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A SPACE-TIME CUT FINITE ELEMENT METHOD FOR CONVECTION-DIFFUSION PROBLEMS ON TIME DEPENDENT DOMAINS

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Problems involving phenomena that take place both on surfaces (or interfaces) and in bulk domains occur in a variety of applications in fluid dynamics and biological applications. The example we consider is given by the modeling of soluble surfactants. Surfactants are important because of their ability to reduce the surface tension and are for example used in detergents, oil recovery, and in treatment of lung diseases.

The model we consider is given by a time-dependent convection-diffusion equation on the interface separating two immiscible fluids coupled with a time-dependent convection-diffusion equation in the bulk. The exchange of surfactants between the interface and the bulk is modeled with a nonlinear term. For existing works on soluble surfactants see e.g. [1, 2]. We present a space-time cut finite element method for the coupled bulk-surface problem modeling soluble surfactants.

Our strategy is to embed the time-dependent domain where the PDE has to be solved, in a fixed background grid equipped with a standard finite element space and then take the restriction of the finite element functions to this domain. This idea was first proposed for an elliptic problem with a stationary fictitious boundary in [3] and for the Laplace-Beltrami operator on a stationary interface in [4]. We call this type of methods cut finite element methods since the interface cuts through the background grid in an arbitrary fashion. An advantage with these methods is that the finite element space defined on the background grid can be used for solving both the PDE in the bulk region and on the interface.

We present an optimal order accurate cut finite element method based on a space-time approach with continuous linear elements in space and discontinuous linear elements in time. The interface can be arbitrarily located with respect to a fixed background mesh. To ensure well posedness of the resulting algebraic systems of equations, independent of the position of the interface in the background grid, we add a consistent stabilization term [5]. In addition, the total mass of surfactant can be accurately conserved using a Lagrange multiplier. Numerical results indicate that the method is optimal order accurate. Optimal order of accuracy has been shown analytically for a stationary coupled bulk-surface problem with a linear coupling term [6].

The proposed finite element method is very convenient for two phase flow simulations since the same finite element spaces can be used for solving problems both in the bulk domain and on the interfaces separating immiscible fluids. Our method works both with explicit and implicit interface representation techniques.

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