The 18th International Conference on Finite Elements in Flow Problems (FEF2015)
16–18 March 2015, Taipei, Taiwan

Incompressible N-Phase Flows: Physical Formulation and Numerical Algorithm

Suchuan Dong
Center for Computational and Applied Mathematics
Department of Mathematics
Purdue University, West Lafayette, IN 47907, USA
sdong@purdue.edu

Keywords: *N-phase flow; large density contrast; phase field; pairwise surface tensions*

The current work concerns the simulation of a mixture of N ($N \geqslant 2$) immiscible incompressible fluids, with possibly very different physical properties such as densities, viscosities and inter-fluid surface tensions. Such incompressible N-phase flows admit multiple types of fluid interfaces and three-phase lines or junctures, and they underline many phenomena and processes of practical engineering significance and fundamental physical interest.

One faces several issues when simulating N-phase flows. A foremost issue confronting N-phase simulations is how to formulate the system in a physically consistent manner. Another crucial outstanding issue with regard to phase field formulations, for situations with more than three fluid phases, concerns how to determine the mixing energy density coefficients involved in the model based on known physical parameters. Apart from the issues regarding the physical formulations, one also faces significant algorithmic challenges when numerically simulating the N-phase system, such as that posed by the strong couplings among the $(N-1)$ phase field variables and those associated with the variable mixture density/viscosity.

In the current paper we present the following [1]:

- A thermodynamically consistent physical formulation for incompressible N-phase flows. The formulation (iso-thermal) honors the conservations of mass and momentum, the second law of thermodynamics, and Galilean invariance.

- A method to uniquely determine the mixing energy density coefficients involved in the N-phase model based on the $\frac{N(N-1)}{2}$ pairwise surface tensions. Our method leads to an explicit form of a system of $\frac{N(N-1)}{2}$ linear algebraic equations, with the pairwise surface tensions on the right hand sides, which can be solved for the mixing energy density coefficients.

- An efficient numerical algorithm for simulating incompressible N-phase flows. With our algorithm the computations for different flow variables are completely de-coupled. For each flow variable the linear algebraic system after discretization involves only *constant* and *time-independent* coefficient matrices, which can be pre-computed during pre-processing, despite the variable and time-dependent nature of the mixture physical properties such as density and dynamic viscosity.

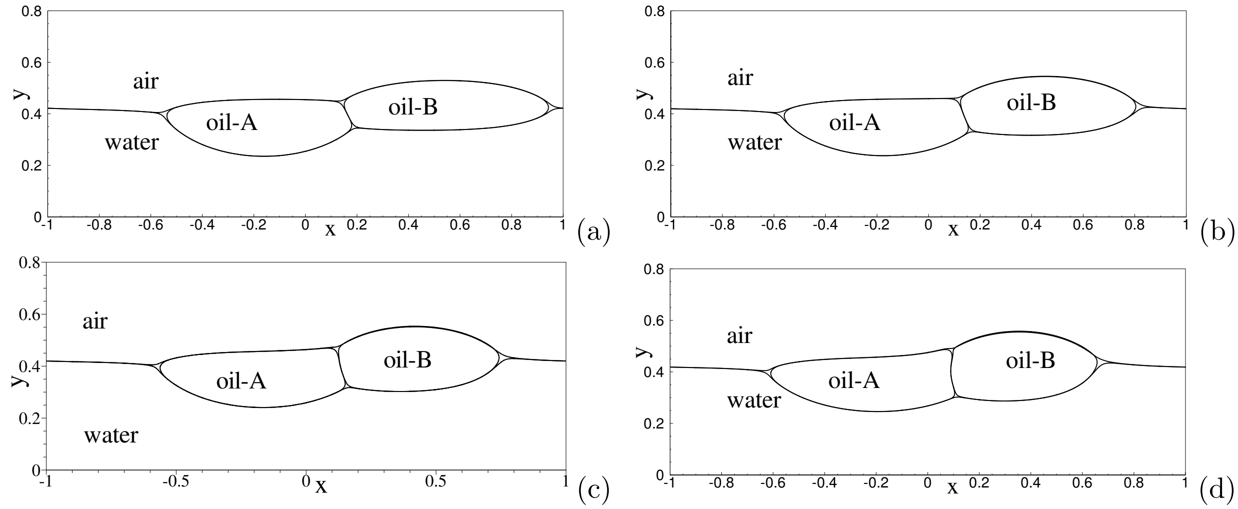

Figure 1: 4 fluid phases: Effect of air/oil-B surface tension on the equilibrium configuration of two types of oil drops at the air-water interface. Air/oil-B surface tensions (kg/s^2): (a) 0.05, (b) 0.06, (c) 0.07, (d) 0.08.

Here we illustrate the method using a four-phase problem. Figure 1 shows the equilibrium configurations of two types of oil drops (oil-A and oil-B) on the air-water interface, which are obtained using our method. This problem involves four fluid phases: air, water, oil-A and oil-B. The four panels correspond to different air/oil-B surface tension values, while the rest of the physical parameters of the system are kept the same.

We will present numerical experiments for several problems involving multiple fluid phases, large density contrasts and large viscosity contrasts. In particular, we compare our simulations with the Lagmuir-de Gennes theory, and demonstrate that our method produces physically accurate results for multiple fluid phases. We will show that the method presented herein is capable of dealing with N-phase systems with large density ratios, large viscosity ratios, and pairwise surface tensions, and that it can be a powerful tool for studying the interactions among multiple types of fluid interfaces.

Acknowledgement

The author gratefully acknowledges the support from ONR and NSF.

References

[1] S. Dong. An efficient algorithm for incompressible N-phase flows. *Journal of Computational Physics*, 276:691–728, 2014.