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A PRESSURE-STABILIZED LAGRANGE-GALERKIN FINITE ELEMENT SCHEME FOR AN OSEEN-TYPE DIFFUSIVE PETERLIN MODEL

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Let Ω be a bounded domain in $\Re^d\ (d=2)$ and T be a positive number. We consider an Oseen-type diffusive Peterlin system which describes a motion of an incompressible viscoelastic fluid,

$$
\begin{aligned}
&\frac{D_w u}{Dt} - \nu\Delta u + \nabla p - \nabla(\mathrm{tr}CC) = f && \text{in } \Omega\times(0,T),\\
&\nabla\cdot u = 0 && \text{in } \Omega\times(0,T),\\
&\frac{D_w C}{Dt} - \varepsilon\Delta C - \left\{(\nabla u)C + C(\nabla u)^T\right\} + (\mathrm{tr}C)^2 C - (\mathrm{tr}C)I = F && \text{in } \Omega\times(0,T),\\
&u = 0,\ \ C = 0 && \text{on } \partial\Omega\times(0,T),\\
&u = u^0,\ C = C^0 && \text{in } \Omega \text{ at } t=0,
\end{aligned}
$$

where $u:\Omega\times(0,T)\to\Re^d$ is the velocity, $p:\Omega\times(0,T)\to\Re$ is the pressure, $C:\Omega\times(0,T)\to\Re^{d\times d}$ is the conformation tensor, $\frac{D_w}{Dt}\equiv\frac{\partial}{\partial t}+w\cdot\nabla$, $w:\Omega\times(0,T)\to\Re^d$ is a given velocity, ν and ε are positive constants, $f:\Omega\times(0,T)\to\Re^d$, $F:\Omega\times(0,T)\to\Re^{d\times d}$, $u^0:\Omega\to\Re^d$ and $C^0:\Omega\to\Re^{d\times d}$ are given functions, $\mathrm{tr}\,C$ means the trace of C . For this problem we present a Lagrange-Galerkin finite element scheme with Brezzi-Pitkaranta type pressure stabilization, where all unknown functions (u,p,C) are approximated by P1 elements, and show that the finite element solution (u_h, p_h, C_h) converges to the exact solution (u, p, C) in order $\Delta t + h$, where Δt is the time increment and h is the representative element size.

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