

## SIMULATIONS OF FLOW PAST A CYLINDER WITH ADAPTIVE NONCONFORMING SPECTRAL ELEMENT METHOD

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Flow past a cylinder is an interesting problem for both science and engineering because it releases rich physical phenomena and practical issues such as flow separation, the primary instability with development of the Von Karman vortex streets, etc.

Posdziech and Grundmann [1] used spectral element method to study the effect of extension of the computational domain on the accuracy of drag and lift forces, base-pressure coefficient and Strouhal number. Although spectral element method preserves the nature of high accuracy and fast convergence, it is still expensive to solve practical problems. Maday et al.[2] broke the limit by developing the mortar element methods for nonconforming spectral elements. The results of Dupond and Lin[3] show the spectral element method is superior to finite element methods in energy conservation with no dissipation and higher accuracy within the same CPU time. However, the accuracy-to-cost convergence is not as good as that achieved by the spectral element method. In this study, for investigation of the performance of the adaptive spectral element method and the verification of the accuracy of mesh refinement on curved elements, simulations of flow past a circular cylinder in  $Re=70$  to  $Re=250$  are implemented to validate the accuracy of the conforming and adaptive spectral element methods.

### NUMERICAL METHOD

A direct numerical technique, spectral element method, is used to simulate flow past the tandem array cylinders system by solving the incompressible Navier-Stokes equations.

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u} \quad (1)$$

$$\nabla \cdot \vec{u} = 0 \quad (2)$$

A time splitting scheme [4] is introduced to treat the nonlinear convective terms explicitly while the pressure and velocity diffusion terms may be solved for implicitly. The nonlinear

terms are advanced by a third order Adams-Bashforth scheme. The pressure and velocities are governed by Helmholtz problems which are discretized by the spectral element method.

## RESULTS AND CONCLUSION

The initial coarse meshes composed of 95 elements with shape function order 11 and its results were used as the inputs of the mesh refinement process. After mesh refinement, the nonconforming meshes, as shown in Fig.1, has 249 elements, which make up only 64% of conforming meshes ( $K=388$ ). The vorticity of  $Re=200$  obtained by adaptive nonconforming method can be found in Fig.2. The Strouhal numbers as shown in Fig.3 reveal that the present works by using conforming or adaptive nonconforming spectral element methods both show good agreement with most of the experimental and numerical results, especially, the results of Posdziech [1] in which the upstream of computational domain is extended to 70 or 4000 times the diameter of the cylinder. The mean drag coefficient and maximum lift coefficient obtained by the adaptive nonconforming method deviate slightly by 0.1~0.3% from the results found by Posdziech[1], in which upstream, upper and lower boundaries are the same as those obtained by present computation as shown in Table 1.

The adaptive nonconforming spectral element method is introduced to achieve reduced computational cost without losing accuracy. This study proves the adaptive nonconforming spectral element method is a useful technique for solving dynamic problems. The results also show that this mesh refinement technique is feasible for curved elements.

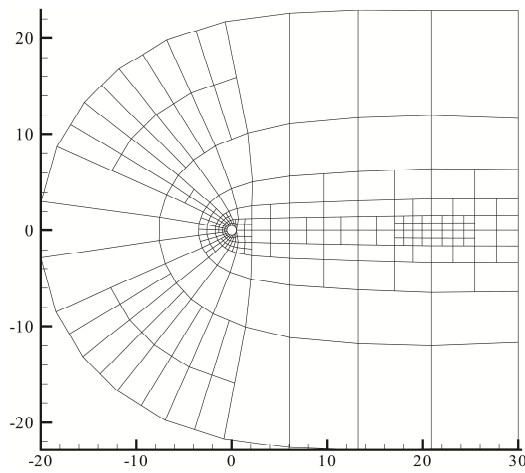


Fig.1 Mesh after refinement,  $k=249$ ,  $N=11$ ,  $Re=200$

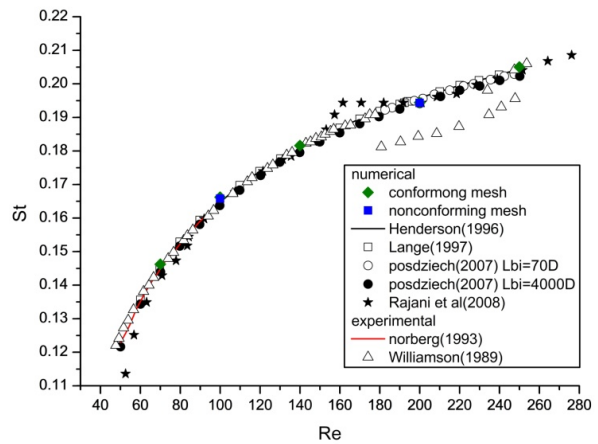


Fig. 3 The comparison of Strouhal number in variant Reynolds numbers

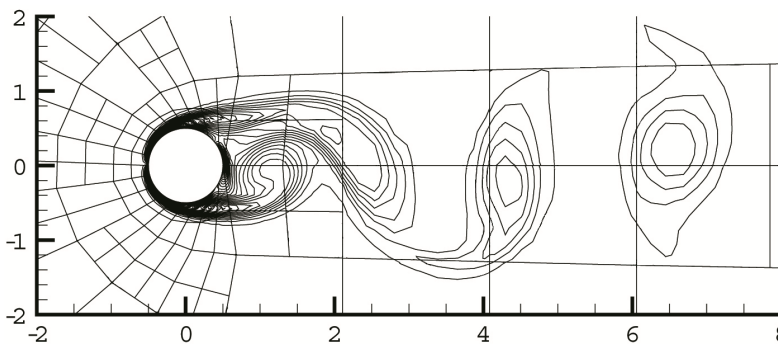


Fig.2 The vorticity contours in  $Re=200$  obtained by adaptive nonconforming method

Table 1 Drag, Lift coefficient and Strouhal number comparison

		Adaptive nonconforming (deviation)	Posdziech[1] ( $L_{bi}/D=20$ )
St.	$Re=100$	0.1658 (0.5%)	0.1667
	$Re=200$	0.1943 (1.67%)	0.1976
Cd	$Re=100$	1.3466 (0.28%)	1.3504
	$Re=200$	1.350 (0.1%)	1.3490
Cl	$Re=100$	0.3299 (0.3%)	0.3309
	$Re=200$	0.6872 (0.1%)	0.6880

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