

APPLICATION OF AN INTEGRAL VORTICITY BOUNDARY CONDITION TO SOLVE THE VORTICITY-STREAMFUNCTION EQUATIONS

Yannick Deleuze^{*,1,2,3}, Yasunori Maekawa⁴, and Tony W.H. Sheu^{2,3}

¹ Sorbonne Universités, UPMC Univ Paris 06, UMR 7598, Laboratoire Jacques-Louis Lions,
Paris, France

² National Taiwan University, Department of Engineering Science and Ocean Engineering,
Scientific Computing and Cardiovascular Simulation Laboratory, Taipei, Taiwan

³Center for Advanced Study in Theoretical Sciences, National Taiwan University, No. 1, Sec.
4, Roosevelt Road, Taipei, Taiwan

⁴Mathematical Institute, Tohoku University, 6-3, Aoba, Aramaki, Aoba-ku, Sendai 980-8578,
Japan

Key words: *Acupuncture, integral vorticity boundary condition, FreeFem++*

In this study, we present finite element solutions of incompressible Navier-Stokes equations cast in the vorticity and stream function. The transport equations for vorticity and stream function under investigation are subject to boundary conditions of integral character for non-primitive vorticity variable.

In the first part, we formulate the integral vorticity boundary conditions for the elliptic-parabolic differential system of vorticity-streamfunction equations [1, 2]. Then, we assess the formulation with the explicit vorticity boundary condition on a simple example. We quantify the convergence behavior with respect to the mesh size and evaluate the error on the velocity with respect to the solution of the Navier-Stokes equations cast in primitive variables. All computations presented and commented are realized by means of the code FreeFem++ developed by Hecht [3]. Finally, we present some numerical applications where the domain of interest $\Omega \in \mathbb{R}^2$ is a bounded multi-connected domain implying that the boundary $\partial\Omega$ has connected components which are disjoint closed curves.

E-mail: yannick.deleuze@ljl.math.upmc.fr

REFERENCES

- [1] Maekawa, Yasunori. On the Inviscid Limit Problem of the Vorticity Equations for Viscous Incompressible Flows in the Half-Plane. *Communications on Pure and Applied Mathematics* 67.7 (2014): 1045-1128.

- [2] Maekawa, Yasunori. Solution formula for the vorticity equations in the half plane with application to high vorticity creation at zero viscosity limit. *Advances in Differential Equations* 18.1/2 (2013): 101-146.
- [3] Hecht, Frédéric. New development in freefem++. *Journal of Numerical Mathematics* 20.3-4 (2012): 251-266.