The 18th International Conference on Finite Elements in Flow Problems (FEF2015)
16-18 March 2015, Taipei, Taiwan

A UNILATERAL BOUNDARY CONDITION FOR THE STOKES EQUATIONS WITH APPLICATION TO NUMERICAL OUTFLOW BOUNDARY CONDITIONS

Norikazu Saito, Guanyu Zhou and Yoshiki Sugitani

Graduate School of Mathematical Sciences,
The University of Tokyo, Tokyo 153-8914, Japan
norikazu@ms.u-tokyo.ac.jp

Key words: *nonlinear boundary condition, outflow boundary condition, error estimate*

We first consider the Stokes equations in a bounded domain Ω with a unilateral boundary condition of Sinorini type. We pose the usual Dirichlet and unilateral boundary conditions, respectively, on S and Γ, where the boundary of Ω is composed of two parts S and Γ. We are interested in the case where there is a strictly inflow at S. That is,

$$u = b \quad \text{on} \quad S \quad \text{with} \quad \int_S b \, dS < 0.$$

Under this assumption, the coupled inf-sup lemmas ([4, Lemmas 7.2, 7.6]) are not available so that we need a new idea to examine the well-posedness and convergence of FEM for the problem. We briefly review our method of analysis based on the theory of variational inequalities with a technique to control the constant part of the pressure.

Then, we apply the unilateral boundary condition to the Navier-Stokes equations as a reasonable outflow boundary condition. One of the main issues in simulations of blood flow in arteries is a proper setting of the outflow boundary condition at artificial boundaries (cf. [2, 3]). The common outflow boundary conditions are a prescribed constant pressure/traction, prescribed velocity profiles. In particular, the so-called do-nothing condition is popular. However, the flow distribution and pressure field are unknown and cannot be prescribed at the outflow boundary in many simulations. We show that, with the unilateral boundary condition, we obtain an energy inequality so that numerical solutions are expected to be stable. On the other hand, we cannot obtain the energy inequality with the do-nothing condition.

Finally, we report that the penalty formulation of our unilateral boundary condition is closely related with the regularized traction boundary condition in [1].

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