High-Order Kinetic Solver of Semi-classical Boltzmann-BGK Equation for Rarefied Gases of Arbitrary Statistics and Arbitrary Flow Regimes

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Abstract

Current progress on numerical methods for full semiclassical Boltmann equation [1, and references therein] and semiclassical Boltzmann model equations, [2, 3, 4, 5, 6], shows that model equations are a more tractable way to obtain smooth and high-quality deterministic solutions for classical and quantum ideal gases in classical and/or degenerate regimes. Moreover, as the solution could be in both, hydrodynamic and kinetic flow regimes, multiscale capabilities for a solver are a must. Based on previous works [3, 6], we introduce a hybrid scheme based on finite difference discretization of the physical domain that takes a discrete velocity formulation. The method differs from the method proposed by Shan, et. al. [7] as no expansion of the semiclassical equilibrium distribution function in Hermite polynomials is requiered. The present formulation is high-order in time and space and recovers the hydrodynamic limit of the quantum Euler/Navier-Stokes equations [8]. Moreover by complying with the Asymptotic-Preserving (AP) properties as proposed by Jin [9], we ensure that in hydrodynamic limit this scheme can capture macroscopic gas dynamics even if the small scale determined by the relaxation time τ is not numerically resolved. The above proposal is achieved by taking advantage of the following ingredients: A weigthed essentially non-oscillatory (WENO) methods for discretizing our physical domain [10], the discrete velocity method by Watari and Tsutahara [11] and the asymptotic preserving scheme of Pierachini & Puppo [12] by means of Additive Runge-Kutta (ARK) schemes of Karpenter & Kennedy [13]. Numerical investigations of 1-D and 2-D Riemann problems [14] for different configurations and flow regimes will be shown to illustrate the AP high-order semiclassical Boltzmann-BGK method and compare with the corresponding classical Euler and Quantum Euler solutions [4].

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