The 18th International Conference on Finite Elements in Flow Problems (FEF2015)
16-18 March 2015, Taipei, Taiwan

A COMPARATIVE STUDY ON THE COMPUTATIONAL EFFICIENCY OF LOCALLY CONSERVATIVE FINITE ELEMENT METHODS FOR SUBSURFACE FLOWS

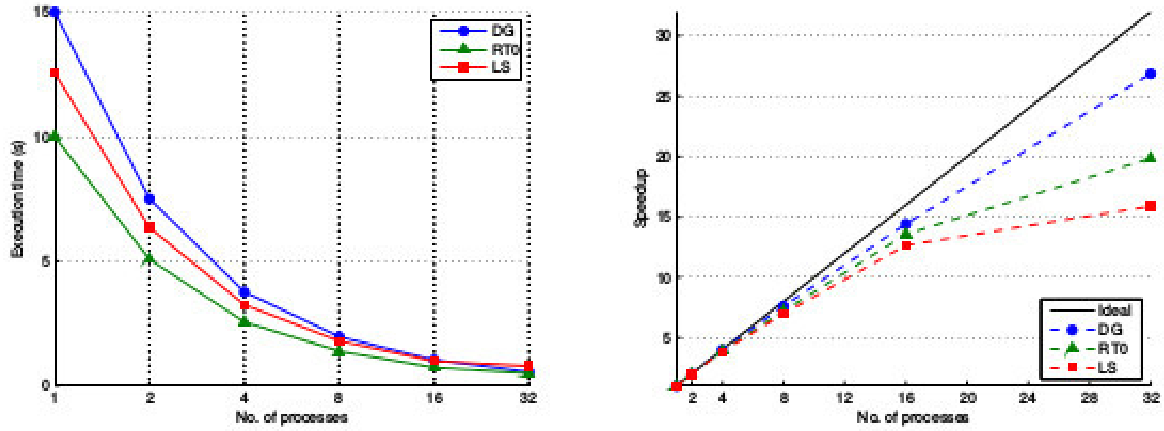
Justin Chang[1*], Kalyana Nakshatrala[1], and Lennart Johnsson[2]

[1] Department of Civil and Environmental Engineering,
University of Houston, Houston, TX 77004
jychang48@gmail.com
[2] Department of Computer Science, Mathematics, and Electrical Engineering,
University of Houston, Houston, TX 77004, USA
johnsson@cs.uh.edu

Key words: *Finite element methods, PETSC, ParMETIS, high performance computing, parallel computing, local mass balance, discontinuous Galerkin, Raviart-Thomas, quadratic optimization, linear solvers, porous media flows*

The finite element method (FEM) is one of the most popular numerical techniques used in solving problems in the field of computational fluid mechanics. However, it is important to note that several FEM formulations do not possess element-wise mass / species balance property. For flow and transport problems through porous media, this can lead to inaccurate measurements of fluid velocity and concentration of chemical species. Several studies over the years have focused on developing locally conservative FEM [1] some of the more popular methodologies include: the discontinuous Galerkin method [2], the classical mixed Galerkin using lowest order Raviart-Thomas space [3], and a recently proposed FEM methodology based on convex optimization [4]. The numerical accuracy of these methods has been rigorously studied and examined for small and 2D academic problems, but when large-scale simulations of subsurface flows have to be conducted, it is important to know how these three FEMs behave in the context of high performance computing.

Technological applications of subsurface flows tend to be extremely large-scale and can easily require the computational domain to consist of millions to billions of degrees of freedom. Some examples include enhanced oil recovery, CO2 migration in carbon sequestration, groundwater contamination, and thermo-hydrological and biogeochemical effects on the Arctic permafrost due to climate change. In order to achieve optimal performance for such problems, it is important for computational scientists to understand three different aspects: (a) the data structures needed to discretize the mathematical models, (b) performance of linear (direct and iterative) solvers for resulting system of equations, and (c) how these various discretization methods behave with respect to the state-of-the-art



(a) Execution time (b) Speed-up

Figure 1: Parallel erformance metrics for the three locally conservative FEM. See [4] for details

high performance computing systems. Herein, a systematically study on the computational performance of the aforementioned three FEM is conducted by solving practical field-scale problems. Popular scientific libraries such as PETSc and ParMETIS for parallel analyses and mesh partitioning respectively were utilized on Los Alamos National Laboratory's Mustang and Wolf supercomputers. Details concerning the computational algorithms, solvers, and computing resources used are presented.

REFERENCES

[1] G. Lin, J Liu, and F. S. Marandi (2015) A comparative study on the weak Galerkin, discontinuous Galerkin, and mixed finite element methods, *Journal of Computational and Applied Mathematics* **273**:346-362.

[2] T. J. R. Hughes, A Masud, and J. Wan (2006) A stabilized mixed discontinuous galerkin method for darcy flow, *Computer Methods in Applied Mechanics and Engineering* **195**:3347-3381.

[3] D. Z. Turner, K. B. Nakshatrala, M. J. Martinez, and P. K. Notz (2012) Modeling subsurface water resource systems involving heterogeneous porous media using the variational multiscale formulation, *Journal of Hydrology* **428-429**:1-14.

[4] J. Chang, K. B. Nakshatrala, and J. N. Reddy (2014) Modification to Darcy model for high pressure and high velocity and associated mixed formulations, *Communications in Nonlinear Science and Numerical Simulation* [under review].