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ERROR ESTIMATES OF STABILIZED GALERKIN-CHARACTERISTICS FINITE ELEMENT SCHEMES FOR INCOMPRESSIBLE FLOW PROBLEMS

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In this paper stabilized Galerkin-characteristics finite element schemes for the Oseen and the Navier-Stokes equations are mathematically analyzed. Stability and convergence results with optimal error estimates for both of the schemes are proved.

They are combined finite element schemes with a characteristics method, cf. [1], and Brezzi-Pitkäranta's (pressure-) stabilization [2]. The idea of the characteristics method is to consider the trajectory of the fluid particle and discretize the material derivative term along the trajectory. The procedure is natural from the physical point of view and the method works well for convection-dominated problems. Moreover, the matrix of the derived system of linear equations is symmetric and we can use efficient iterative solvers for symmetric linear systems. Brezzi-Pitkäranta's stabilization has been originally proposed for the stationary Stokes equations and is one of the simplest stabilized methods. We note that it enables us to use a cheap P1/P1-element and keeps the symmetry of the system of linear equations.

Many characteristics schemes combined with finite element and finite difference methods are considered and analyzed, e.g., [3, 4]. In [5] we have proposed a stabilized Galerkin-characteristics finite element scheme for the Navier-Stokes equations, while its mathematical analysis has not been completed at that time. Here, in addition to the scheme we consider a corresponding stabilized Galerkin-characteristics finite element scheme for the Oseen equations, and show stability and error estimates for both schemes [6, 7]. The main difference between the Oseen and the Navier-Stokes equations is nonlinearity whose effect appears in the main mathematical results as follows. The scheme for the Oseen equations is unconditionally stable and its error estimates are optimal. On the other hand, the scheme for the Navier-Stokes equations is conditionally stable and its error estimates are optimal under that condition.

We prove the main results of the scheme for the Oseen equations by the Gronwall inequality in the framework of L^2-theory. The nonlinearity of the Navier-Stokes equations is overcome by the mathematical induction, while a condition between time increment and mesh size is required.

Two and three-dimensional numerical results are shown and the numerical convergence orders are recognized to be consistent with the theoretical results. Consequently, the schemes lead to efficient computation especially in three dimensions, which produces mathematically sound and numerically good solutions.

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