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FINITE ELEMENT APPROXIMATION OF THE VISCOELASTIC FLOW PROBLEM: A NON-RESIDUAL BASED STABILIZED FORMULATION

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The finite element approximation of the flow of viscoelastic fluids presents several numerical difficulties. It inherits obviously the problems associated with the approximation of the incompressible Navier-Stokes equations, mainly the compatibility between the velocity-pressure approximation and the treatment of the nonlinear advective term. But, on top of that, now the constitutive equation is highly nonlinear, with an advective term that may lead to both global and local oscillations in the numerical approximation. Moreover, even in the case of smooth solutions it is necessary to meet some additional compatibility conditions between the velocity and the stress interpolation in order to control velocity gradients. Elements that satisfy the compatibility requirements velocity-pressure and stress-velocity are rare.

The advective nature of the constitutive equation makes it necessary to use a stabilized finite element formulation to avoid global oscillations. In the present work, we apply two stabilized formulations based on the VMS framework to control the convective nature of the viscoelastic constitutive equation, different to those available in the literature. The use of discontinuity-capturing (DC) techniques is not a popular topic in the analysis of viscoelastic flows, but the high elastic stress gradients that appear when the Weissenberg number is increased make it a typical situation where the application of a DC scheme can help. In the present work we propose a numerical diffusion based on the orthogonal projection of the elastic stress gradient, which represents the *non-captured* part in the finite element approximation.

Referring to the compatibility conditions of inf-sup type for the viscoelastic three-field approximation, they consist of two restrictions on the interpolation spaces, one between pressure and velocity and the other between velocity and the elastic stress. These two restrictions reduce drastically the choices of stable finite element spaces that allow one to discretize the unknowns.

The stabilized formulations proposed in this work have their roots in the context of VMS methods introduced by Hughes et al. in [1] for the scalar convection-diffusion-reaction problem, and extended later to the vectorial Stokes problem in [2], where the space of the sub-grid scales is taken as orthogonal to the finite element space. As we shall see, this is an important ingredient in the design of our formulations. The purpose of the present work is precisely to design and test numerically stabilized formulations for the viscoelastic fluid flow problem, permitting the use of equal interpolation between the unknowns (deviatoric elastic stress, velocity and pressure) even in cases where the elastic stress gradients and the elastic component of the fluid are important.

The starting point of a VMS approach is to split the unknowns of the problem into two components, namely, the component that can be approximated by the finite element mesh and the unresolvable one, called sub-grid scale or simply sub-scale in what follows. The latter needs to be approximated in a simple manner in terms of the former, so as to capture its main effect and yield a stable formulation for the finite element unknown. There are different ways to approximate the sub-scale and, in particular, to choose the space where it is taken. We will describe two formulations which precisely differ in this choice. Both formulations will allow one to deal with the instabilities of the three-field viscoelastic formulation described earlier. There will be no need to meet the inf-sup conditions for the interpolation spaces and it will be possible to solve convection dominated problems both in the momentum and in the constitutive equation. For the latter, these methods have been found to work well. However, for the momentum equation we have observed that *they are not robust in the presence of high gradients of the unknowns*, and therefore we have had to modify them. The modification consists in designing a sort of *term-by-term stabilization based on the choice of subscales orthogonal to the finite element space.* We will describe in detail this method and the need for it.

The work to be presented has been published in [3, 4].

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