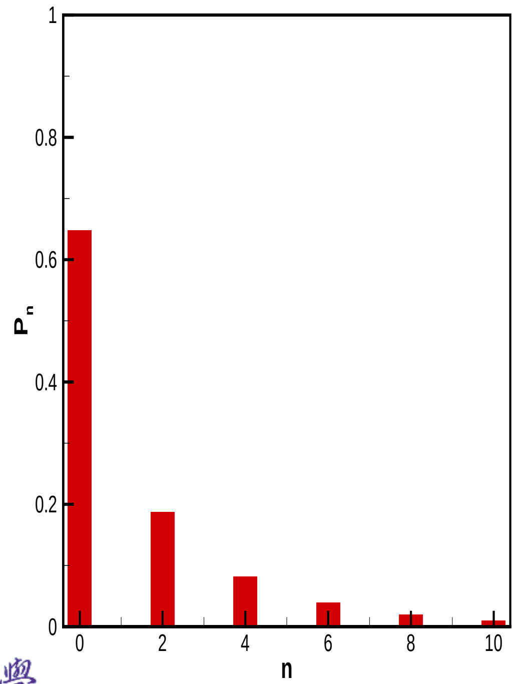
$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{m=0}^{\infty} (-1)^m \frac{\sqrt{(2m)!}}{2^m m!} e^{im\theta} \tanh^m r |2m\rangle,$$

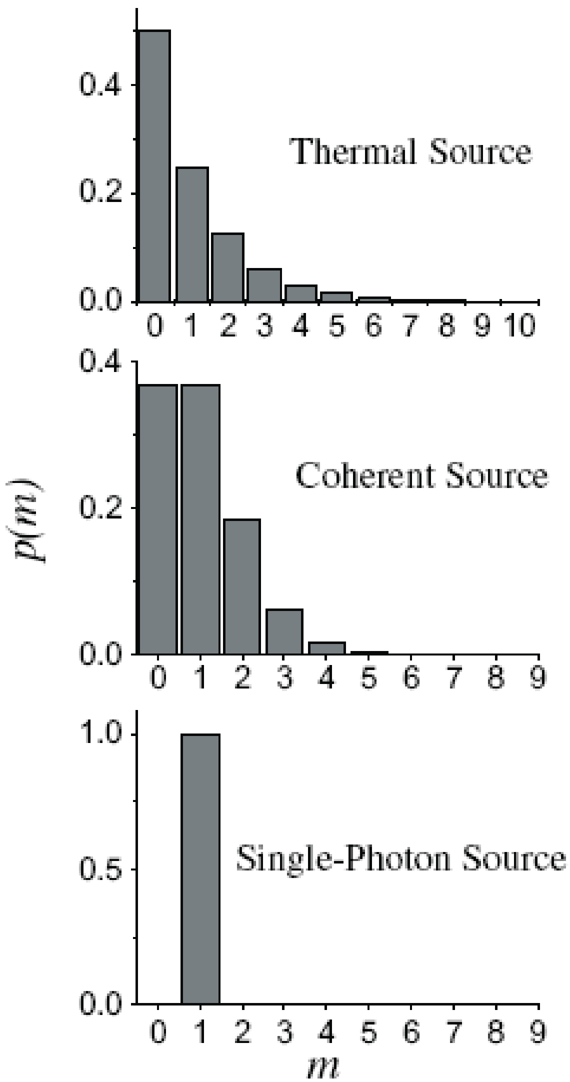
- the probability of detecting $2m$ photons in the field is

$$P_{2m} = |\langle 2m|\xi\rangle|^2 = \frac{(2m)!}{2^{2m}(m!)^2} \frac{\tanh^{2m} r}{\cosh r},$$

- for detecting $2m+1$ states $P_{2m+1} = 0$,
- the photon probability distribution for a squeezed vacuum state is *oscillatory*, vanishing for all odd photon numbers,
- the shape of the squeezed vacuum state resembles that of thermal radiation.

Number distribution of the Squeezed State





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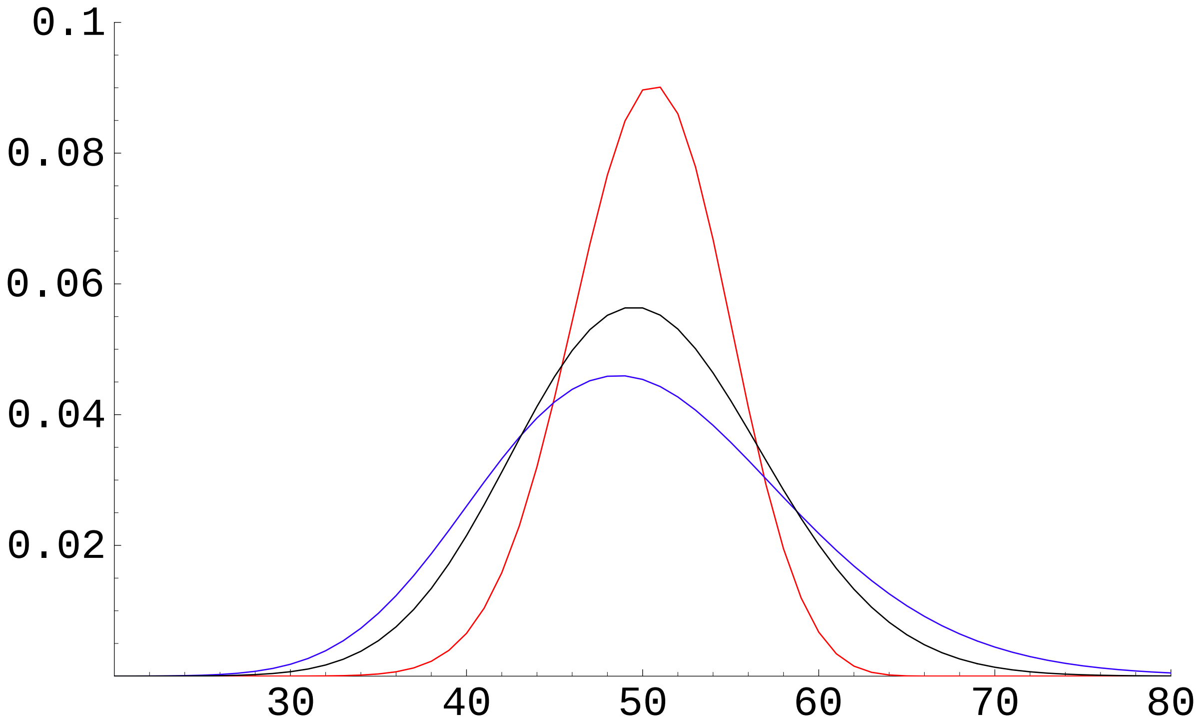
Number distribution of the Squeezed Coherent State

For a squeezed coherent state,

$$P_n = |\langle n|\alpha,\xi\rangle|^2 = \frac{(\frac{1}{2}\tanh r)^n}{n!\cosh r}\exp[-|\alpha|^2 - \frac{1}{2}(\alpha^{*2}e^{i\theta} + \alpha^2 e^{-i\theta})\tanh r] \mathrm{H}_n^2(\gamma(e^{i\theta}\sinh(2r))^{-1/}$$



Ref:

Ch. 5, 7 in *"Introductory Quantum Optics,"* by C. Gerry and P. Knight.

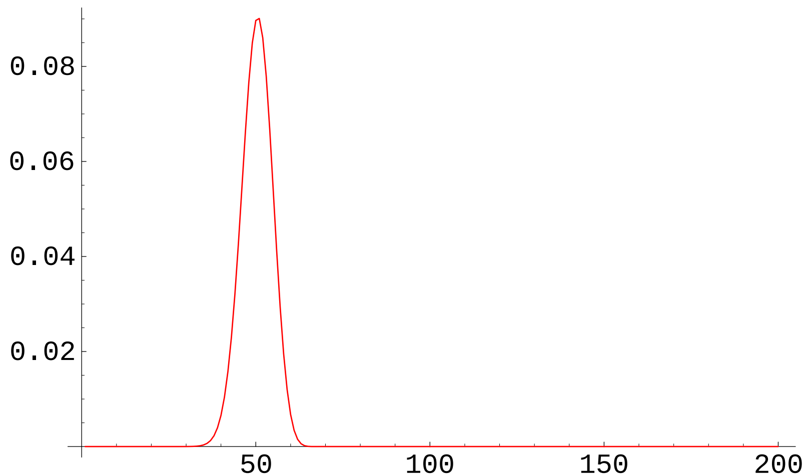
Number distribution of the Squeezed Coherent State

- A squeezed coherent state $|\alpha, \xi\rangle$ is obtained by first acting with the displacement operator $\hat{D}(\alpha)$ on the vacuum followed by the squeezed operator $\hat{S}(\xi)$, i.e.

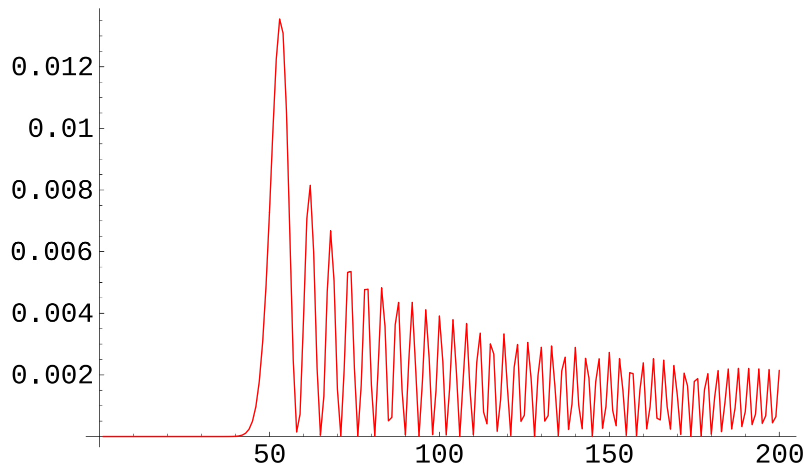
$$|\alpha, \xi\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle,$$

- the expectation values,

$$\langle \hat{a}^\dagger \hat{a}\rangle = |\alpha|^2 + \sinh^2 r,$$



$|\alpha|^2 = 50, \theta = 0, r = 0.5$



$|\alpha|^2 = 50, \theta = 0, r = 4.0$

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Generations of Squeezed States

- Generation of quadrature squeezed light are based on some sort of *parametric process* utilizing various types of nonlinear optical devices.
- for degenerate parametric down-conversion, the nonlinear medium is pumped by a field of frequency ω_p and that field are converted into pairs of identical photons, of frequency $\omega = \omega_p/2$ each,

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_p\hat{b}^\dagger\hat{b} + i\hbar\chi^{(2)}(\hat{a}^2\hat{b}^\dagger - \hat{a}^{\dagger 2}\hat{b}),$$

  where b is the pump mode and a is the signal mode.
- assume that the field is in a coherent state $|\beta e^{-i\omega_p t}\rangle$ and approximate the operators $\hat{b}$ and $\hat{b}^\dagger$ by classical amplitude $\beta e^{-i\omega_p t}$ and $\beta^* e^{i\omega_p t}$, respectively,
- we have the interaction Hamiltonian for *degenerate parametric down-conversion*,

$$\hat{H}_I = i\hbar(\eta^*\hat{a}^2 - \eta\hat{a}^{\dagger 2}),$$

  where $\eta = \chi^{(2)}\beta$.

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Generations of Squeezed States

- we have the interaction Hamiltonian for *degenerate parametric down-conversion*,

$$\hat{H}_I = i\hbar(\eta^* \hat{a}^2 - \eta \hat{a}^{\dagger 2}),$$

where $\eta = \chi^{(2)}\beta$, and the associated evolution operator,

$$\hat{U}_I(t) = \text{exp}[-i\hat{H}_I t/\hbar] = \text{exp}[(\eta^* \hat{a}^2 - \eta \hat{a}^{\dagger 2})t] \equiv \hat{S}(\xi),$$

with $\xi = 2\eta t$.

- for degenerate four-wave mixing, in which two pump photons are converted into two signal photons of the same frequency,

$$\hat{H} = \hbar\omega \hat{a}^\dagger \hat{a} + \hbar\omega \hat{b}^\dagger \hat{b} + i\hbar\chi^{(3)}(\hat{a}^2 \hat{b}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{b}^2),$$

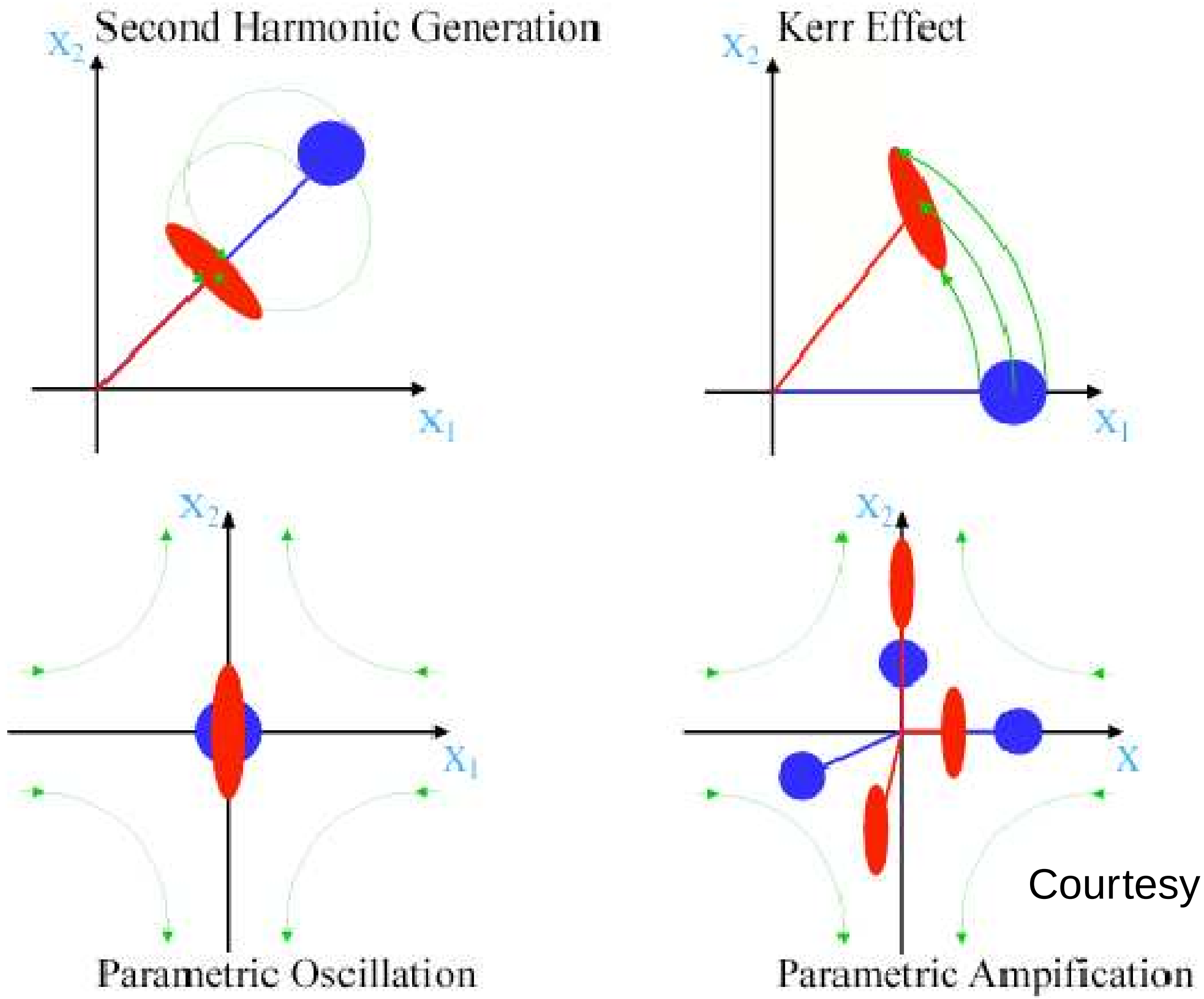
- the associated evolution operator,

$$\hat{U}_I(t) = \text{exp}[(\eta^* \hat{a}^2 - \eta \hat{a}^{\dagger 2})t] \equiv \hat{S}(\xi),$$

with $\xi = 2\chi^{(3)}\beta^2 t$.

Generations of Squeezed States

Nonlinear optics:

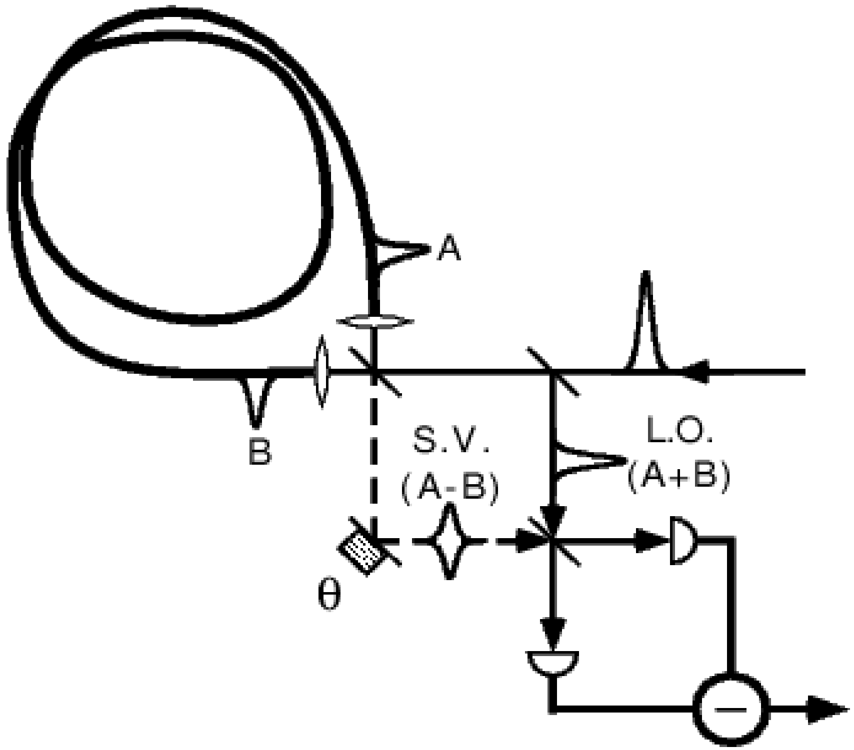


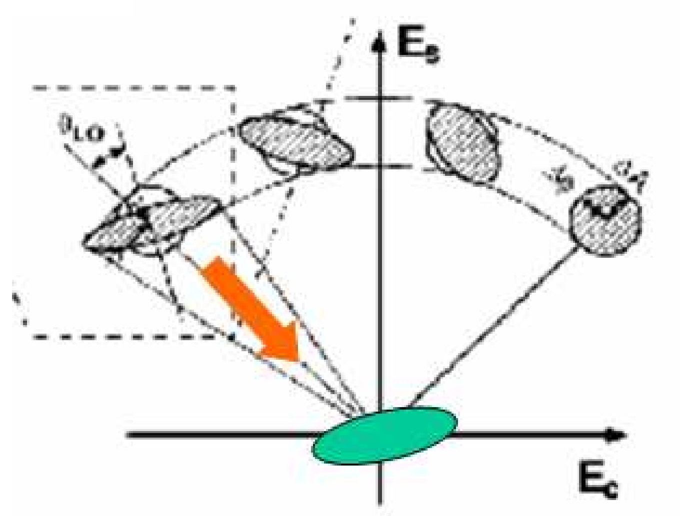
Courtesy of P. K. Lam

Generation and Detection of Squeezed Vacuum

1. Balanced Sagnac Loop (to cancel the mean field),

2. Homodyne Detection.





M. Rosenbluh and R. M. Shelby, *Phys. Rev. Lett.* **66**, 153(1991).

Beam Splitters

- *Wrong* picture of beam splitters,

$$\hat{a}_2 = r\hat{a}_1, \qquad \hat{a}_3 = t\hat{a}_1,$$

where r and t are the complex reflectance and transmittance respectively which require that $|r|^2 + |t|^2 = 1$.

- in this case,

$$[\hat{a}_2, \hat{a}_2^\dagger] = |r|^2[\hat{a}_2, \hat{a}_2^\dagger] = |r|^2, \quad [\hat{a}_3, \hat{a}_3^\dagger] = |t|^2[\hat{a}_2, \hat{a}_2^\dagger] = |t|^2, \quad \text{and} \quad [\hat{a}_2, \hat{a}_3^\dagger] = rt^* \neq 0,$$

this kind of the transformations do not preserve the commutation relations.

- *Correct* transformations of beam splitters,

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} r & jt \\ jt & r \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix},$$

Homodyne detection

- the detectors measure the intensities $I_c = \langle \hat{c}^\dagger \hat{c} \rangle$ and $I_d = \langle \hat{d}^\dagger \hat{d} \rangle$, and the difference in these intensities is,

$$I_c - I_d = \langle \hat{n}_{cd} \rangle = \langle \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} \rangle = i \langle \hat{a}^\dagger \hat{b} - \hat{a} \hat{b}^\dagger \rangle,$$

- assuming the b mode to be in the coherent state $|\beta e^{-i\omega t}\rangle$, where $\beta = |\beta| e^{-i\psi}$, we have

$$\langle \hat{n}_{cd} \rangle = |\beta| \{ \hat{a} e^{i\omega t} e^{-i\theta} + \hat{a}^\dagger e^{-i\omega t} e^{i\theta} \},$$

  where $\theta = \psi + \pi/2$,

- assume that a mode light is also of frequency ω (in practice both the a and b modes derive from the same laser), i.e. $\hat{a} = \hat{a}_0 e^{-i\omega t}$, we have

$$\langle \hat{n}_{cd} \rangle = 2|\beta| \langle \hat{X}(\theta) \rangle,$$

  where $\hat{X}(\theta) = \frac{1}{2}(\hat{a}_0 e^{-i\theta} + \hat{a}_0^\dagger e^{i\theta})$ is the field quadrature operator at the angle θ,

- by changing the phase ψ of the local oscillator, we can measure an arbitrary quadrature of the signal field.

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Detection of Squeezed States

- mode a contains the single field that is possibly squeezed,
- mode b contains a strong coherent classical field, *local oscillator*, which may be taken as coherent state of amplitude β,
- for a balanced homodyne detection, $50:50$ beam splitter,
- the relation between input $(\hat{a}, \hat{b})$ and output $(\hat{c}, \hat{d})$ is,

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + i\hat{b}), \qquad \hat{d} = \frac{1}{\sqrt{2}}(\hat{b} + i\hat{a}),$$

- the detectors measure the intensities $I_c = \langle \hat{c}^\dagger \hat{c} \rangle$ and $I_d = \langle \hat{d}^\dagger \hat{d} \rangle$, and the difference in these intensities is,

$$I_c - I_d = \langle \hat{n}_{cd} \rangle = \langle \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} \rangle = i \langle \hat{a}^\dagger \hat{b} - \hat{a} \hat{b}^\dagger \rangle,$$

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Squeezed States in Quantum Optics

- Generation of squeezed states:
  - nonlinear optics: $\chi^{(2)}$ or $\chi^{(3)}$ processes,
  - cavity-QED,
  - photon-atom interaction,
  - photonic crystals,
  - semiconductor, photon-electron/exciton/polariton interaction,
  - . . .
- Applications of squeezed states:
  - Gravitational Waves Detection,
  - Quantum Non-Demolition Measurement (QND),
  - Super-Resolved Images (Quantum Images),
  - Generation of EPR Pairs,
  - Quantum Informatio Processing, teleportation, cryptography, computing,
  - . . .

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