IPT 5340 (PHYS 6840)

Quantum Optics

Ray-Kuang Lee†

Institute of Photonics Technologies

Department of Electrical Engineering and Department of Physics

National Tsing-Hua University, Hsinchu, Taiwan

†Ext: 42439

Room 523, EECS bldg.

e-mail: rklee@ee.nthu.edu.tw

國立清華大學
National Tsing Hua University

Course info: http://mx.nthu.edu.tw/∼rklee

IPT 5340 (PHYS 6840)

Time: W3W4Wn (10:10 AM-01:00 PM, Wednesday)

Course Description:

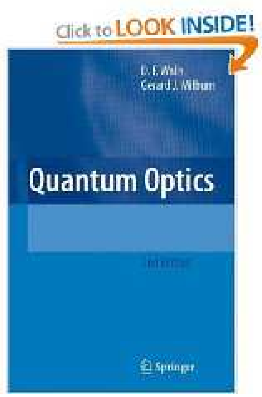
- The field of quantum optics has made a revolution on modern physics, from laser, precise measurement, Bose-Einstein condensates, quantum information process, to the fundamental issues in quantum mechanics.
- Through this course, I want to provide an in-depth and wide-ranging introduction to the fundamental concepts for quantum optics, including physical concepts, mathematical methods, simulation techniques, basic principles and applications.
- Current researches on non-classical state generation, quantum noise measurement, nonlinear quantum pulse propagation, quantum interference, quantum information science, Bose-Einstein condensates, and atom optics would also be stressed.
- Background requirements: Basics of quantum mechanics, electromagnetic theory, and nonlinear optics.

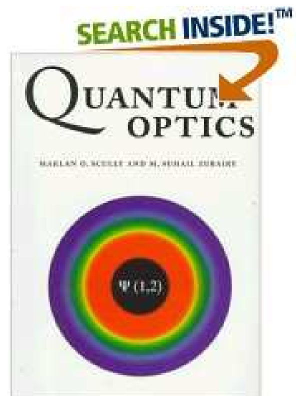
Teaching Method: in-class lectures with discussion and project studies.

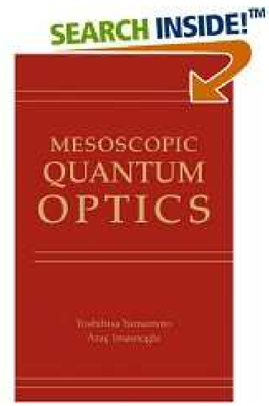
國立清華大學
National Tsing Hua University

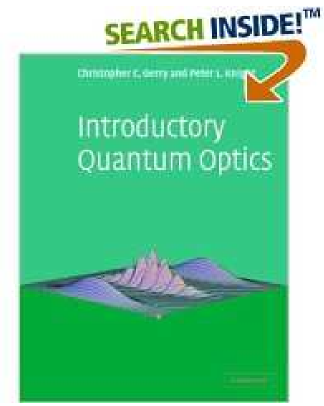
Reference Books

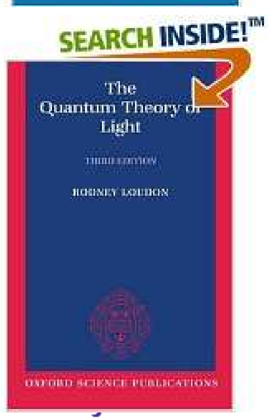
- [Textbook]: D. F. Walls and Gerard J. Milburn, *"Quantum Optics,"* 2nd Ed. Springer (2008); 1st Ed. (1993).
- Marlan O. Scully and M. Suhail Zubairy, *"Quantum Optics,"* Cambridge (1997).
- Yoshihisa Yamamoto and Atac Imamoglu, *"Mesoscopic Quantum Optics,"* Wiley (1999).

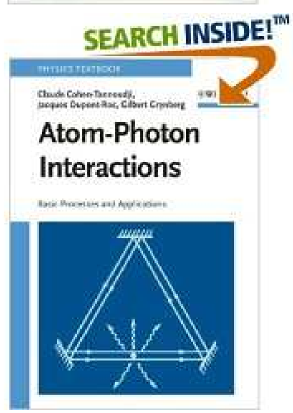


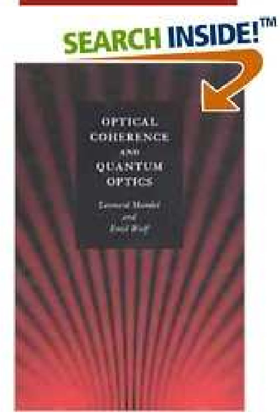


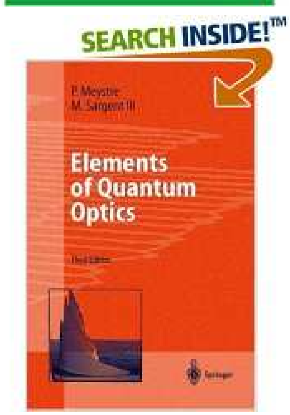






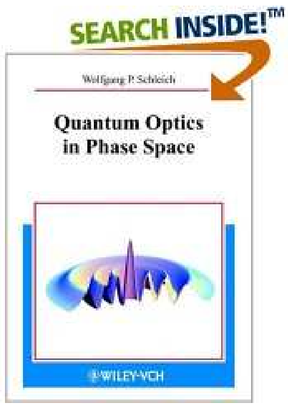


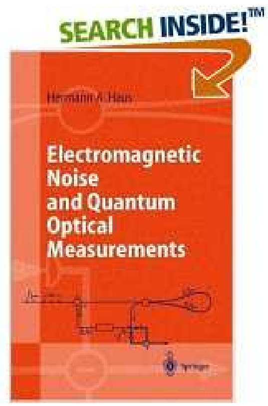


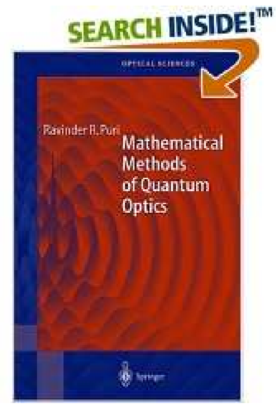


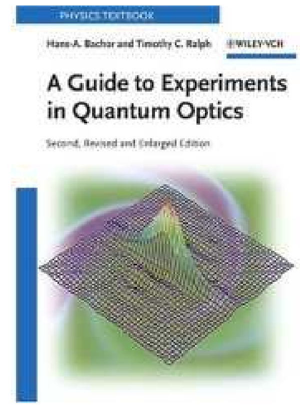
National Tsing Hua University

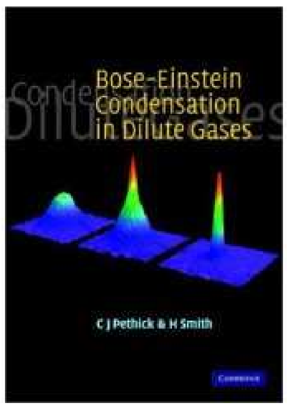
Advanced Reference Books

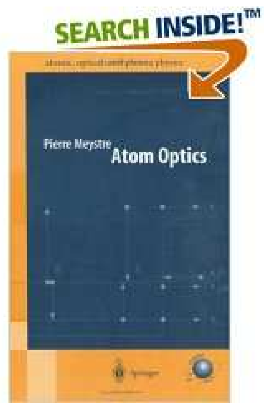


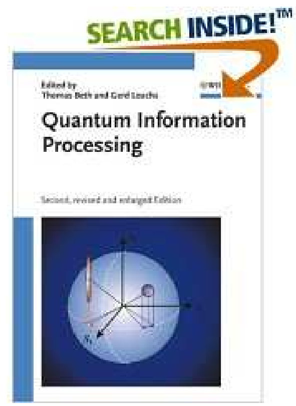


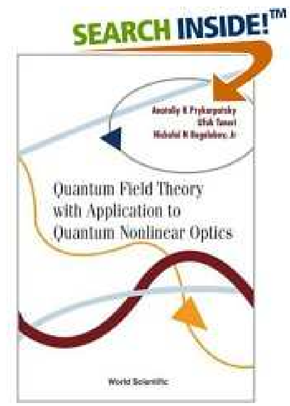












Syllabus

1. Quantization of the Electromagnetic Fields,[Textbook] Ch. 2, (3/4, 3/11)
2. Coherence properties of the EM fields, [Textbook] Ch. 3, (3/18, 3/25)
3. Representation of the EM fields, [Textbook] Ch. 4, (4/1, 4/8)
4. Quantum phenomena in simple nonlinear optics, [Textbook] Ch. 5, (4/15, 4/22)
5. Input-Output Formulation of optical cavities, [Textbook] Ch. 7, (5/6, 5/13)
6. Squeezed lights, [Textbook] Ch. 8, (5/20, 5/27)
7. Atom-field interaction, [Textbook] Ch. 10, (6/3, 6/10)
8. Cavity Quantum ElectroDynamics (Cavity-QED), [Textbook] Ch. 11, (6/17)
9. Midterm, (5/6) and Semester reports, (6/24).
10. Quantum theory of Laser, [Textbook] Ch. 12,
11. Quantum Non-demolition Measurement, [Textbook] Ch. 14,
12. Quantum Coherence and Measurement theory, [Textbook] Ch. 15,
13. Quantum Information, [Textbook] Ch. 13 and Ch. 16,
14. Ion Trap, [Textbook] Ch. 17,
15. Light Forces, [Textbook] Ch. 18,
16. Bose-Einstein Condensation, [Textbook] Ch. 19,

Evaluation

1. Homework $\times 4$ (monthly), 50%
2. Midterm, (take home exam) 30%
3. Semester Report, (oral presentation) 20%
4. Other suggestions

Office hours: 3:00-5:00, Thursday
at Room 523, EECS bldg.

國立清華大學
National Tsing Hua University

2005 Nobel Laureates

Glauber(Harvard)   Hall(JILA)   Hänsch(MPI)

Roy J. Glauber: "for his contribution to the quantum theory of optical coherence,"

John L. Hall and Theodor W. Hänsch: "for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique."

國立清華大學
National Tsing Hua University

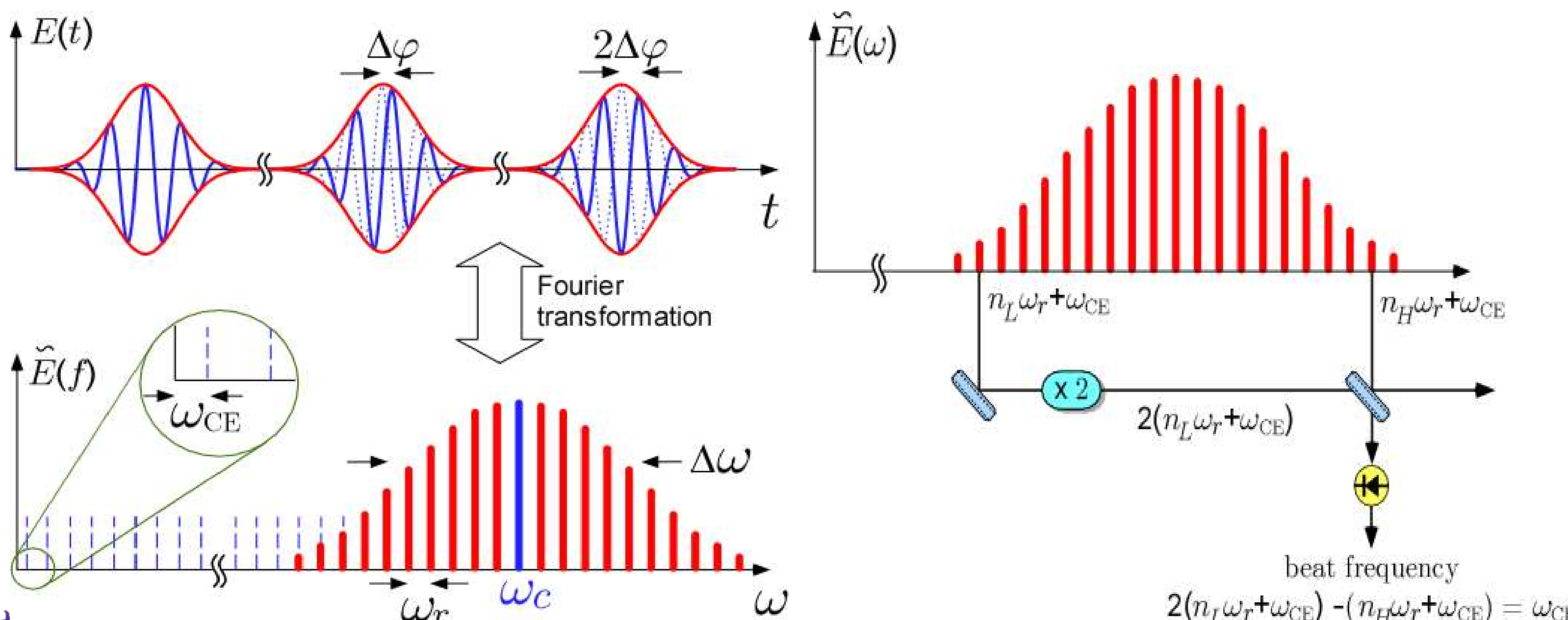
from: http://nobelprize.org/

Coherent states and Comb lasers

coherent Glauber state:

$$|\alpha> = \sum_{n=0} \alpha^n \frac{e^{-\frac{|\alpha|^2}{2}}}{\sqrt{n!}} |n>$$

Self referencing of frequency combs:



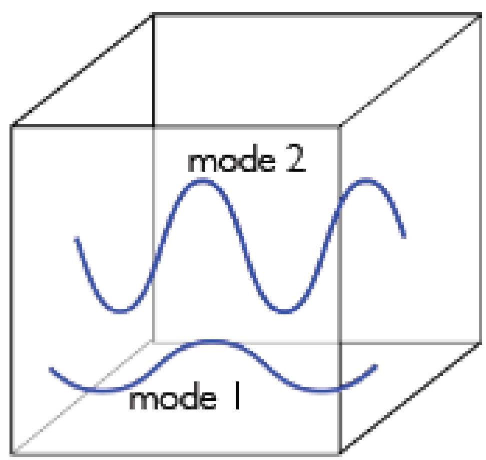
National Tsing Hua University

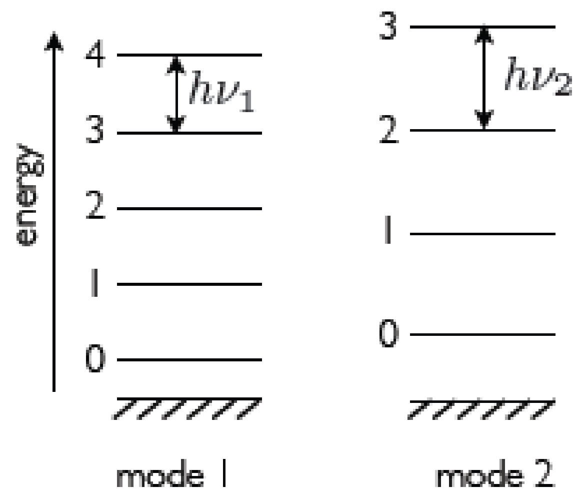
from: http://www.mpq.mpg.de/ haensch/

Quantization of EM fields

- the Hamiltonian for EM fields becomes: $\hat{H} = \sum_j \hbar\omega_j(\hat{a}_j^\dagger \hat{a}_j + \frac{1}{2})$,
- the electric and magnetic fields become,

$$\begin{aligned}\hat{E}_x(z,t) &= \sum_j \left(\frac{\hbar\omega_j}{\epsilon_0 V}\right)^{1/2}[\hat{a}_j e^{-i\omega_j t} + \hat{a}_j^\dagger e^{i\omega_j t}]\sin(k_j z), \\ &= \sum_j c_j[\hat{a}_{1j}\cos\omega_j t + \hat{a}_{2j}\sin\omega_j t]u_j(r),\end{aligned}$$





國立清華大學
National Tsing Hua University

Role of Quantum Optics

- photons occupy an *electromagnetic mode*, we will always refer to modes in quantum optics, typically a plane wave;
- the energy in a mode is not continuous but discrete in quanta of $\hbar\omega$;
- the observables are just represented by probabilities as usual in quantum mechanics;
- there is a *zero point energy* inherent to each mode which is equivalent with fluctuations of the electromagnetic field in vacuum, due to uncertainty principle.

Vacuum

vacuum is not just nothing, it is full of energy.

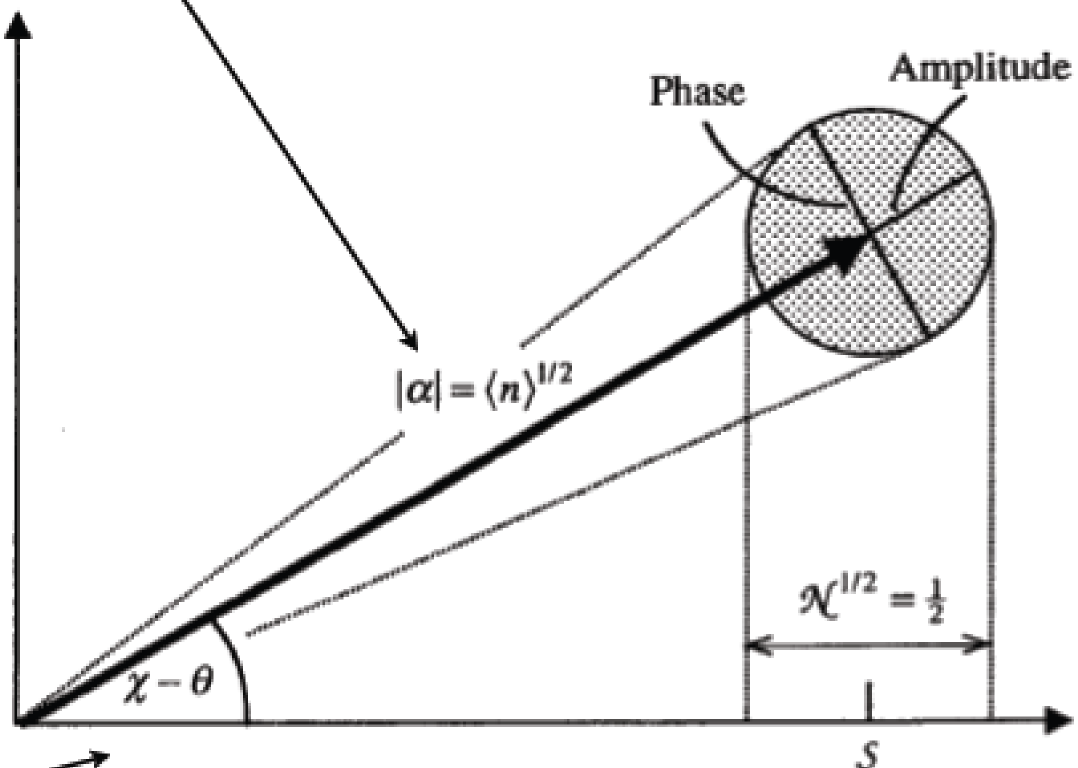
Vacuum

- **spontaneous emission** is actually stimulated by the vacuum fluctuation of the electromagnetic field,
- one can modify vacuum fluctuations by resonators and photonic crystals,
- **atomic stability**: the electron does not crash into the core due to vacuum fluctuation of the electromagnetic field,
- **gravity** is not a fundamental force but a side effect matter modifies the vacuum fluctuations, by Sakharov,
- **Casimir effect**: two charged metal plates repel each other until Casimir effect overcomes the repulsion,
- **Lamb shift**: the energy level difference between $2S_{1/2}$ and $2P_{1/2}$ in hydrogen.
- ...

Phase diagram for coherent states

mean number of photons

$$< \hat{N} >=< \alpha|\hat{N}|\alpha >=< \alpha|\hat{a}^\dagger \hat{a}|\alpha >= |\alpha|^2$$



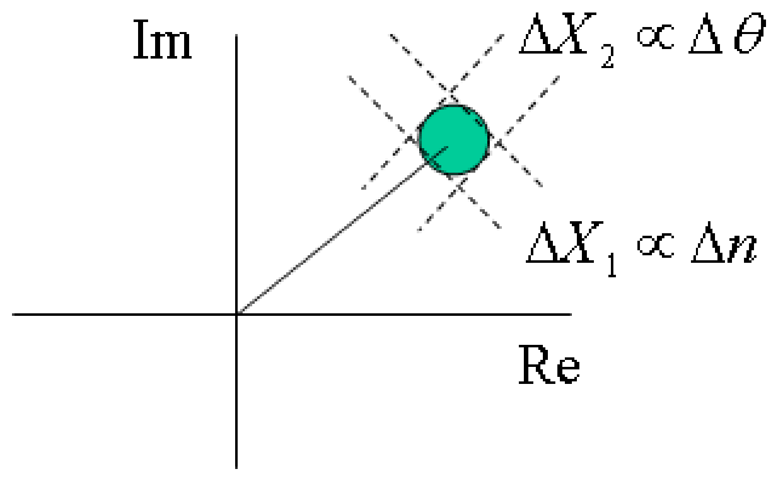
phase of the field

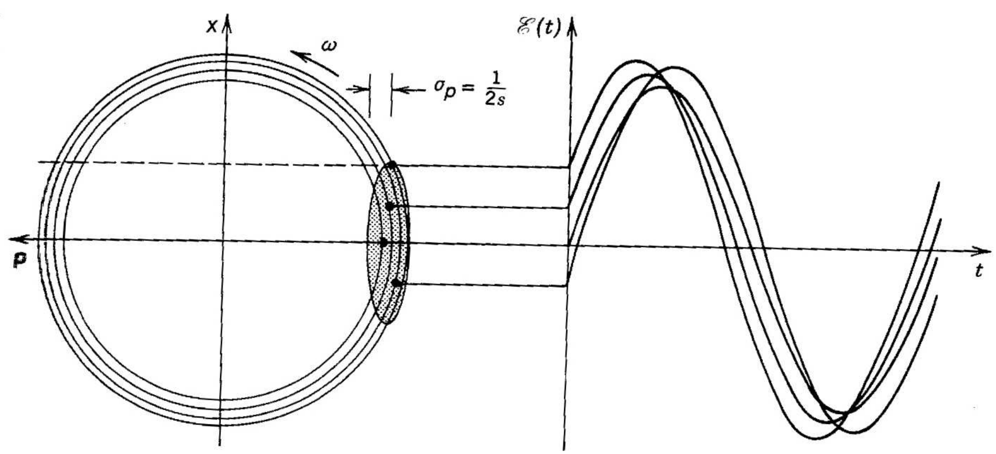
$$\alpha = |\alpha| \exp(i\theta)$$

Coherent and Squeezed States

Uncertainty Principle: $\Delta\hat{X}_1\Delta\hat{X}_2 \geq 1$.

1. Coherent states: $\Delta\hat{X}_1 = \Delta\hat{X}_2 = 1$,
2. Amplitude squeezed states: $\Delta\hat{X}_1 < 1$,
3. Phase squeezed states: $\Delta\hat{X}_2 < 1$,
4. Quadrature squeezed states.

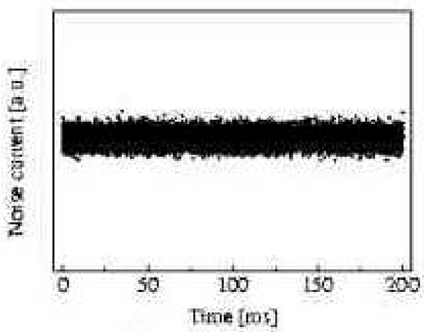




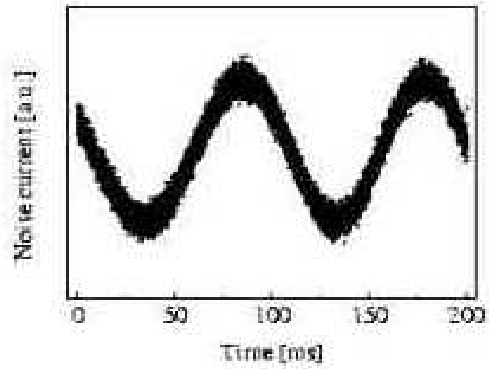
國立清華大學
National Tsing Hua University

Vacuum, Coherent, and Squeezed states

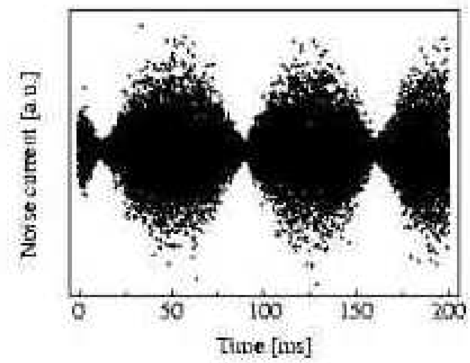
vacuum

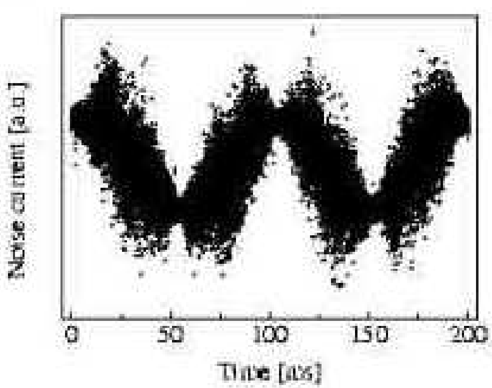


coherent

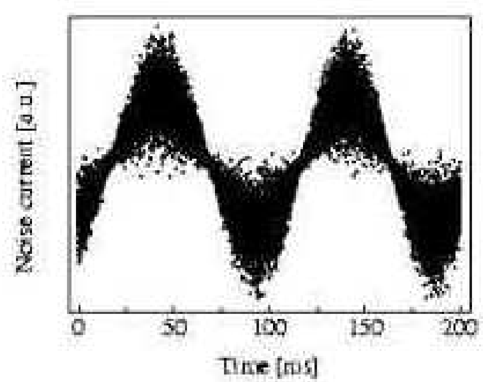


squeezed-vacuum

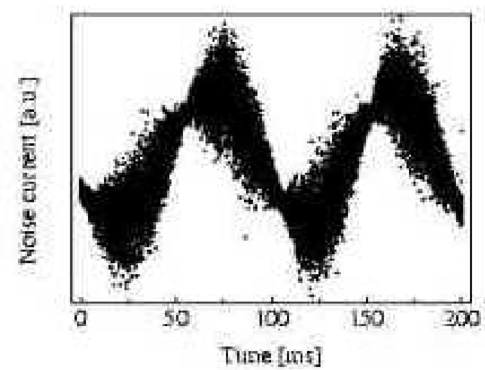




amp-squeezed



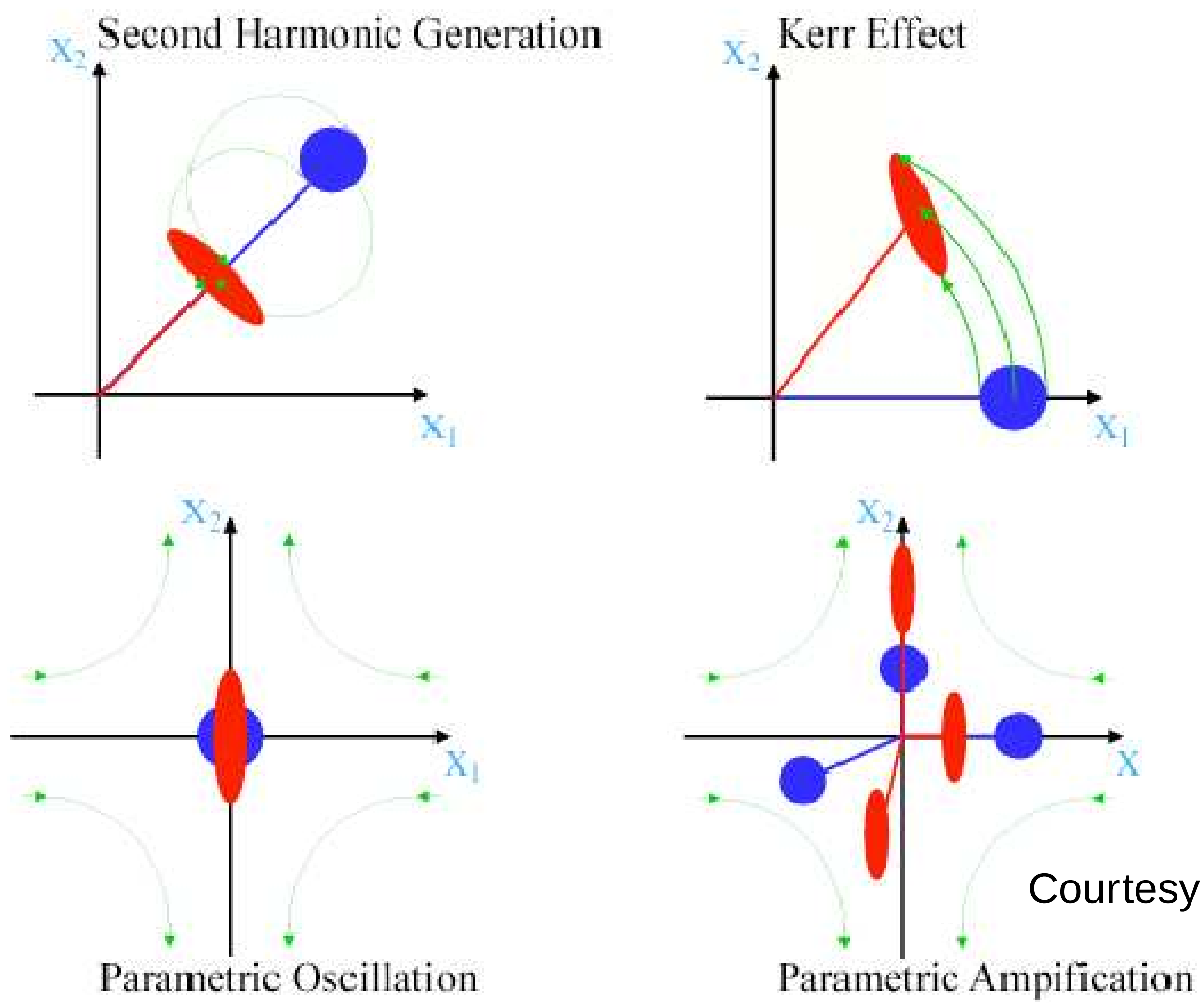
phase-squeezed



quad-squeezed

Generations of Squeezed States

Nonlinear optics:



Courtesy of P. K. Lam

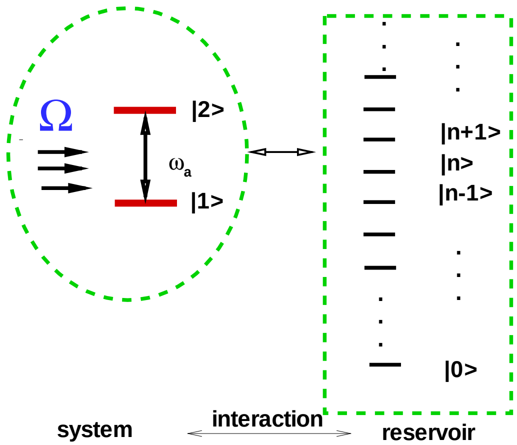
National Tsing Hua University

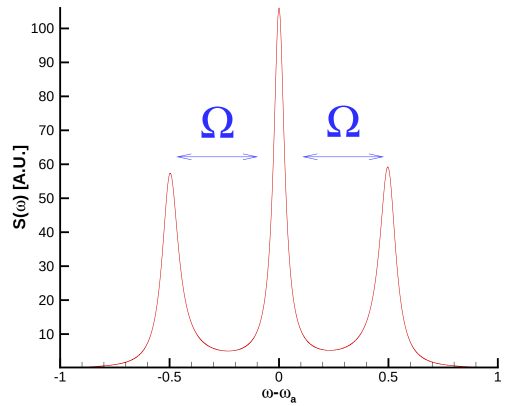
Squeezed States in Quantum Optics

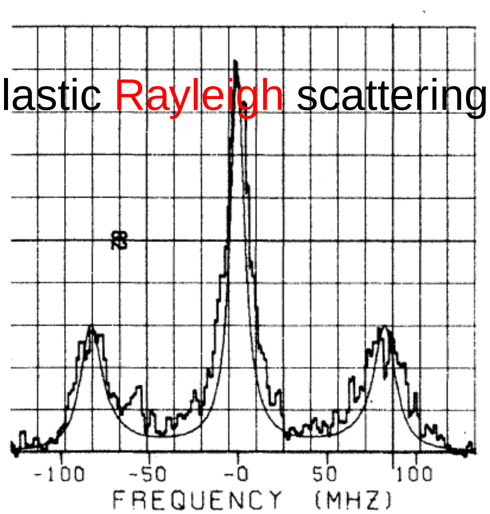
- Generation of squeezed states:
  - nonlinear optics: $\chi^{(2)}$ or $\chi^{(3)}$ processes,
  - cavity-QED,
  - photon-atom interaction,
  - photonic crystals,
  - semiconductor, photon-electron/exciton/polariton interaction,
  - . . .
- Applications of squeezed states:
  - Gravitational Waves Detection,
  - Quantum Non-Demolition Measurement (QND),
  - Super-Resolved Images (Quantum Images),
  - Generation of EPR Pairs,
  - Quantum Informatio Processing, teleportation, cryptography, computing,
  - . . .

國立清華大學
National Tsing Hua University

Mollow's triplet: Resonance Fluorescence Spectrum







elastic Rayleigh scattering and inelastic Raman scattering

Theory: B. R. Mollow, *Phys. Rev.* **188**, 1969 (1969).

Exp: F. Y. Wu, R. E. Grove, and S. Ezekiel, *Phys. Rev. Lett.* **35**, 1426 (1975).

Hamiltonian of atom-light interaction: Jaynes-Cummings model

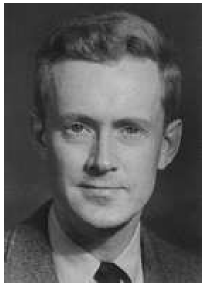
$$\begin{aligned} H &= \frac{\hbar}{2}\omega_a\sigma_z + \hbar\sum_k \omega_k a_k^\dagger a_k + \frac{\Omega}{2}\hbar(\sigma_- e^{i\omega_L t} + \sigma_+ e^{-i\omega_L t}) \\ &+ \hbar\sum_k (g_k\sigma_+ a_k + g_k^* a_k^\dagger \sigma_-) \end{aligned}$$

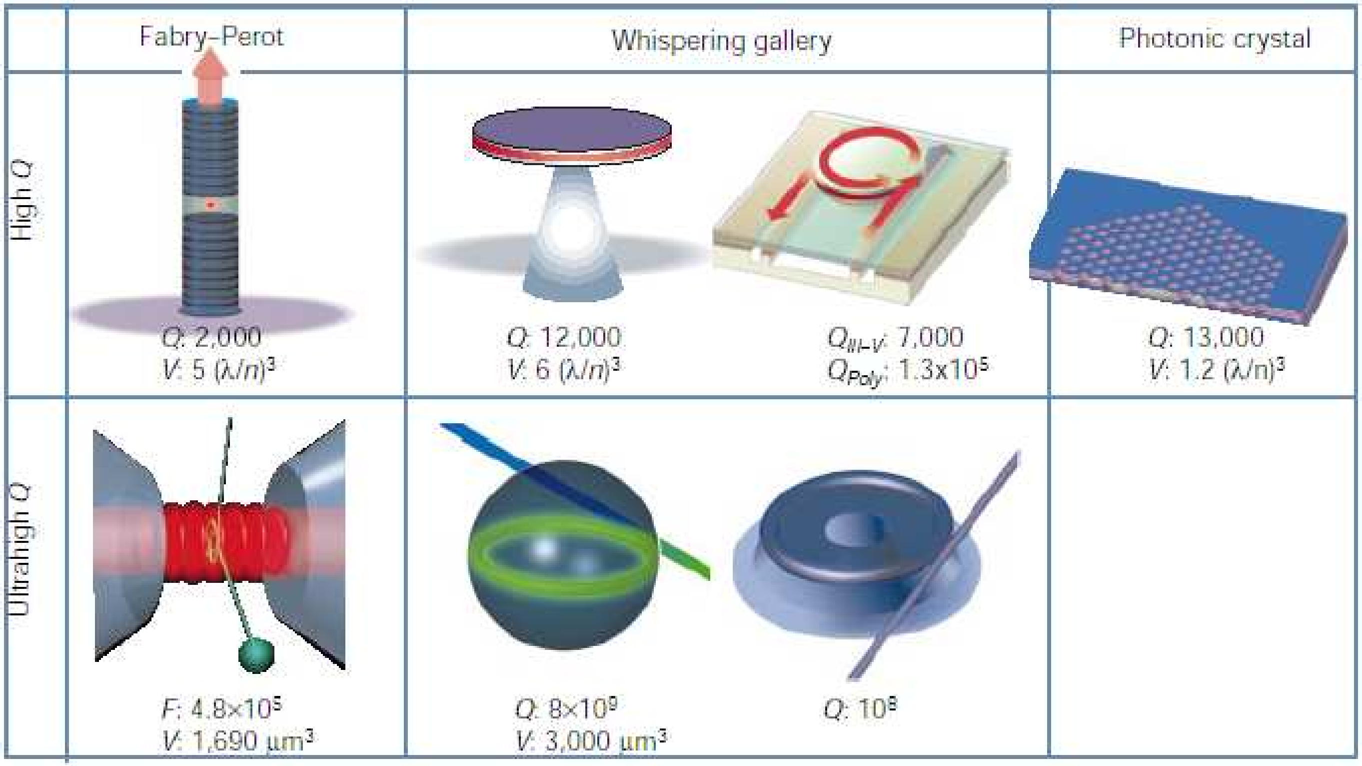
And we want to solve the generalized Bloch equations:

$$\begin{aligned} \dot{\sigma}_-(t) &= i\frac{\Omega}{2}\sigma_z(t)e^{-i\Delta t} + \int_{-\infty}^{t} dt' G(t-t')\sigma_z(t)\sigma_-(t') + n_-(t) \\ \dot{\sigma}_+(t) &= -i\frac{\Omega}{2}\sigma_z(t)e^{i\Delta t} + \int_{-\infty}^{t} dt' G_c(t-t')\sigma_+(t')\sigma_z(t) + n_+(t) \\ \dot{\sigma}_z(t) &= i\Omega(\sigma_-(t)e^{i\Delta t} - \sigma_+(t)e^{-i\Delta t}) + n_z(t) \\ &- 2\int_{-\infty}^{t} dt'[G(t-t')\sigma_+(t)\sigma_-(t') + G_c(t-t')\sigma_+(t')\sigma_-(t)] \end{aligned}$$

國立清華大學
National Tsing Hua University

Purcell effect: Cavity-QED (Quantum ElectroDynamics)





E. M. Purcell, *Phys. Rev.* **69** (1946).

Nobel laureate Edward Mills Purcell (shared the prize with Felix Bloch) in 1952,

for their contribution to nuclear magnetic precision measurements.

國立清華大學 National Tsing Hua University

from: K. J. Vahala, *Nature* **424**, 839 (2003).

Quantum theory for Nonlinear Pulse Propagation

1. Quantum Nonlinear Schrödinger Equation
2. Quadrature Squeezing of Optical Solitons
3. Amplitude Squeezing of Bragg Solitons
4. Quantum Correlation of Solitons
5. Quantum theory for Bound-State Solitons

Ref:

"Electromagnetic Noise and Quantum Optical Measurements," by H. Haus.

R.-K. Lee and Y. Lai, *Phys. Rev. A* **69**, 021801(**R**) (2004);

R.-K. Lee and Y. Lai, *J. Opt. B* **6**, S638 (2004);

R.-K. Lee, Y. Lai and B. A. Malomed, *J. Opt. B* **6**, 367 (2004);

R.-K. Lee, Y. Lai and B. A. Malomed, *Phys. Rev. A* **70**, 063817 (2004);

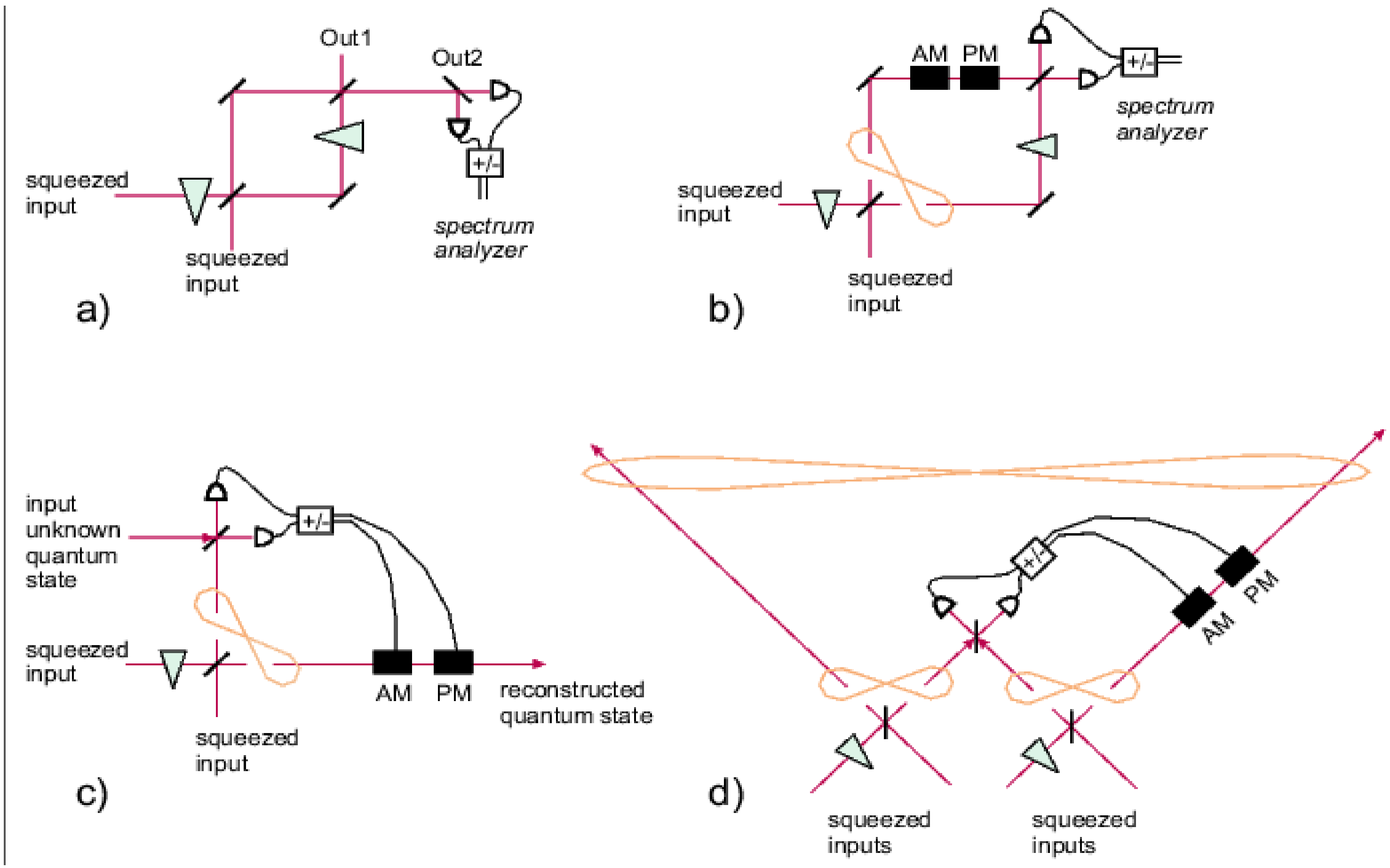
R.-K. Lee, Y. Lai and Yu. S. Kivshar, *Phys. Rev. A* **71**, 035801 (2005);

R.-K. Lee and Y. Lai, SIT solitons, to *Phys. Rev. A* (2009);

Y. Lai and R.-K. Lee, entangled solitons, to *Phys. Rev. Lett.* (2009);

Applications of EPR Pairs by Using Squeezed States

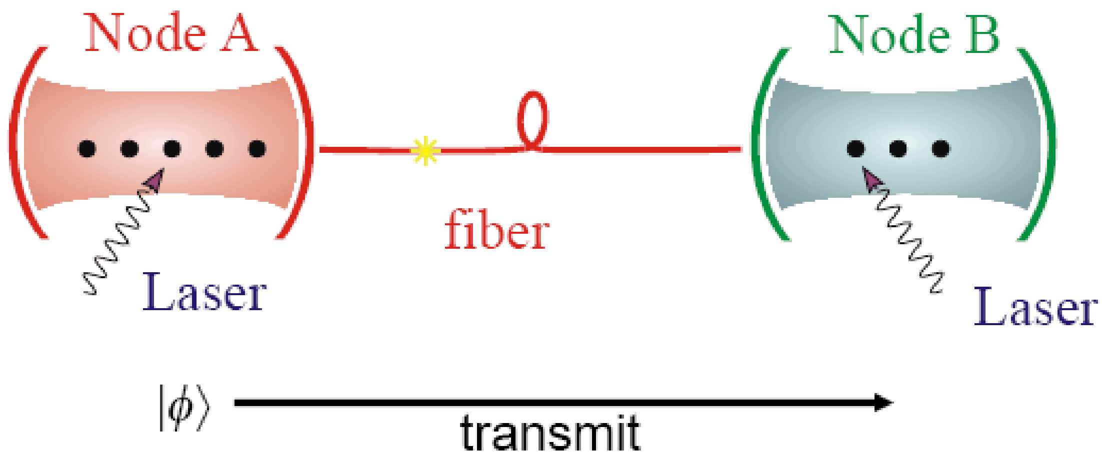
(a)entanglement; (b)quantum dense coding; (c) teleportation; (d)entangle swapping.



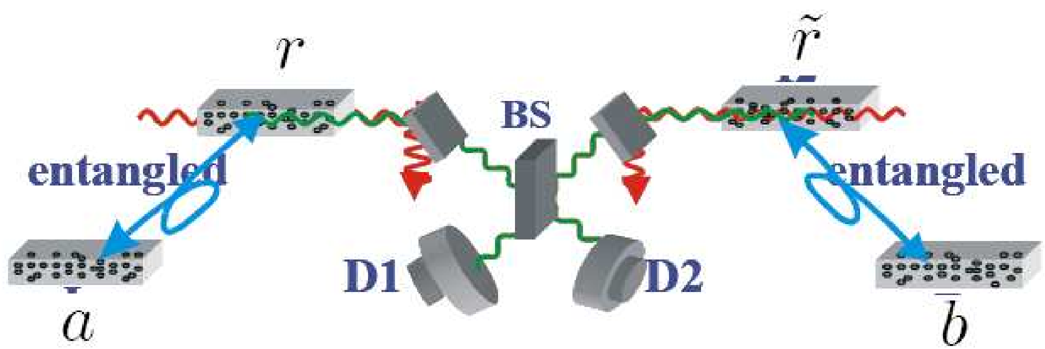
国立清華大學
National Tsing Hua University

G. Leuchs and N. Korolkova, *Optics & Photonics News* **Feb.**, 64 (2002).

Quantum State Transfer

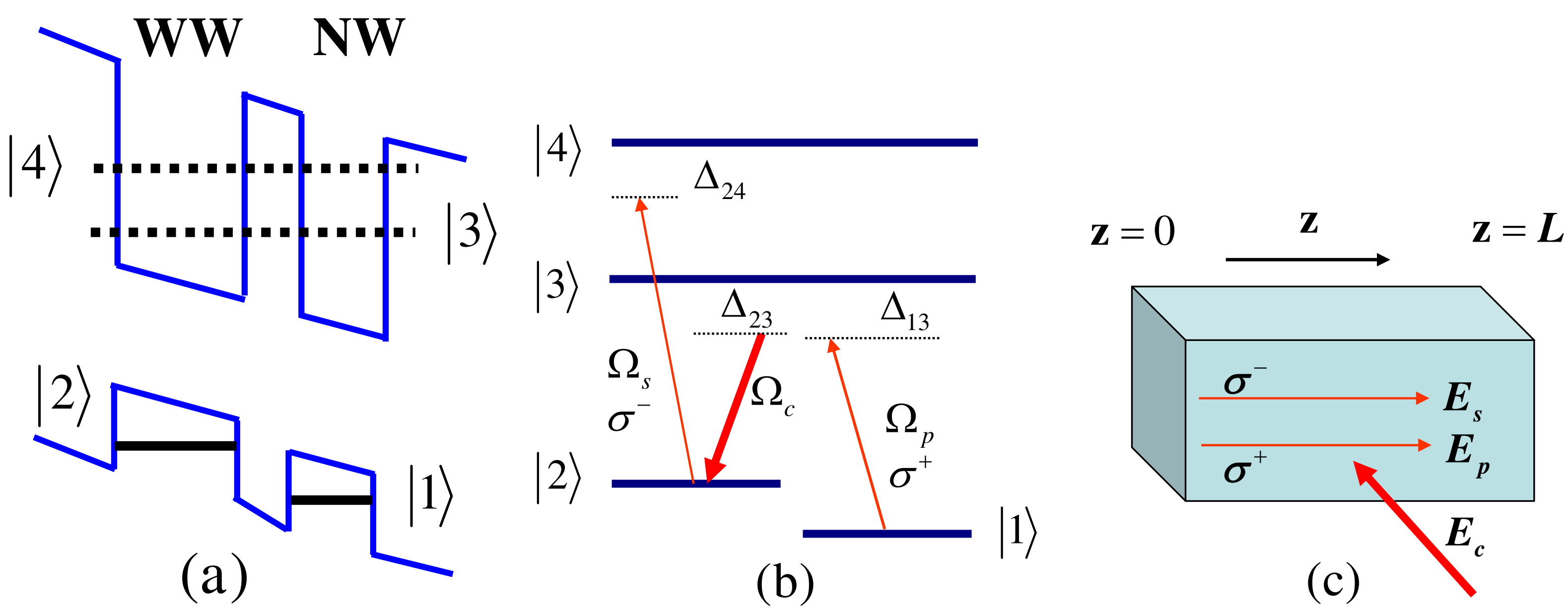


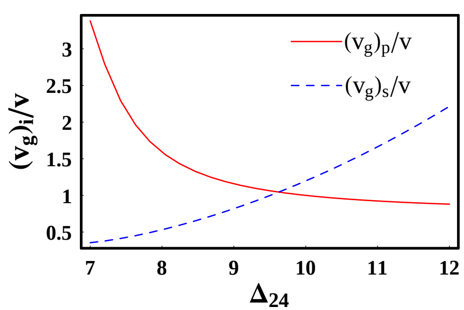
- memory: atoms → • databus: photons → • memory: atoms



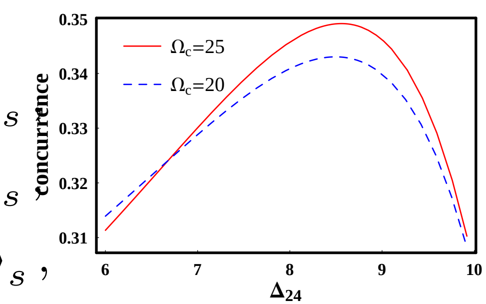
L.-M. Duan, M. D. Lukin, J. I. Cirac, P. Zoller, *nature* **414**, 413 (2001).

Controllable entanglement and polarization phase gate, 2-qubit





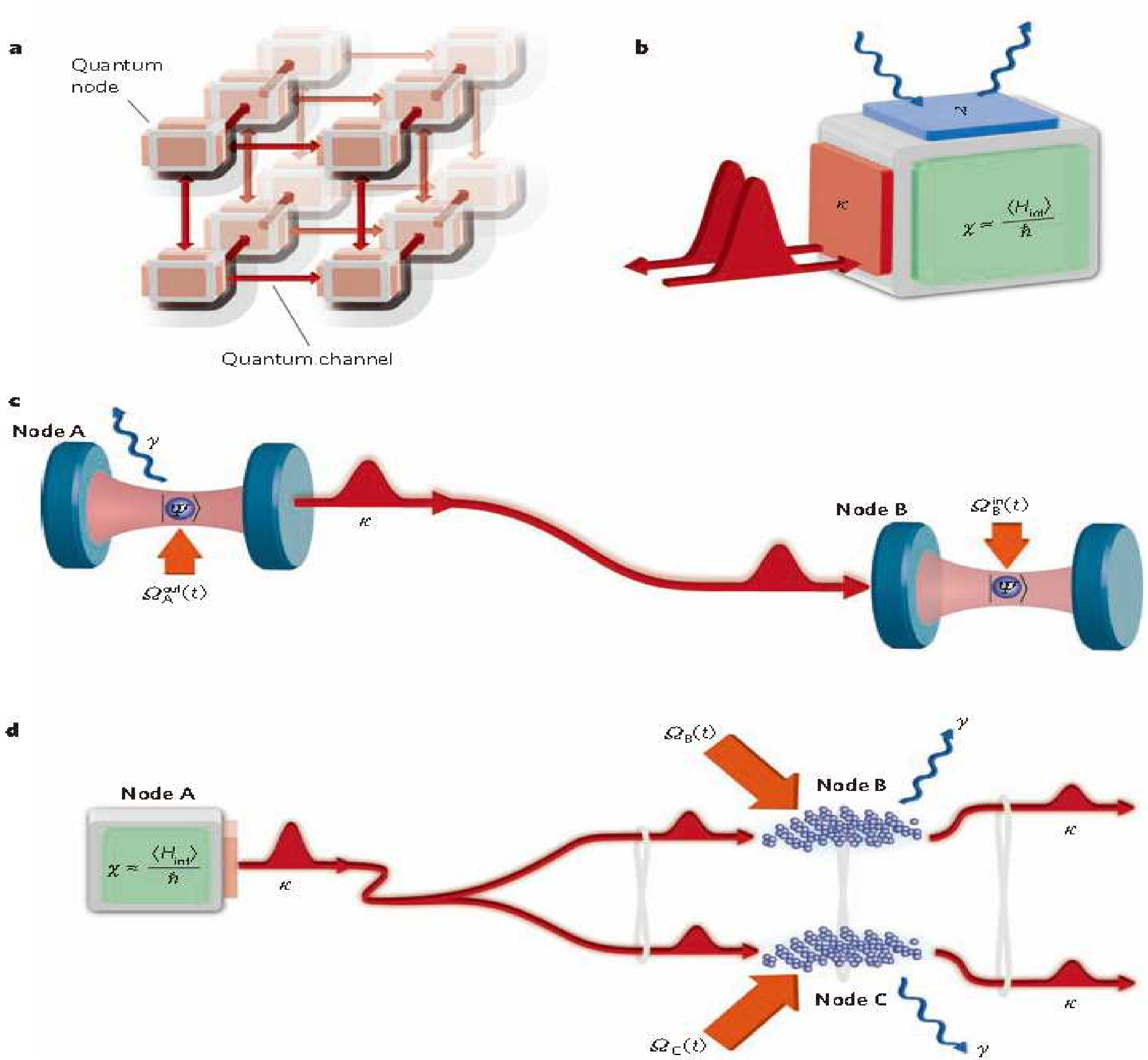
$$
\begin{aligned}
|0\rangle_p\,|0\rangle_s &\rightarrow \exp[-i(\phi_0^p+\phi_0^s)]\,|0\rangle_p\,|0\rangle_s\\
|0\rangle_p\,|1\rangle_s &\rightarrow \exp[-i(\phi_0^p+\phi_0^s)]\,|0\rangle_p\,|1\rangle_s\\
|1\rangle_p\,|1\rangle_s &\rightarrow \exp[-i(\phi_\Lambda^p+\phi_0^s)]\,|1\rangle_p\,|1\rangle_s\,,\\
|1\rangle_p\,|0\rangle_s &\rightarrow \exp[-i(\phi_{total}^p+\phi_0^s)]\,|1\rangle_p\,|0\rangle_s\,.
\end{aligned}
$$



國立清華大學 National Tsing Hua University

W.-X. Yang and RKL, *Opt. Express* **16**, 17616 (2008).

Quantum network



H. J. Kimble, *nature* **453**, 1023 (2008).

Quantum images

Entangled Images from Four-Wave Mixing

Vincent Boyer,* Alberto M. Marino, Raphael C. Pooser, Paul D. Lett*

Two beams of light can be quantum mechanically entangled through correlations of their phase and intensity fluctuations. For a pair of spatially extended image-carrying light fields, the concept of entanglement can be applied not only to the entire images but also to their smaller details. We used a spatially multimode amplifier based on four-wave mixing in a hot vapor to produce twin images that exhibit localized entanglement. The images can be bright fields that display position-dependent quantum noise reduction in their intensity difference or vacuum twin beams that are strongly entangled when projected onto a large range of different spatial modes. The high degree of spatial entanglement demonstrates that the system is an ideal source for parallel continuous-variable quantum information protocols.

Entangling the Spatial Properties of Laser Beams

Katherine Wagner,[1] Jiri Janousek,[1] Vincent Delaubert,[1,2] Hongxin Zou,[1] Charles Harb,[3] Nicolas Treps,[2] Jean François Morizur,[1,2] Ping Koy Lam,[1] Hans A. Bachor[1]*

Position and momentum were the first pair of conjugate observables explicitly used to illustrate the intricacy of quantum mechanics. We have extended position and momentum entanglement to bright optical beams. Applications in optical metrology and interferometry require the continuous measurement of laser beams, with the accuracy fundamentally limited by the uncertainty principle. Techniques based on spatial entanglement of the beams could overcome this limit, and high-quality entanglement is required. We report a value of 0.51 for inseparability and 0.62 for the Einstein-Podolsky-Rosen criterion, both normalized to a classical limit of 1. These results are a conclusive optical demonstration of macroscopic position and momentum quantum entanglement and also confirm that the resources for spatial multimode protocols are available.

國立清華大學
National Tsing Hua University

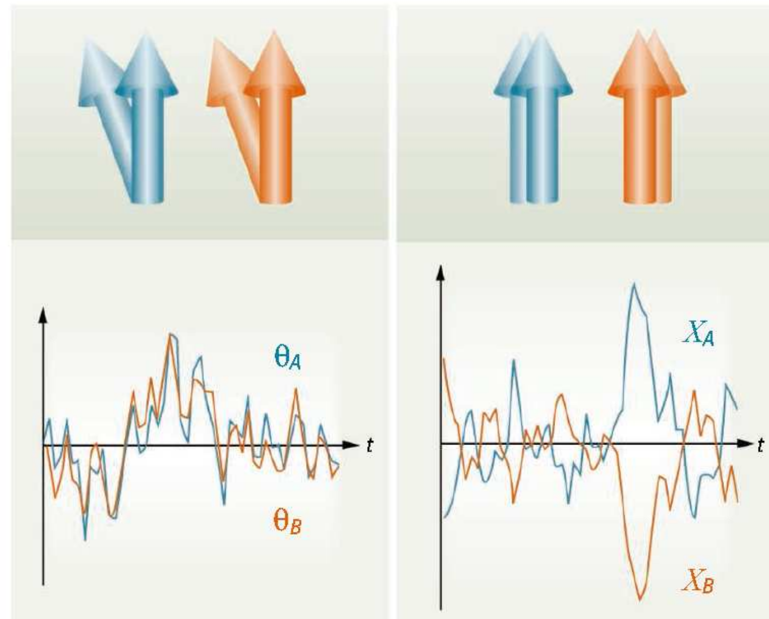
K. Wagner *et al.*, *Science* **321**, 541 (2008).
V. Boyer, A. M. Marino, R. C. Pooser, P. D. Lett, *Science* **321**, 544 (2008).

Quantum images

Entangled Images from Four-Wave Mixing

Vincent Boyer,* Alberto M. Marino, Raphael C. Pooser, Paul D. Lett*

Two beams of light can be quantum mechanically er
phase and intensity fluctuations. For a pair of spatia
concept of entanglement can be applied not only to
details. We used a spatially multimode amplifier bas
produce twin images that exhibit localized entangler
display position-dependent quantum noise reduction
beams that are strongly entangled when projected o
The high degree of spatial entanglement demonstrat
parallel continuous-variable quantum information pr



Entangled light beams. Entanglement is illustrated between two laser beams (A in blue and B in red). (**Left**) The direction of propagation fluctuates, but the fluctuations are highly correlated, so that the directions change in unison. (**Right**) Two beams whose positions are anticorrelated—as one beam moves left, the other moves to the right. For either beam studied individually, the position fluctuations ΔX and direction fluctuation $\Delta\theta$ are related by diffraction laws and obey an uncertainly relation $\Delta X\,\Delta\theta > \lambda/4\pi$, where λ is the wavelength of light. However, because of the correlations between the two beams, the uncertainty product of the differences $[\Delta(X_A + X_B)][\Delta(\theta_A - \theta_B)]$ can be much smaller than $\lambda/4\pi$

Entangling the Spati
of Laser Beams

Katherine Wagner,[1] Jiri Janousek,[1] Vincent Delaul
Nicolas Treps,[2] Jean François Morizur,[1,2] Ping Koy

Position and momentum were the first pair of conju
the intricacy of quantum mechanics. We have extenc
bright optical beams. Applications in optical metrolo
measurement of laser beams, with the accuracy funda
Techniques based on spatial entanglement of the be
quality entanglement is required. We report a value
Einstein-Podolsky-Rosen criterion, both normalized t
conclusive optical demonstration of macroscopic positi
and also confirm that the resources for spatial multim

国立清華大學
National Tsing Hua University

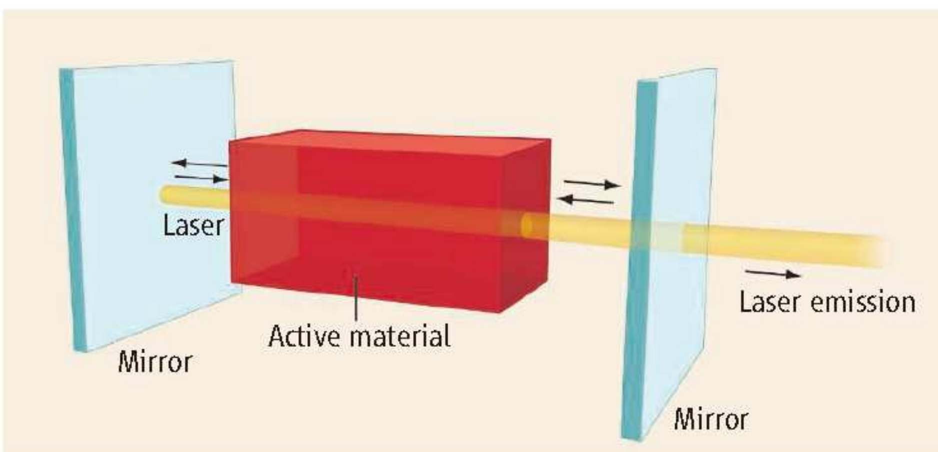
K. Wagner *et al.*, *Science* **321**, 541 (2008).
V. Boyer, A. M. Marino, R. C. Pooser, P. D. Lett, *Science* **321**, 544 (2008).

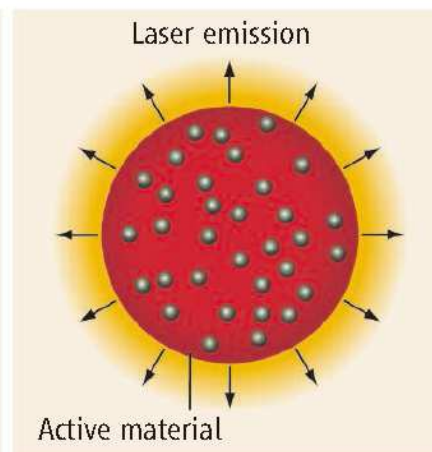
Random lasers

Strong Interactions in Multimode Random Lasers

Hakan E. Türeci,[1*] Li Ge,[2] Stefan Rotter,[2†] A. Douglas Stone[2]

Unlike conventional lasers, diffusive random lasers (DRLs) have no resonator to trap light and no high-Q resonances to support lasing. Because of this lack of sharp resonances, the DRL has presented a challenge to conventional laser theory. We present a theory able to treat the DRL rigorously and provide results on the lasing spectra, internal fields, and output intensities of DRLs. Typically DRLs are highly multimode lasers, emitting light at a number of wavelengths. We show that the modal interactions through the gain medium in such lasers are extremely strong and lead to a uniformly spaced frequency spectrum, in agreement with recent experimental observations.

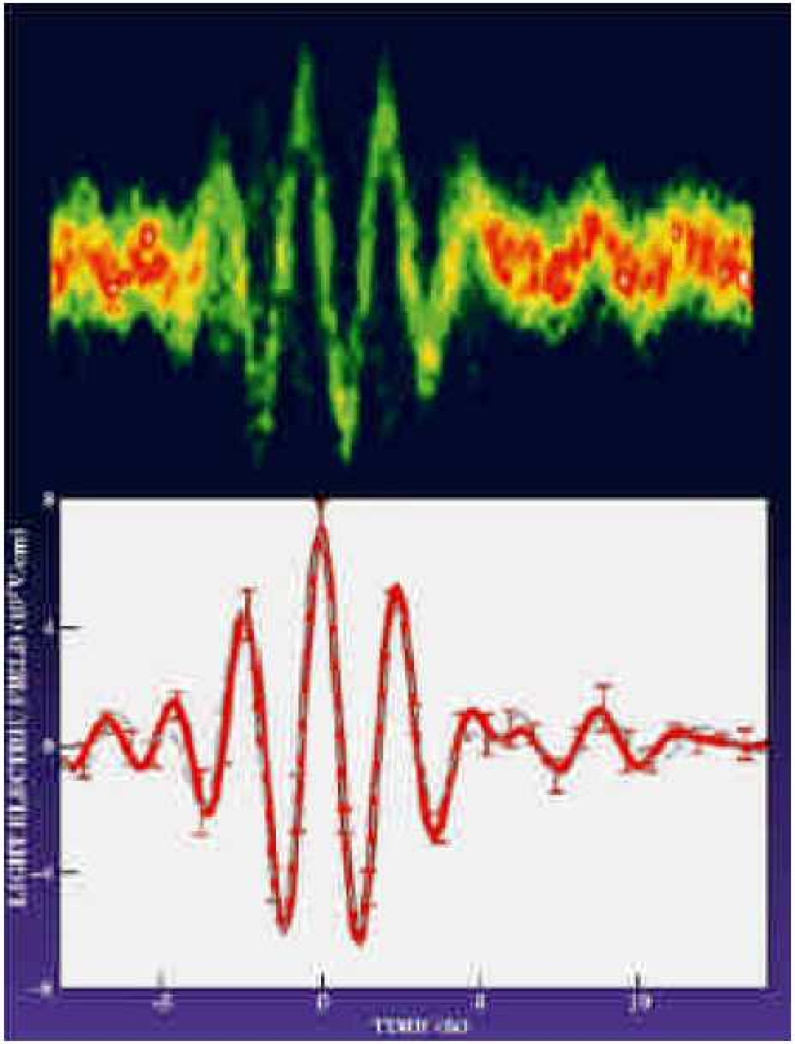


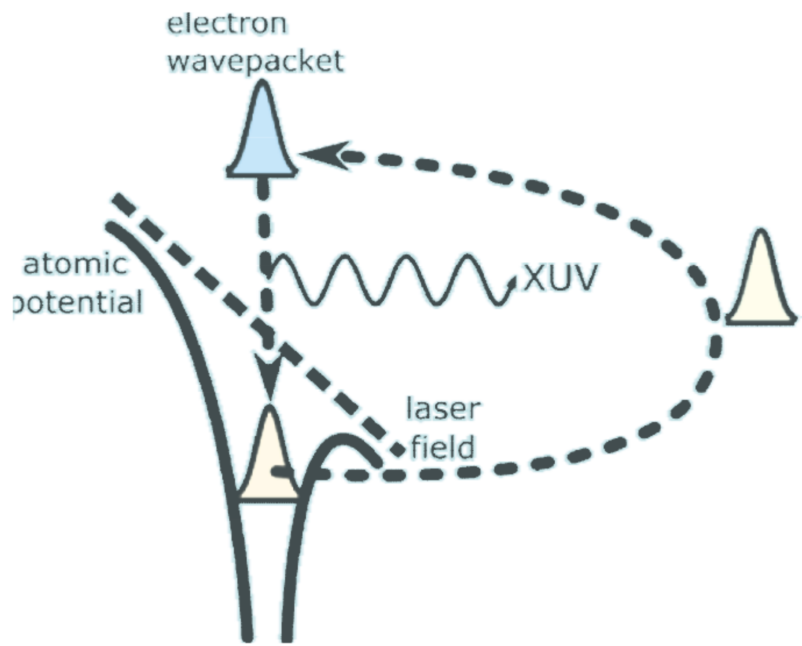


國立清華大學 National Tsing Hua University

H. E. Türeci, L. Ge, S. Rotter, A. D. Stone, *Science* **320**, 643 (2008).

Attosecond Physics



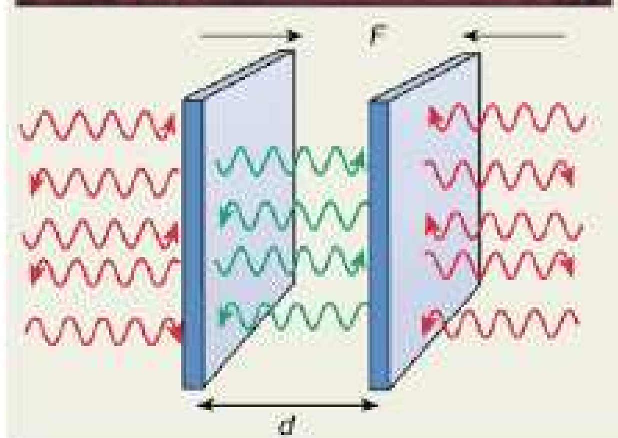


Courtesy of C.D. Lin, K-State.

國立清華大學
National Tsing Hua University

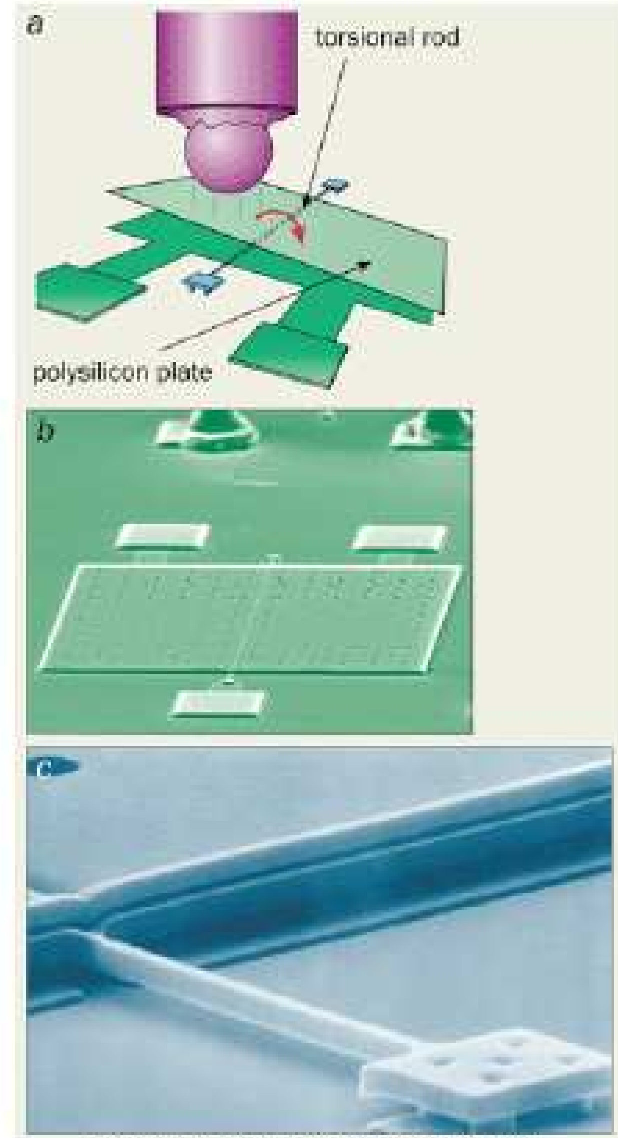
Casimir effect

Hendrik Casimir (1909-2000)



there is a force between
two metal slabs if brought
in close vicinity

important for micromechanical devices (MEMS)



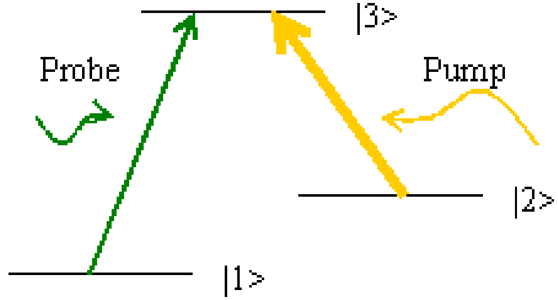
http://physicsweb.org/articles/world/15/9/6

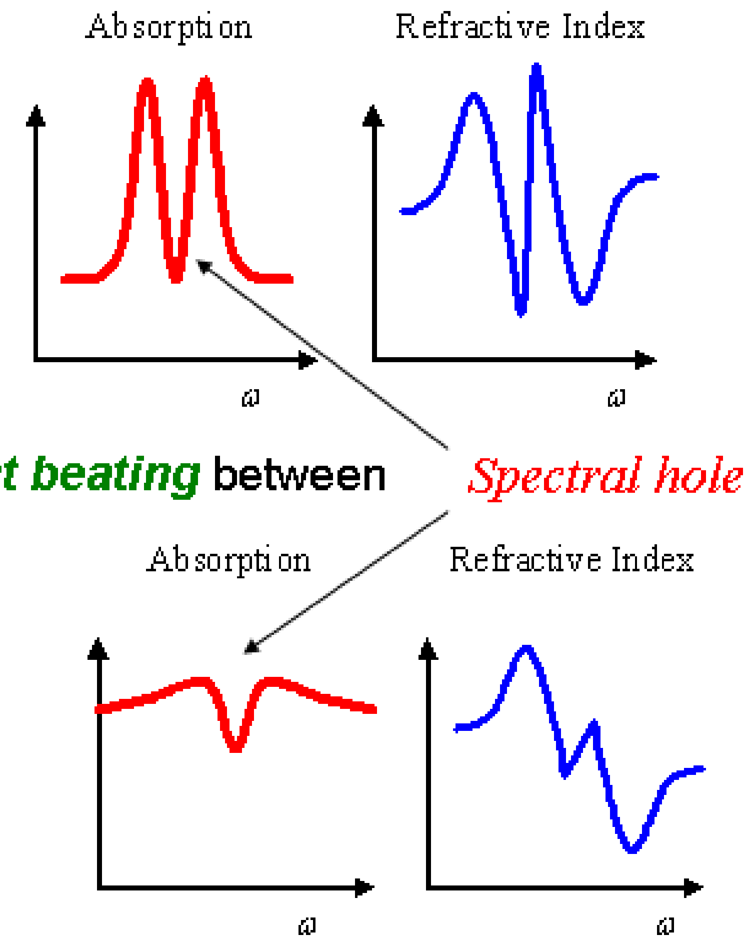
force is due to vacuum fluctuations of
the electromagnetic field

S. K. Lamoreaux, "Demonstration of the Casimir Force in the 0.6 to 6 μm Range"
Phys. Rev. Lett. 78, 5–8 (1997)

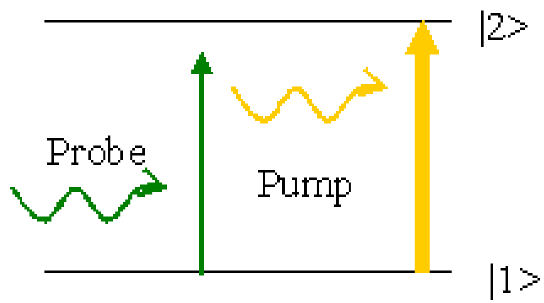
Slow-light via EIT and CPO

- **Electromagnetically Induced Transparency (EIT)**
  — Formation of ***dark state*** by intense pump in a three-level system





- **Population Oscillation**
  — Absorption dip generated by ***coherent beating*** between pump and probe in a two-level system

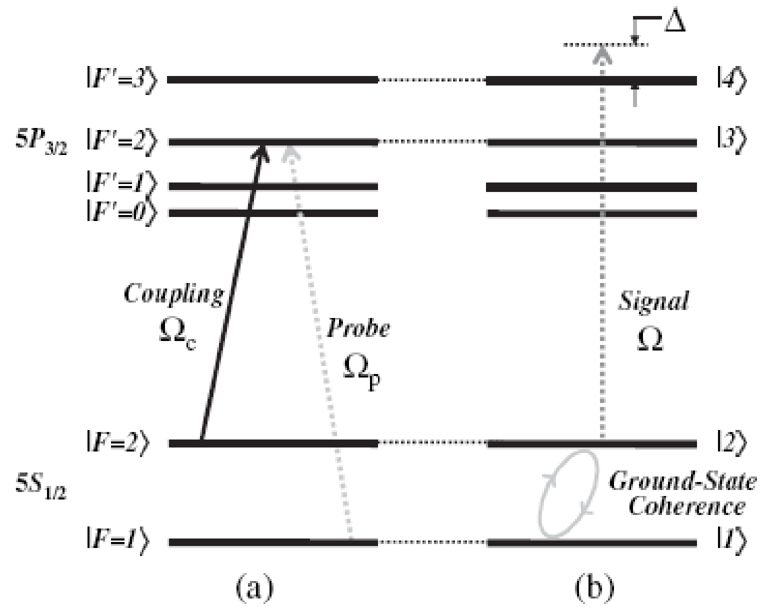


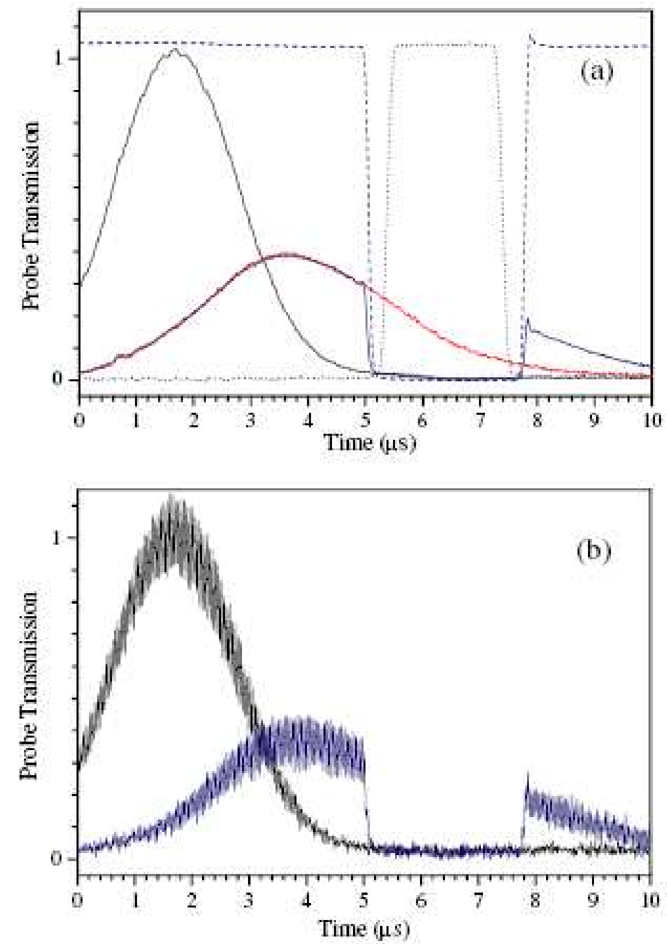
國立清華大學 National Tsing Hua University

Courtesy of Shun Lien Chuang, UIUC/USA

Low-Light-Level Cross-Phase-Modulation Based on Stored Light Pulses

less than one photon per $\lambda^2/2\pi$ in probe pulse and π phase shift for XPM,



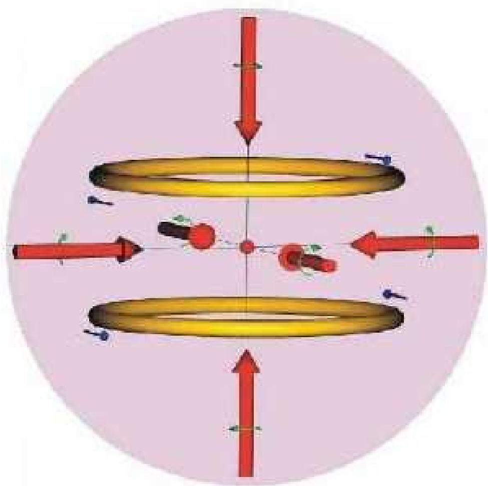


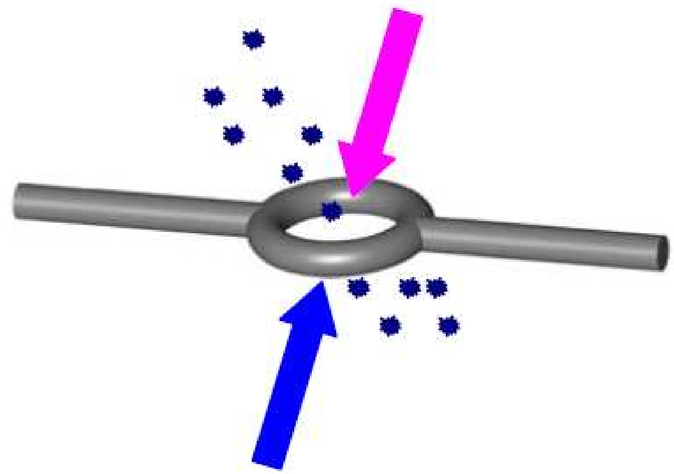
Courtesy: I.A. Yu (NTHU)

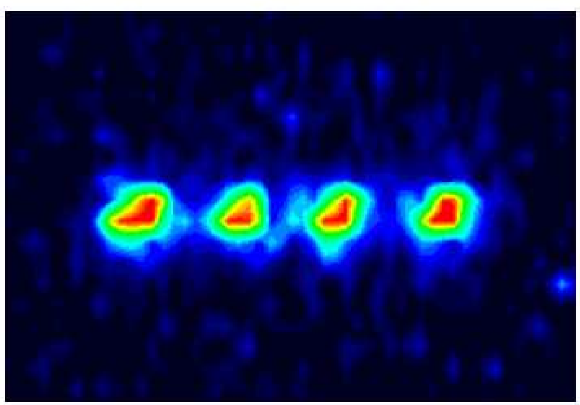
National Tsing Hua University

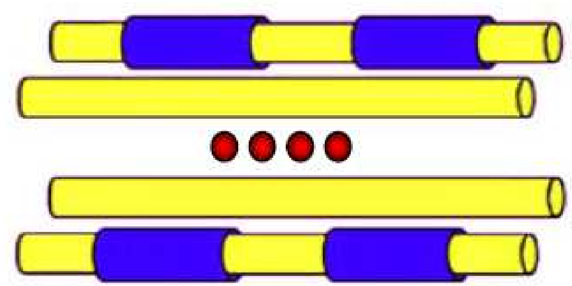
Y.F. Chen, C.-Y. Wang, S.-H. Wang, and I.A. Yu, *Phys. Rev. Lett.* **96**, 043603 (2006).

Ion-trap





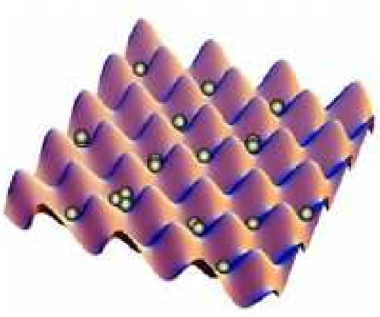




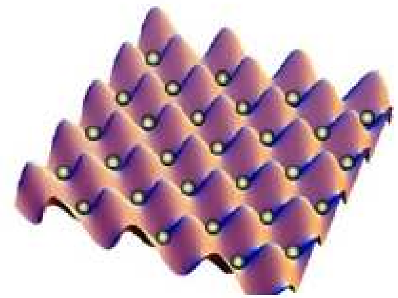
Courtesy of L.B. Wang, NTHU.

國立清華大學
National Tsing Hua University

Quantum Phase Transitions of BEC in Optical Lattices

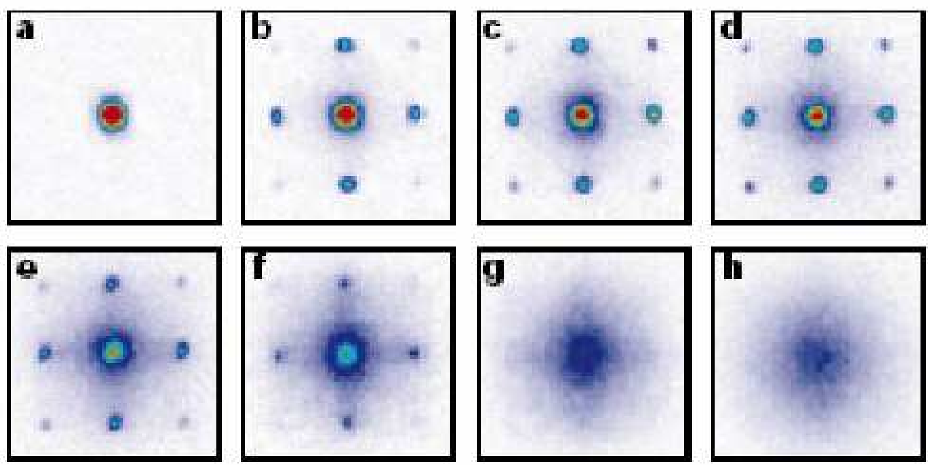


raising up the lattice potential!



BEC superfluid state

insulator state (1 atom /lattice site)



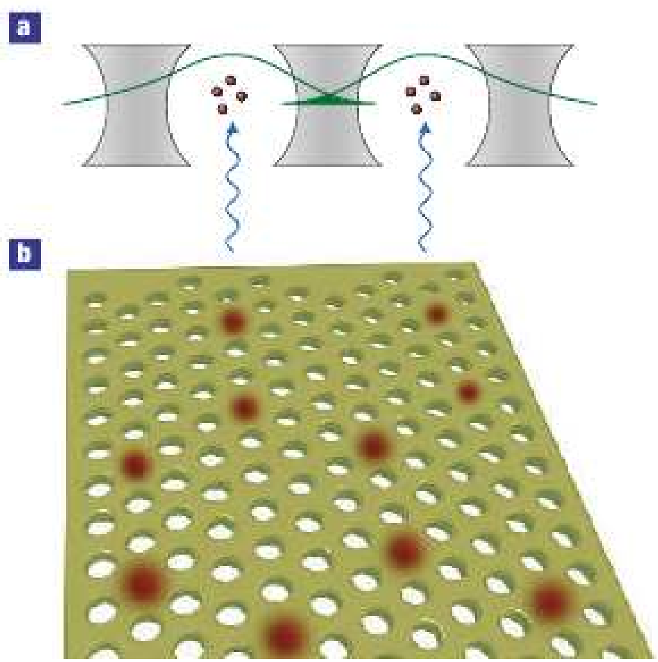
raising up lattice potential by increasing light intensity.

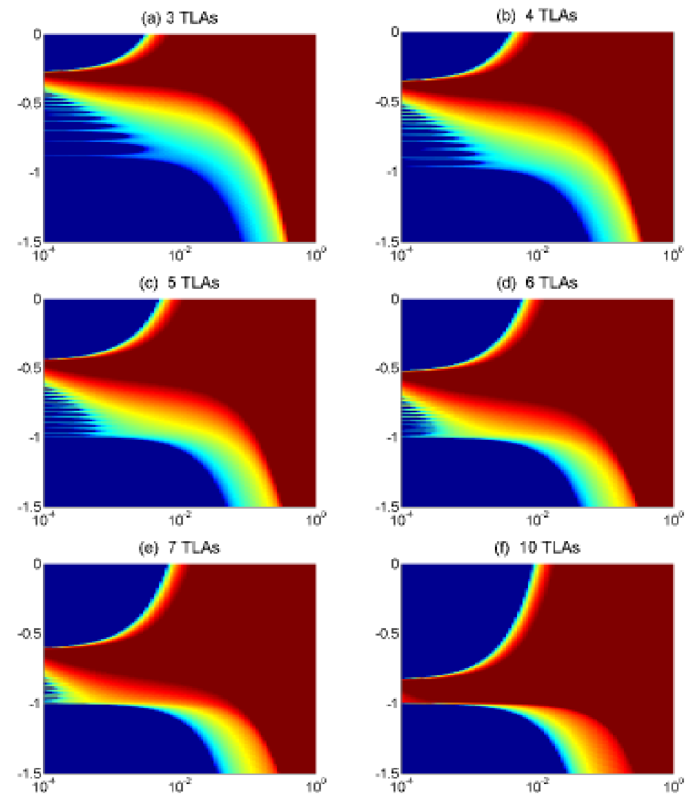
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Optics in 2008, Quantum Optics

$$\hat{H} = \sum_i H_i^{\mathrm{DM}} - \kappa \sum_{ij} a_i^+ a_j - \mu \sum_i N_i,$$

$$H_i^{DM} = \varepsilon J_i^+ J_i^- + \omega a_i^+ a_i + \beta (a_i J_i^+ + a_i^+ J_i^-),$$





Soi-Chan Lei and RKL, *Optics & Photonics News*, Dec., 44 (2008).

Wavelike energy transfer in photosynthetic systems

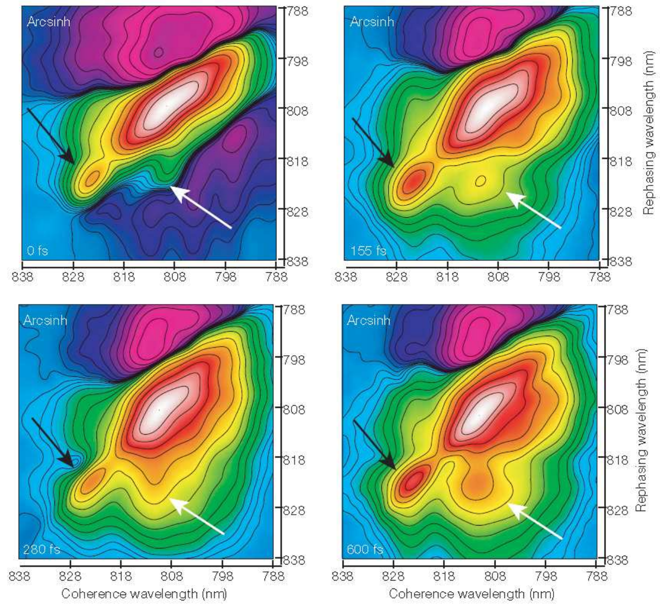


Figure 1 | Two-dimensional electronic spectra of FMO. Selected two-dimensional electronic spectra of FMO are shown at population times from $T = 0$ to 600 fs demonstrating the emergence of the exciton 1–3 cross-peak (white arrows), amplitude oscillation of the exciton 1 diagonal peak (black arrows), the change in lowest-energy exciton peak shape and the oscillation of the 1–3 cross-peak amplitude. The data are shown with an arcsinh coloration to highlight smaller features: amplitude increases from blue to white (for a three-dimensional representation of the coloration see Fig. 3a).

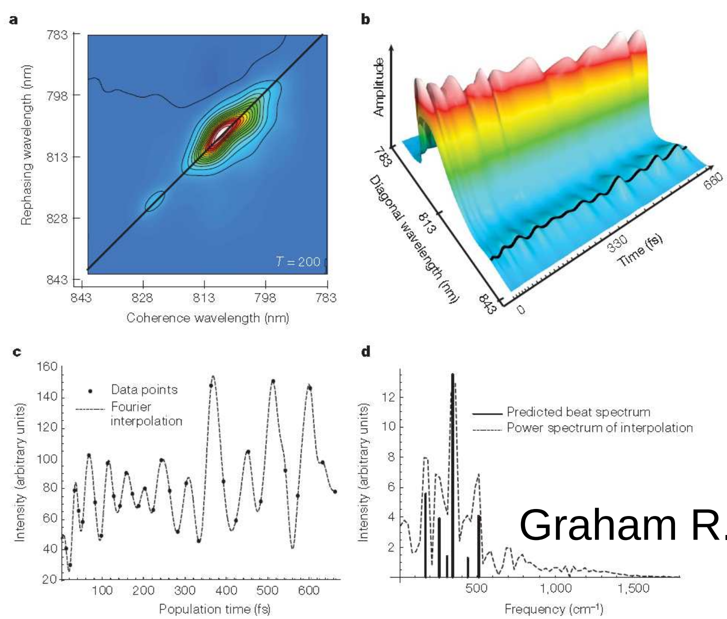


Figure 2 | Electronic coherence beating. a, A representative two-dimensional electronic spectrum with a line across the main diagonal peak. The amplitude along this diagonal line is plotted against population time in **b** with a black line covering the exciton 1 peak amplitude; the data are scaled by a smooth function effectively normalizing the data without affecting oscillations. A spline interpolation is used to connect the spectra; the times at which spectra were taken are denoted by tick marks along the time axis. **c,** The amplitude of the peak corresponding to exciton 1 shown with a dotted Fourier interpolation. **d,** The power spectrum of the Fourier interpolation in **c** is plotted with the theoretical spectrum showing beats between exciton 1 and excitons 2–7.

Graham R. Fleming, *Nature* **446**, 782 (2007).

國立清華大學
National Tsing Hua University